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Bell Inequalities for Entangled Pairs of Neutral Kaons

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We extend the use of Bell inequalities to $\Phi \to K^0\bar{K}^0$ decays by exploiting analogies and differences to the well-known and experimentally verified singlet-spin case. Contrasting with other analyses, our Bell inequalities are violated by quantum mechanics and can strictly be derived from local realistic theories. In principle, quantum mechanics could then be tested using unstable, oscillating states governed by a CP-violating Hamiltonian.

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Quantum entanglement, as shown by the separate parts of nonfactorizable composite systems, is an extremely peculiar feature of quantum mechanics to which much attention has been devoted. Since the paper by Einstein, Podolsky, and Rosen (EPR) [1], quantum entanglement has been also a continuous source of speculations on the "spooky action-at-a-distance," better characterized as nonlocality in the correlations of an EPR pair [2]. Well-known and useful tools to probe into this nonlocality are the original Bell inequalities [3] and their reformulated versions [4,5], as reviewed, for instance, in [6,7].

Bell inequalities have been subjected to experimental tests with the general outcome that they are violated [8,9]; i.e., local realistic theories fail and nature is indeed nonlocal. However, possible loopholes in the tests have been pointed out [10]. There is therefore a continuous interest to test Bell inequalities in different experiments and, more importantly, in different branches of physics. One such possible place is an e^+e^- machine copiously producing EPR-entangled $K^0\bar{K}^0$ pairs through the reaction $e^+e^- \to \Phi \to K^0\bar{K}^0$. Such a Φ or "entanglement" factory, DaΦne, will be soon operating in Frascati [11]. Because of the negative charge conjugation of the Φ meson, the EPR entanglement of the neutral kaon pair can be explicitly written as

$$|\Phi(0)\rangle = \frac{1}{\sqrt{2}}[|K^0\rangle \otimes |\bar{K}^0\rangle - |\bar{K}^0\rangle \otimes |K^0\rangle]. \quad (1)$$

Starting with this initial state, the two neutral kaons—denoted by the kets at the left and right hand sides of the direct product symbol $\otimes$ in (1)—fly apart thus defining after collimation a left and a right hand kaon beam. Their time evolution is given by (see Appendix and [12])

$$|\Phi(t)\rangle = \frac{N(t)}{\sqrt{2}}[|K_S\rangle \otimes |K_L\rangle - |K_L\rangle \otimes |K_S\rangle] \quad (2)$$

with $|N(t)| = (1 + |\epsilon|^2)/(|1 - \epsilon^2|) \times e^{-\frac{1}{2}(\Gamma_S+\Gamma_L)t}$ reflecting the extinction of the beams via weak kaon decays but without modifying the perfect antisymmetry of the initial state. This is then in close analogy to the singlet-spin state usually considered in the Bohm reformulation of the EPR configuration (EPRB). But there is also a substantial difference: while most of the experimental tests favoring quantum nonlocality have been successfully performed in the EPRB configuration, early [13,14] and more recent [15,16] attempts to check similar Bell inequalities in $e^+e^- \to \Phi \to K^0\bar{K}^0$ either fail in showing their violation by quantum mechanics or seem to be affected by serious difficulties (see below and [17]). Our purpose in this Letter is to fill this gap by exploiting the analogies between the $\Phi \to K^0 \bar{K}^0$ and the singlet EPRB cases.

In the well-known EPRB configuration one deals with the singlet state

$$|0,0\rangle = \frac{1}{\sqrt{2}} [|+\rangle \otimes |-\rangle - |-\rangle \otimes |+\rangle], \quad (3)$$

