Observing the Quantum Limit of an Electron Cyclotron: QND Measurements of Quantum Jumps between Fock States

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Quantum jumps between Fock states of a one-electron oscillator reveal the quantum limit of a cyclotron. With a surrounding cavity inhibiting synchrotron radiation 140-fold, the jumps show a 13 s Fock state lifetime and a cyclotron in thermal equilibrium with 1.6 to 4.2 K blackbody photons. These disappear by 80 mK, a temperature 50 times lower than previously achieved with an isolated elementary particle. The cyclotron stays in its ground state until a resonant photon is injected. A quantum cyclotron offers a new route to measuring the electron magnetic moment and the fine structure constant.

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The quantum limit of an electron cyclotron accelerator is demonstrated for the first time. When the cyclotron is cooled to 80 mK, 50 times lower than previously realized with an isolated elementary particle, quantum nondemolition (QND) measurements show that the electron stays in the ground state of its cyclotron motion for hours, leaving only in response to resonant photons deliberately introduced from outside. At higher temperatures, blackbody photons are present in sufficient numbers to occasionally excite the electron cyclotron motion. QND measurements show the cyclotron oscillator remains in an excited energy eigenstate for many seconds before making an abrupt quantum jump to an adjacent state. The striking isolation of the electron from its environment is due to a 140-fold cavity-induced suppression of the spontaneous emission of synchrotron radiation. Analysis of the quantum jumps provides a way to measure the temperature of the electron, the average number of blackbody photons, and the spontaneous emission rate. Quantum jump spectroscopy provides a way to precisely measure the frequency separation of the lowest quantum states. A variety of applications are mentioned in conclusion.

The quantum cyclotron provides an unusual opportunity to observe and manipulate long lived states of a harmonic oscillator. When written in terms of raising and lowering operators, the Hamiltonian of the two dimensional cyclotron $H_c = h\nu_c(a^\dagger a + 1/2)$ is formally equivalent to that of the familiar one dimensional harmonic oscillator. The energy eigenstates of the electron cyclotron ($|n = 0\rangle$, $|n = 1\rangle$, ... in Fig. 1a) are often called Landau levels. They are formally equivalent to the familiar number states of the harmonic oscillator, often called Fock states in quantum optics. Though these states are well-known to every student of quantum mechanics, the production, observation, and use of Fock states in experiments is surprisingly difficult and rare. The unusually high probability $P > 0.999$ to be in the ground state of the quantum cyclotron, and the extremely long lifetime of the Fock states, should make it possible to excite any superposition of the lowest Fock states with a properly tailored sequence of drive pulses.

We report the nondestructive observation of Fock states as high as $|n = 4\rangle$. Only zero- and one-photon Fock states, $|n = 0\rangle$ and $|n = 1\rangle$, have previously been observed for a radiation mode of a cavity [1,2], though efforts are under way to observe two-photon and higher Fock states [3]. A ground state occupation fraction $P = 0.95$ was reported. Vibrational Fock states of a laser-cooled Be^+ ion in a potential well have also been selectively excited, starting from a similar ground state occupation of $P = 0.95$ [4]. The formation of these Fock states was deduced destructively, from repeated measurements which transferred the population of identically prepared states to internal energy levels, whose monitored time evolution revealed the original state. Very recently, the $|n = 0\rangle$ and $|n = 1\rangle$ Fock states of neutral atoms oscillating in a one dimensional harmonic well were also observed [5] with $P = 0.92$ for the ground state.

The quantum cyclotron is realized with a single electron stored in a cylindrical Penning trap [6,7] that is cooled by a dilution refrigerator. The trap cavity (Fig. 1b) is a good approximation to a cylindrical microwave cavity at

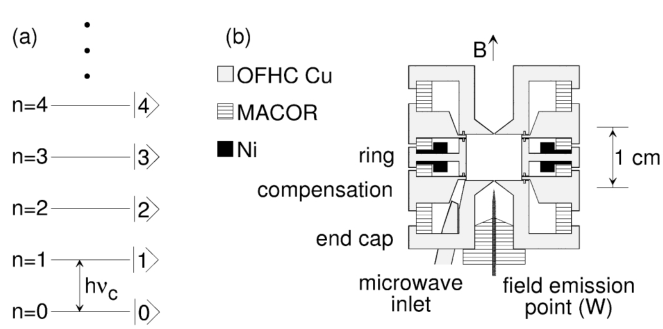

FIG. 1. (a) Energy levels of the one-electron cyclotron oscillator. (b) Electrodes of the cylindrical Penning trap cavity.

