Quintessence, Cosmic Coincidence, and the Cosmological Constant

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Recent observations suggest that a large fraction of the energy density of the Universe has negative pressure. One explanation is vacuum energy density; another is quintessence in the form of a scalar field slowly evolving down a potential. In either case, a key problem is to explain why the energy density nearly coincides with the matter density today. The densities decrease at different rates as the Universe expands, so coincidence today appears to require that their ratio be set to a specific, infinitesimal value in the early Universe. In this paper, we introduce the notion of a "tracker field," a form of quintessence, and show how it may explain the coincidence, adding new motivation for the quintessence scenario. [S0031-9007(98)08257-X]

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A number of recent observations suggest that Ω_m, the ratio of the (baryonic plus dark) matter density to the critical density, is significantly less than unity [1]. Either the Universe is open or there is some additional energy density ρ sufficient to reach $\Omega_{\rm total} = 1$, as predicted by inflation. Measurements of the cosmic microwave background, the mass power spectrum [1–3], and, most explicitly, the luminosity-redshift relation observed for Type Ia supernovae [4] all suggest that the missing energy should possess negative pressure (p) and equation of state ($w \equiv p/\rho$). One candidate for the missing energy is vacuum energy density or cosmological constant Λ for which $w = -1$. The resulting cosmological model, ΛCDM, consists of a mixture of vacuum energy and cold dark matter. Another possibility is QCDM cosmologies based on a mixture of cold dark matter and quintessence ($-1 < w \leq 0$), a slowly varying, spatially inhomogeneous component [5]. An example of quintessence is the energy associated with a scalar field (Q) slowly evolving down its potential $V(Q)$ [6–8]. Slow evolution is needed to obtain negative pressure, $p = \frac{1}{2}\dot{Q}^2 - V(Q)$, so that the kinetic energy density is less than the potential energy density.

Two difficulties arise from all of these scenarios. The first is the fine-tuning problem: Why is the missing energy density today so small compared to typical particle physics scales? If $\Omega_m \sim 0.3$ today the missing energy density is of order 10^{-47} GeV^4, which appears to require the introduction of a new mass scale 14 or so orders of magnitude smaller than the electroweak scale. A second difficulty is the "cosmic coincidence" problem [9]: Since the missing energy density and the matter density decrease at different rates as the Universe expands, it appears that their ratio must be set to a specific, infinitesimal value in the very early Universe in order for the two densities to nearly coincide today, some 15 billion years later.

What seems most ideal is a model in which the energy density in the Q component is comparable to the radiation density (to within a few orders of magnitude) at the end of inflation, say. If there was some rough equipartition of energy following reheating among several thousands of degrees of freedom, one might expect the energy density of the Q component to be 2 or so orders of magnitude smaller than the total radiation density. One would want that the energy density of the Q component somehow tracks below the background density for most of the history of the Universe, and, then, only recently, grows to dominate the energy density and drive it into a period of accelerated expansion. The models we present will do all this and more even though there is only one adjustable parameter. The models are extremely insensitive to initial conditions—variations in the initial ratio of the Q-energy density to the matter density by nearly 100 orders of magnitude do not affect the cosmic history. The models are in excellent agreement with current measurements of the cosmic microwave background, large scale structure, and cosmic acceleration. We also find that the models predict a relation between Ω_m and the acceleration of the Universe. These properties suggest a new perspective for the quintessence models, perhaps placing them on equal footing with the more conventional Λ models.

The models considered in this Letter are based on the notion of "tracker fields," a form of quintessence in which the tracker field Q rolls down a potential $V(Q)$ according to an attractorlike solution to the equations of motion. The tracker solution is an attractor in the sense that a very wide range of initial conditions for Q and $\dot{Q}$ rapidly approach a common evolutionary track, so that the cosmology is insensitive to the initial conditions. Tracking has an advantage similar to inflation in that a wide range of initial conditions is funneled into the same final condition. This contrasts with most quintessence potentials studied previously in the literature [5,7] which require very fine adjustment of the initial value of Q (as well as parameters in the potential) to obtain a suitable cosmology. We introduce the term "tracker" because there is a subtle but important difference from attractor solutions in dynamical systems. Unlike a standard attractor, the tracker solution is not a fixed point (in the sense of a

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fixed point solution of a system of autonomous differential equations of motion [8]): The ratio of the Q energy to the background matter or radiation density changes steadily as Q proceeds down its track. This is desirable because one is interested in having the Q energy ultimately overtake the background density and drive the Universe towards an accelerating phase. This contrasts with the "self-adjusting" solutions recently discussed by Ferreira and Joyce [8] based on $V(Q) = M^4 \exp(\beta Q)$ potentials. Self-adjusting solutions are more nearly true attractors in that Ω_Q remains constant for a constant background equation of state (Ω_Q changes slightly when the Universe transforms from radiation to matter domination). This means, for example, that Ω_Q is constant throughout the matter-dominated epoch. For constant Ω_Q, satisfying the constraints from structure formation requires that Ω_Q be less than 0.2 and Ω_m exceed 0.8, which runs into conflict with current best estimates [1,3,5] of Ω_m and produces a decelerating universe in conflict with recent supernovae results [4]. So, the interesting and significant features of tracking are that (a) as for the self-adjusting case, a wide range of initial conditions is drawn towards a common cosmic history, but (b) the tracking solutions do not "self-adjust" to the background equation of state, but, instead, maintain some finite difference in the equation of state such that the Q energy ultimately dominates and the Universe enters a period of acceleration. Compared to the self-adjusting case, tracking does not require any additional parameters and allows a much wider range of potentials.