i.e., an antisymmetric system consisting of two separating components, exactly as in Eq. (1). Also, each one of these two components (say, electrons) is assumed to have spin-$\frac{1}{2}$ thus belonging to a dimension-two Hilbert space with basis vectors $|+\rangle$ and $|-\rangle$, in close analogy to the basis vectors $|K^0\rangle$ and $|\bar{K}^0\rangle$ in Eq. (1). In the EPRB configuration, the experimentalist is supposed to be able to measure *at will* the spin components along different directions in both beams, as explicitly required to derive Bell inequalities in local realistic contexts. More precisely, we assume that on the left (right) beam one can adjust these measurement directions either along $\boldsymbol{a}$ or along $\boldsymbol{a}'$ ($\boldsymbol{b}$ or $\boldsymbol{b}'$). In the appropriate units, the outcomes of these measurements are simply $\pm$ signs, i.e., $\sigma_i = \pm$ for $i = \boldsymbol{a}, \boldsymbol{a}', \boldsymbol{b}, \boldsymbol{b}'$. The probability to obtain specific outcomes (say, σ_a and σ_b) when measuring along given directions (say, $\boldsymbol{a}$ on the left and $\boldsymbol{b}$ on the right) will be denoted by $P(\boldsymbol{a}, \sigma_a; \boldsymbol{b}, \sigma_b)$ and by similar expressions for alternative outcomes and orientations.

In the context of quantum mechanics, all these probabilities can be unambiguously computed. For the singlet state one obtains

$$\begin{aligned} P(\boldsymbol{a},+;\boldsymbol{b},+) = P(\boldsymbol{a},-;\boldsymbol{b},-) &= \frac{1}{4}(1 - \cos\theta_{ab}) \\ &= \frac{1}{2}\sin^2(\theta_{ab}/2), \\ P(\boldsymbol{a},+;\boldsymbol{b},-) = P(\boldsymbol{a},-;\boldsymbol{b},+) &= \frac{1}{4}(1 + \cos\theta_{ab}) \\ &= \frac{1}{2}\cos^2(\theta_{ab}/2), \end{aligned} \quad (4)$$

where θ_{ab} is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$. In the context of local realistic theories, rather than explicitly computing probabilities, one can establish several Bell inequalities to be satisfied by these probabilities in alternative experimental setups,

$$P(\boldsymbol{a},+;\boldsymbol{b},+) \le P(\boldsymbol{a},+;\boldsymbol{c},\sigma_c) + P(\boldsymbol{c},\sigma_c;\boldsymbol{b},+), \quad (5)$$

where σ_c can be either $+$ or $-$ and $\boldsymbol{c}$ stands for a given direction common to both left ($\boldsymbol{c} = \boldsymbol{a}'$) and right ($\boldsymbol{c} = \boldsymbol{b}'$) hand sides. This is the Wigner version of the Bell inequality and can easily be derived (for details, see [5,6,15]) for deterministic, local hidden-variable theories. It holds for the most interesting case in which one has spacelike separation between the left and right spin-measurement events. This is simply achieved working in a symmetric configuration, i.e., placing the detectors at equal (time-of-flight) distances from the origin. Bell's theorem then establishes the incompatibility between these theories and quantum mechanics by simply proving that the probabilities in Eq. (4) can violate the Wigner inequality (5).

Before entering into this violation, let us reformulate our simple EPRB analysis in a slightly different configuration. Assume now that the experimentalist is constrained to measure spin projections of the spin-$\frac{1}{2}$ particles along a single and fixed direction common to both left and right hand beams (say, the vertical or z direction). All the discussion of the previous paragraph can be maintained if the experimentalist is allowed to introduce magnetic field(s) along the electron path(s). Indeed, if the magnetic field B is adjusted to produce a rotation of the spinor around the propagation axis of angle $\theta_{ab} \equiv \theta_B \equiv \omega_B \Delta t$ on only one of the two electrons, then the same expressions (4) are the correct quantum mechanical predictions for the different probabilities to measure the left and right vertical spin components. This can be immediately seen substituting the effects of the rotation,

$$\begin{aligned} |+\rangle &\to \cos(\theta_{ab}/2)\,|+\rangle + \sin(\theta_{ab}/2)\,|-\rangle, \\ |-\rangle &\to \cos(\theta_{ab}/2)\,|-\rangle - \sin(\theta_{ab}/2)\,|+\rangle, \end{aligned} \quad (6)$$