frequencies up to 160 GHz [8]. Tiny slits (125 μm) in the walls of the cavity make it possible to apply a trapping potential between the central ring electrode and the two flat end cap electrodes. The small slits include quarter wave "choke flanges" to minimize the loss of microwave radiation from the cavity. The potential is made a better approximation to a harmonic potential along the central symmetry axis of the trap by tuning an additional voltage applied to the two compensation electrodes.

Cavity radiation modes that couple to the cyclotron oscillator [8,9] have quality factors as high as $Q = 5 \times 10^4$. The energy in a 150 GHz mode with this Q value damps exponentially with a 50 ns time constant that is very short compared to all relevant time scales. (The frequency widths of the cavity mode resonances, for example, are much wider than the oscillator's cyclotron resonance width.) The radiation modes of the cavity are thus thermal states with the temperature of the trap cavity. Thermal contact to a dilution refrigerator allows us to adjust the trap temperature between 4.2 K and 70 mK (only to 80 mK when our detector is on). We detune the frequency of the one-electron cyclotron oscillator away from the radiation modes to decrease the spontaneous emission rate.

Two of the three motions of a trapped electron (charge $-e$ and mass m) in a Penning trap [10] are relevant to this work. Our central focus is upon the circular cyclotron motion, perpendicular to a vertical 5.3 T magnetic field, with cyclotron frequency $\nu_c = eB/2\pi m = 147$ GHz and energy levels separated by $h\nu_c$. The Fock states $|n\rangle$, often called Landau states for the particular case of a charged particle in a magnetic field, decay via spontaneous emission to $|n - 1\rangle$ at a rate $n\gamma$, where γ is the classical decay rate of the oscillator. In free space for our field, $\gamma = (4\pi\epsilon_o)^{-1}16\pi^2\nu_c^2e^2/3mc^3 = (94 \text{ ms})^{-1}$. This is the rate that is inhibited by the trap cavity.

The electron is also free to oscillate harmonically along the direction of the vertical magnetic field, $\hat{z}$, at a frequency $\nu_z = 64$ MHz $\approx \nu_c/1000$. We drive this axial motion by applying an oscillatory potential between the ring and an end cap electrode and detect the oscillatory current induced through a resonant tuned circuit attached between the ring and the other end cap. The electron axial motion damps as energy dissipates in the detection circuit, yielding an observed resonance width of 5 Hz for the driven axial motion. With appropriate amplification and narrow bandwidth detection we are able to measure small (1 Hz) shifts in ν_z. A heterostructure field effect transistor (HFET), constructed with Harvard collaborators just for these experiments [11], provides the radiofrequency gain that is needed while dissipating only 4.5 μW. The dilution refrigerator had difficulty with the nearly 700 times greater power dissipation (3 mW) of the conventional MESFET used initially.

The cyclotron and axial motions of the electron would be uncoupled except that we incorporate two small nickel rings into the ring electrode of the trap (Fig. 1b). These saturate in and distort the otherwise homogeneous magnetic field. The resulting "magnetic bottle,"

$$\Delta\vec{B} = B_2\{[z^2 - (x^2 + y^2)/2]\hat{z} - z(x\hat{x} + y\hat{y})\}, \quad (1)$$

is similar to but much bigger than what was used to determine an electron spin state [12]. Coupling the combined cyclotron and spin magnetic moment $\vec{\mu}$ to $\Delta\vec{B}$ gives a term in the Hamiltonian that is harmonic in z,

$$V = -\vec{\mu} \cdot \Delta\vec{B} = 2\mu_B B_2(a^\dagger a + 1/2 + S_z/\hbar)z^2, \quad (2)$$

where μ_B is the Bohr magneton, S_z is the spin operator, and the electron g value is taken to be 2. This V makes ν_z shift in proportion to the energy in the cyclotron and spin motions,

$$\Delta\nu_z = \delta(n + 1/2 + m_s). \quad (3)$$

A one quantum excitation of the cyclotron oscillator shifts the monitored ν_z by $\delta = 2\mu_B B_2/(m\omega_z) = 12.4$ Hz, substantially more than the 5 Hz axial linewidth and the 1 Hz resolution.