in the second kets (say) of Eq. (3). One then finds

$$\begin{aligned} |0,0\rangle \to \frac{1}{\sqrt{2}} [&\cos(\theta_{ab}/2)\,(|+\rangle \otimes |-\rangle - |-\rangle \otimes |+\rangle) \\ &- \sin(\theta_{ab}/2)\,(|+\rangle \otimes |+\rangle + |-\rangle \otimes |-\rangle)], \end{aligned} \quad (7)$$

thus recovering the quantum mechanical probabilities in Eq. (4):

$$\begin{aligned} P(0,+;B,+) = P(0,-;B,-) &= \frac{1}{4}(1 - \cos\theta_B) \\ &= \frac{1}{2}\sin^2(\theta_B/2) \\ P(0,+;B,-) = P(0,-;B,+) &= \frac{1}{4}(1 + \cos\theta_B) \\ &= \frac{1}{2}\cos^2(\theta_B/2), \end{aligned} \quad (8)$$

with a new notation indicating explicitly the presence of the magnetic field B on the right and its absence on the left. In the context of local realistic theories, two Wigner inequalities (5) can be derived,

$$\begin{aligned} P(0,+;2B,+) &\le P(0,+;B,+) + P(B,+;2B,+), \\ P(0,+;2B,+) &\le P(0,+;B,-) + P(B,-;2B,+). \end{aligned} \quad (9)$$

The first (second) inequality implies $\frac{1}{2}\sin^2(\theta_B) \le \sin^2(\theta_B/2)\,[\frac{1}{2}\cos^2(\theta_B) \le \cos^2(\theta_B/2)]$ and is violated for rotation angles $0° < \theta_B < 90°$ $[128° \simeq \cos^{-1}(1/2 - \sqrt{5}/2) < \theta_B < 180°]$. Care has to be taken, however, to concentrate the magnetic field in a small region just before detection in such a way that the spin-measurement event on the left is spacelike separated from the whole rotation interval Δt and spin measurement on the right. A tiny violation of the first inequality (9) persists even for small values of θ_B.

We now turn to the $\Phi \to K^0\bar{K}^0$ case, where one is really constrained to the situation of the preceding paragraph. Indeed, only the two basis states $|K^0\rangle$ and $|\bar{K}^0\rangle$ can be unambiguously identified on both sides by means of their distinct strangeness-conserving strong interactions on nucleons (see [14]; see also [17] for a discussion on this issue). One then needs to mimic the preceding effects of an adjustable magnetic field. A thin, homogeneous slab of ordinary (nucleonic) matter placed just before one of the two $K^0\bar{K}^0$ detectors will do the job. The effects of this slab—a neutral kaon regenerator with regeneration parameter ρ—on the entering, freely propagating $|K_{S/L}\rangle$ states are (see Appendix)

$$\begin{aligned} |K_S\rangle &\to |K_S\rangle + r|K_L\rangle, \\ |K_L\rangle &\to |K_L\rangle + r|K_S\rangle, \end{aligned} \tag{10}$$

where only first order terms in the (small) parameter r have been kept. Notice the strong similarity between $r \equiv (im_S - im_L + \frac{1}{2}\Gamma_S - \frac{1}{2}\Gamma_L) \times \rho\Delta t$ and the previous $\theta_B \equiv \omega_B\Delta t = (ge\hbar/2m) \times B\Delta t$ in that both expressions contain a first factor characterizing the propagating particles times a second one allowing for different choices of external intervention. But notice also that the transformation (10) with a complex r is not a true rotation in contrast to (6). Introducing the regenerator on the right beam, as before, and finally reverting to the $K^0\bar{K}^0$ basis, Eq. (2) becomes

$$\begin{aligned} |\Phi(0)\rangle \to |\Phi(t)\rangle \simeq \frac{1}{\sqrt{2}}\, e^{-i(\lambda_S+\lambda_L)t} \\ \times [(1 - r)\,|K^0\rangle \otimes |\bar{K}^0\rangle \\ - (1 + r)\,|\bar{K}^0\rangle \otimes |K^0\rangle]. \end{aligned} \tag{11}$$

As in the spin case, the antisymmetry of the initial state has been lost although not in the same way, as expected from the differences between (6) and (10).