The measurement of the cyclotron energy is an example of a QND measurement [13,14] in that V and H_c commute, $[V, H_c] = 0$. The desirable consequence is that a second measurement of the cyclotron energy at a later time will give the same answer as the first (unless a change is caused by another source). This is not generally true for measurements with a quantum system. For example, measuring the position of a free particle would make its momentum completely uncertain. After additional time evolution a second measurement of the particle's position would give a different outcome.

Five one-hour sequences of QND measurements of the one-electron oscillator's energy are shown in Fig. 2. Each is for a different cavity temperature T, as measured with a ruthenium oxide sensor attached to the ring electrode. Greatly expanded views of several quantum jumps are shown in Fig. 3. Energy quantization is clearly visible, as are the abrupt quantum jumps between Fock states. The

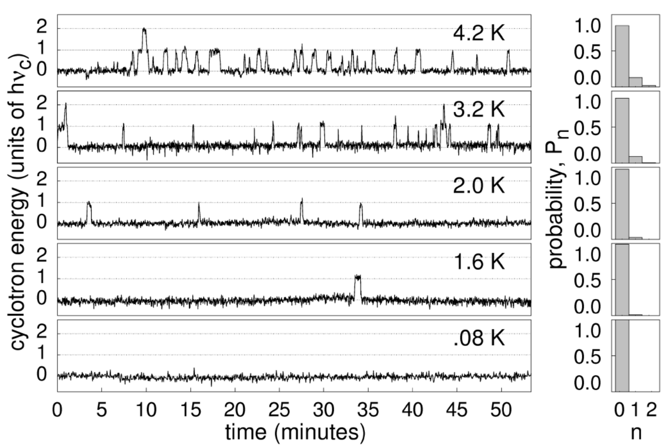

FIG. 2. Quantum jumps between the lowest states of the one-electron cyclotron oscillator decrease in frequency as the cavity temperature is lowered.

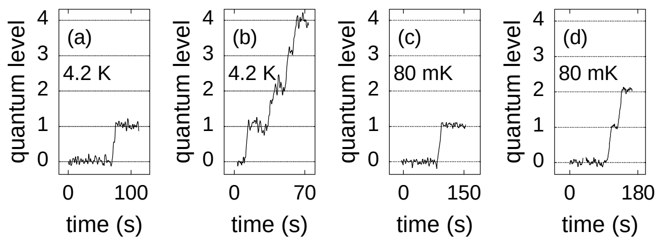

FIG. 3. Excitations to excited Fock states which are stimulated by 4.2 K blackbody photons in (a) and (b), and by an externally applied microwave field in (c) and (d).

upward quantum jumps are absorptions stimulated by the blackbody photons in the trap cavity. The downward transitions are spontaneous or stimulated emissions. Mostly we see the oscillator in its ground state $|n = 0\rangle$, with occasional quantum jumps to excited Fock states. Figure 3b shows a rare event in which 4.2 K blackbody photons sequentially excite the one-electron cyclotron oscillator to the Fock state $|n = 4\rangle$. It takes of order 2 s of signal averaging for us to ascertain the quantum state of the cyclotron oscillator. This true measurement time is less, being the time required to establish the quantum state in principle. An estimate of this time [15] unfortunately uses assumptions that do not correspond well to the experimental conditions.

We analyze the quantum jumps to measure the temperature of the cyclotron oscillator, T_c. The measured probabilities P_n for occupying Fock states $|n\rangle$, averaged over many hours, are shown to the right in Fig. 2 for each cavity temperature. The measured P_n fit well to the Boltzmann factors $P_n = Ae^{-nh\nu_c/kT_c}$ which pertain for thermal equilibrium, demonstrating that averaged over hours the oscillator is in a thermal state. The fit determines T_c. Measurements with this "quantum Boltzmann thermometer" (solid points in Fig. 4a) shows that T_c is equal to the cavity temperature T; the cyclotron oscillator is in thermal equilibrium with the blackbody photons in the cavity. The solid points in Fig. 4b show the measured average quantum number superimposed upon the curve $\bar{n} = [e^{h\nu_c/kT} - 1]^{-1}$ which pertains for an oscillator in thermal equilibrium at the measured cavity temperature T. For temperatures of 4.2 K, 1 K, and 80 mK, $\bar{n}$ varies dramatically from 0.23, to 9×10^{-4}, to 6×10^{-39}.

Below 1 K the oscillator resides in its ground state for so long (we estimate 10^{32} years for 80 mK) that it is difficult to directly measure the oscillator temperature T_c. The best we can do is to establish that at some confidence level C, this temperature is below a limit given by $kT_c \le h\nu_c/\ln[1 - \gamma t/\ln(1 - C)]$ if we observe no excitation for time t. When no excitation is observed for $t = 5$ h, for example, we establish that $T_c < 1.0$ K at the $C = 68\%$ confidence level. For temperatures below 1 K, blackbody photons have been essentially eliminated, and the one-electron cyclotron oscillator is virtually isolated from its environment.

We can separately measure the rate $\Gamma_{\rm abs}$ for the upward jumps (corresponding to stimulated absorption), and the rate $\Gamma_{\rm em}$ for downward jumps (corresponding to stimulated and spontaneous emission together). For $T = 1.6$ K, Fig. 5 shows a histogram of the dwell times in $|n = 0\rangle$ in (a) and for $|n = 1\rangle$ in (b). Both histograms decrease exponentially, indicating random processes, so the fitted lifetimes $(\Gamma_{\rm abs})^{-1}$ and $(\Gamma_{\rm em})^{-1}$ are just the average values of the dwell times. The rates for stimulated emission from $|n\rangle$ to $|n - 1\rangle$ and for stimulated absorption from $|n - 1\rangle$ to $|n\rangle$ are expected to be equal by the principle of detailed balance. Thus the spontaneous emission rate is simply the difference between the observed emission rate and the observed absorption rate, $\gamma = \Gamma_{\rm em} - \Gamma_{\rm abs}$. At $T = 1.6$ K (Fig. 5) the measured stimulated absorption rate is negligibly smaller so that $\gamma^{-1} \approx \Gamma_{\rm em}^{-1} = 13$ s.

Comparing the 13 s spontaneous emission lifetime that is measured with the 94 ms expected for free space shows that spontaneous emission of synchrotron radiation is strongly suppressed. The 140-fold inhibition is due to the copper trap cavity that encloses the electron oscillator [16]. By adjusting the magnetic field, the frequency of the cyclotron oscillator is tuned away from resonance with the

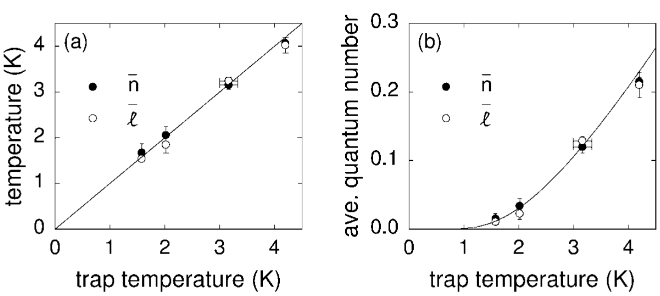

FIG. 4. (a) The oscillator temperatures deduced from the measured occupation times in each number state (solid points) and deduced from the transition rates (open points) are compared to the temperature of a ruthenium oxide thermometer attached to a trap electrode. (b) Measured average values $\bar{n}$ and $\bar{\ell}$ as a function of cavity temperature.