In the context of quantum mechanics one can unambiguously compute the detection probabilities by simply projecting Eq. (11) over the appropriate states

$$\begin{aligned} P(0,K^0;r,\bar{K}^0) &= P(r,\bar{K}^0;0,K^0) \\ &\simeq e^{-(\Gamma_S+\Gamma_L)t}[1/2 - \mathrm{Re}(r)], \\ P(0,\bar{K}^0;r,K^0) &= P(r,K^0;0,\bar{K}^0) \\ &\simeq e^{-(\Gamma_S+\Gamma_L)t}[1/2 + \mathrm{Re}(r)], \\ P(0,\bar{K}^0;r,\bar{K}^0) &= P(r,\bar{K}^0;0,\bar{K}^0) \simeq 0, \\ P(0,K^0;r,K^0) &= P(r,K^0;0,K^0) \simeq 0, \end{aligned} \tag{12}$$

where the left equalities are an obvious consequence of rotation invariance and the approximated ones in the right are valid at first order in r. The notation follows closely that in Eq. (8) with the K^0 or $\bar{K}^0$ indicating the outcome of the measurement and r or 0 indicating the presence or absence of the regenerator. Under the same conditions as before, one can now invoke local realistic theories to establish Wigner inequalities such as

$$\begin{aligned} P(0,K^0;0,\bar{K}^0) &\le P(0,K^0;r,K^0) \\ &\quad + P(r,K^0;0,\bar{K}^0), \\ P(0,K^0;0,\bar{K}^0) &\le P(0,K^0;r,\bar{K}^0) \\ &\quad + P(r,\bar{K}^0;0,\bar{K}^0). \end{aligned} \tag{13}$$

The incompatibility between quantum mechanics and local realism appears when introducing the probabilities (12) in (13): the first inequality leads to $1 \le 1 + 2\,\mathrm{Re}(r)$, while the second one leads to $1 \le 1 - 2\,\mathrm{Re}(r)$. Hence, in any case (i.e., independently of the specific properties of the regenerator) one of the Wigner inequalities (13) is violated by quantum mechanics. According to the detailed analysis by Di Domenico [15], detection of such a violation is feasible even in this simplest version of the experiment involving thin regenerators which are only a few millimeters thick. For usual materials this implies $2|\mathrm{Re}(r)| \simeq 10^{-2}$ [15] and thus violation of the inequality by some 1%. Because of the high kaon detection efficiency [18] and the high luminosity of Φ factories like DaΦne (around 10^{10} neutral-kaon pairs will be yearly produced quite soon [11]) the required accuracy does not seem to escape present day capabilities even for this simplest version of the experiment [19]. In our case, however, thicker regenerators can be used (as suggested by Eberhard [16] for asymmetric Φ factories) thus enlarging the effect but requiring a more involved theoretical treatment (now in progress).

These results contrast with the previously mentioned ones coming from early attempts to check local realistic theories in $e^+e^- \to \Phi \to K^0\bar{K}^0$ [13,14], where interesting Bell inequalities involving different $K^0\bar{K}^0$ detection *times* were proposed. Choosing among these different times entails the required active intervention of the experimentalist (as particularly emphasized in [20]), but the inequalities so derived failed in showing their violation by quantum mechanics due to the specific values of kaon masses and widths. More recently, there has been a renewed interest in this subject [15,16] but, in spite of several claims, we believe that the proposed Bell inequalities do not follow strictly from local realism. Indeed, detection of kaonic states other than $K^0, \bar{K}^0$ is required and their identification via their associated decay modes is proposed (see [17] for details). But the simple observation and counting of these decay events offer no option for an active intervention of the experimentalist, as required to establish these inequalities in a local realistic context. This clearly contrasts with our proposal, where freely adjustable regenerators are involved in alternative experimental setups. Finally, we emphasize that in spite of certain analogies the K^0-$\bar{K}^0$ system displays interesting differences as compared to the usually considered photons or electrons. Indeed, the K^0-$\bar{K}^0$ system has some unique and peculiar quantum mechanical properties: it is unique as it is the only place in nature where *CP* violation has been detected so far; it is peculiar since the mass eigenstates are unstable and manifest $K^0\bar{K}^0$ oscillations in space-time. This could add some relevance to our results.