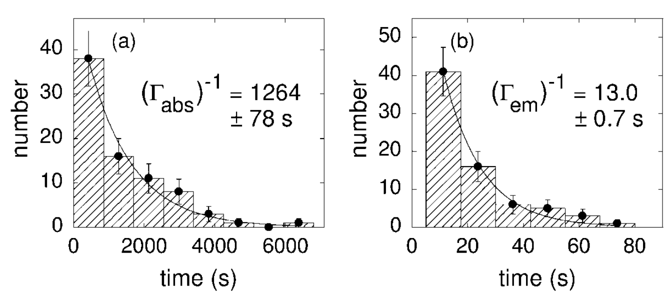

FIG. 5. Histograms of the dwell times preceding stimulated absorption from $n = 0$ to $n = 1$ in (a), and for spontaneous and stimulated emissions from $n = 1$ to $n = 0$ in (b), both for $T = 1.6$ K. Dwell times less than 5 s are excluded since short dwell times are obscured by detection time constants.

radiation modes of the trap cavity. The electron oscillator then couples only very weakly to the modes of the radiation field, and spontaneous emission is suppressed. We would not otherwise be able to signal average sufficiently to observe the quantum jumps so distinctly, nor would the excited Fock states persist so long.

The measured emission and absorption rates determine the average number $\bar{\ell}$ of resonant blackbody photons within the cavity. Quantum electrodynamics indicates that stimulated emission from $|n\rangle$, and stimulated absorption into $|n\rangle$, both have the same rate given by $\bar{\ell} n \gamma$. Applied to $n = 1$, this means that $\Gamma_{\rm abs} = \bar{\ell}\gamma$ and $\Gamma_{\rm em} = (1 + \bar{\ell})\gamma$. The average number of blackbody photons in terms of measurable quantities is thus given by $\bar{\ell} = \Gamma_{\rm abs}/(\Gamma_{\rm em} - \Gamma_{\rm abs})$. The measured open points in Fig. 4b agree well with the expected curve $\bar{\ell} = [e^{h\nu_c/kT} - 1]^{-1}$, and $\bar{n} = \bar{\ell}$ as predicted. Fitting to the measured $\bar{\ell}$ gives an independent measurement of the temperature of the cavity (open points in Fig. 4a). These agree well with the directly measured cavity temperature.

Extremely precise quantum jump spectroscopy of the lowest levels of the quantum cyclotron should become possible with blackbody photons eliminated from the trap cavity. Quantum jumps (e.g., Figs. 3c and 3d) will take place only when externally generated microwave photons are introduced into the trap cavity, increasing in rate as the drive frequency is swept through resonance. One challenge is that the z^2 term in the magnetic bottle [Eq. (1)] not only couples ν_z to the cyclotron energy [Eq. (3)] as is desired for good detection sensitivity. It also shifts the cyclotron frequency in proportion to the axial energy E_z with $\Delta\nu_c = \delta E_z/h\nu_z$. The measured distribution of cyclotron frequencies shows that the current axial detector heats the axial motion of the electron to 17 K, well above the 80 mK temperature of the trap and cyclotron motion. However, the long lifetime of the first excited Fock state should make it possible to introduce microwave photons while the axial motion is cooled to 80 mK, before turning on the axial detector to observe whether a cyclotron excitation has been made.

In conclusion, a quantum cyclotron is demonstrated using one electron in a cylindrical Penning trap cavity. QND measurements of quantum jumps between cyclotron Fock states shows that the temperature of the cyclotron motion tracks the cavity temperature where this can be measured, from 4.2 to 1.6 K. At 80 mK the electron is 50 times colder than previously demonstrated for an isolated elementary particle. Blackbody photons are completely absent and the cyclotron remains in its quantum ground state. The jumps also show that the Fock states are long lived; the cavity suppresses the spontaneous emission of synchrotron radiation 140-fold.

The quantum cyclotron is so well prepared in its ground state, and so well isolated from its environment, that it may be possible to excite any desired superposition of excited states, to probe the nature of decoherence and quantum measurement. Quantum jump spectroscopy offers the prospect to measure the frequency between the lowest Fock states (and spin states) with the exquisite precision required to significantly improve the very accurate measurement of the electron magnetic moment reported 12 years ago [12]. Spectacular theoretical advances made in recent quantum electrodynamics calculations [17] should allow a 10-fold improvement in the determination of the fine structure constant. A better lepton CPT test, comparing the magnetic moments of the electron and positron, should be possible, along with a better measurement of the proton-to-electron mass ratio.

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