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Appendix.—We define the $CP = \pm 1$ eigenstates $K_{1/2}$ by $|K_{1/2}\rangle = \frac{1}{\sqrt{2}}[|K^0\rangle \pm |\bar{K}^0\rangle]$. The mass eigenstates $K_{S/L}$ in terms of $K_{1/2}$ and the *CP* violation parameter ϵ are

$$\begin{aligned} |K_S\rangle &= \frac{1}{\sqrt{1 + |\epsilon|^2}}[|K_1\rangle + \epsilon|K_2\rangle], \\ |K_L\rangle &= \frac{1}{\sqrt{1 + |\epsilon|^2}}[|K_2\rangle + \epsilon|K_1\rangle]. \end{aligned} \tag{14}$$

The time development of these nonoscillating mass eigenstates is given by $|K_{S/L}(t)\rangle = e^{-i\lambda_{S/L}t}|K_{S/L}\rangle$, with $\lambda_{S/L} \equiv m_{S/L} - \frac{i}{2}\Gamma_{S/L}$, and $m_{S/L}$ and $\Gamma_{S/L}$ being the mass and width of K_S and K_L, respectively.

For kaon regeneration in homogeneous nucleonic media we follow [15], [21], and [22]. The eigenstates of the mass matrix inside matter are

$$\begin{aligned} |K'_S\rangle &\simeq |K_S\rangle - \varrho|K_L\rangle, \\ |K'_L\rangle &\simeq |K_L\rangle + \varrho|K_S\rangle, \end{aligned} \tag{15}$$

where only first order terms in ϱ have been retained. This regeneration parameter is $\varrho = \pi\nu(f - \bar{f})/m_K \times (\lambda_S - \lambda_L)$, where $m_K = (m_S + m_L)/2$, $f(\bar{f})$ is the forward scattering amplitude for $K^0(\bar{K}^0)$ on nucleons and ν is the nucleonic density. The time evolution inside matter for the eigenstates $|K'_{S/L}\rangle$ follows the standard exponential form, $|K'_{S/L}(t)\rangle = e^{i\lambda'_{S/L}t}|K'_{S/L}\rangle$, where $\lambda'_{S/L} = \lambda_{S/L} - \pi\nu(f + \bar{f})/m_K + \mathcal{O}(\varrho^2)$. To compute the net effect of a thin regenerator slab over the entering $|K_{S/L}\rangle$ states one simply expresses these states in the $|K'_{S/L}\rangle$ basis using (15), introduces their time evolution in Δt, and reverts to the initial $|K_{S/L}\rangle$ basis (see, for instance, [15] and [22]). One finds

$$\begin{aligned} |K_S\rangle &\to e^{-i\lambda'_S\Delta t}[|K_S\rangle + i\varrho(\lambda'_S - \lambda'_L)\Delta t|K_L\rangle] \\ &\simeq |K_S\rangle + r|K_L\rangle, \\ |K_L\rangle &\to e^{-i\lambda'_L\Delta t}[|K_L\rangle + i\varrho(\lambda'_S - \lambda'_L)\Delta t|K_S\rangle] \\ &\simeq |K_L\rangle + r|K_S\rangle, \end{aligned} \tag{16}$$

where Δt is short enough to justify the systematic use of first order approximations. Equation (16) defines the parameter r entering Eqs. (10)–(13); in a first approximation we have $r \simeq i(\lambda_S - \lambda_L) \times \rho\Delta t$, as quoted in the main text. Appropriate values for r can be adjusted choosing among different materials (ρ) and their thickness (Δt).

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