Potentials in which $d \ln V/dQ$ is decreasing as Q rolls down the potential admit tracker solutions [10]. (The self-adjusting potentials correspond to constant $d \ln V/dQ$.) The energy density of the tracker field decreases as $1/a^{3(1+w_Q)}$, where w_Q remains constant or varies slowly in each epoch of the Universe but changes sharply when the background expansion of the Universe changes from radiation, to matter, to quintessence dominated. The value of w_Q differs from the background equation of state such that the value of Ω_Q increases as the Universe ages and, for most potentials, increases more rapidly as the Universe ages. Hence, it is more likely that Ω_Q grows to order unity late in the history in the Universe compared to earlier.

We will consider two examples: $V(Q) = M^{(4+\alpha)}Q^{-\alpha}$ and $V(Q) = M^4[\exp(M_p/Q) - 1]$, where M is the one free parameter and M_p is the Planck mass. For any given V, there is a family of tracker solutions parametrized by M. The value of M is fixed by the measured value of Ω_m. The potentials are suggested by particle physics models with dynamical symmetry breaking or nonperturbative effects [11–15], although we consider it premature to justify our concept at this formative stage on the basis of fundamental physics. Our purpose, rather, is to show that a simply parametrized fluid with the desired properties is physically possible. Pioneering studies of the inverse power-law case have been done by Peebles and Ratra [6]. Here we point out some additional important properties and generalizations and, then, explain how all of these properties are relevant to quintessence and the coincidence problem and possibly the fine-tuning problem.

The tracker field Q satisfies the equation of motion,

$$\ddot{Q} + 3H\dot{Q} + V'(Q) = 0\,, \qquad (1)$$

where $V'(Q)$ is the derivative of V with respect to Q; H is the Hubble parameter which satisfies the Friedmann equation,

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_B + \rho_Q)\,, \qquad (2)$$

where G is Newton's constant and ρ_B is the background energy density. We assume $H = 100h$ (km/sec)/Mpc with $h = 0.65$ in our computations. For the inverse power-law potential, Q has a tracker solution [6] which maintains the condition

$$V'' = (9/2)(1 - w_Q^2)[(\alpha + 1)/\alpha]H^2. \qquad (3)$$

The condition that ρ_Q is beginning to dominate today means that Q must be $\mathcal{O}(M_p)$ today since $V'' \approx \rho_Q/Q^2$ and $H^2 \approx \rho_Q/M_p^2$.

The one free parameter, M, is determined by the observational constraint, $\Omega_Q \approx 0.7$ today. Here is where the fine-tuning issue must be considered. To have $\Omega_Q \approx 0.7$ today requires $V(Q \approx M_p) \approx \rho_m$, where $\rho_m \approx 10^{-47}$ GeV^4 is the current matter density; this imposes the constraint $M \approx (\rho_m M_p^{\alpha})^{1/(\alpha+4)}$. For low values of α or for the exponential potential, this forces M to be a tiny mass as low as 1 meV for the exponential case. However, we note that $M > 1$ GeV—comparable to particle physics scales—is possible for $\alpha \gtrsim 2$. Hence, while this is not our real aim, it is interesting to note that the tracker solution does not require the introduction of a new mass hierarchy in fundamental parameters.

To address the coincidence problem—removing the need to tune initial conditions in order for the matter and missing energy densities to nearly coincide today—our proposal relies on the tracking behavior of Q in a background of standard cosmology. Let us first consider $V(Q) = M^{(4+\alpha)}Q^{-\alpha}$ for $\alpha \geq 1$. For any fixed M, the tracker solution is determined by Eq. (3). We shall call the energy density in the Q field as a function of z along the tracker solution $\bar{\rho}_Q(z)$. If initial conditions are set at $z = z_i$, at the end of inflation, say, then one possibility is that the initial energy density in Q is less than the attractor value, $\rho_Q(z_i) < \bar{\rho}_Q(z_i)$. In this case, the field remains frozen until H^2 decreases to the point where Eq. (3) is satisfied. See Fig. 1. After that point, Q begins rolling down the potential, maintaining the condition in Eq. (3) as it rolls along. A second possibility is that the initial energy density in Q is greater than the tracker value but less than the background radiation density, $\bar{\rho}_Q(z_i) < \rho_Q(z_i) < \rho_B(z_i)$. This includes the case of equipartition after reheating. In this case Q starts rolling down the potential immediately and so rapidly that its

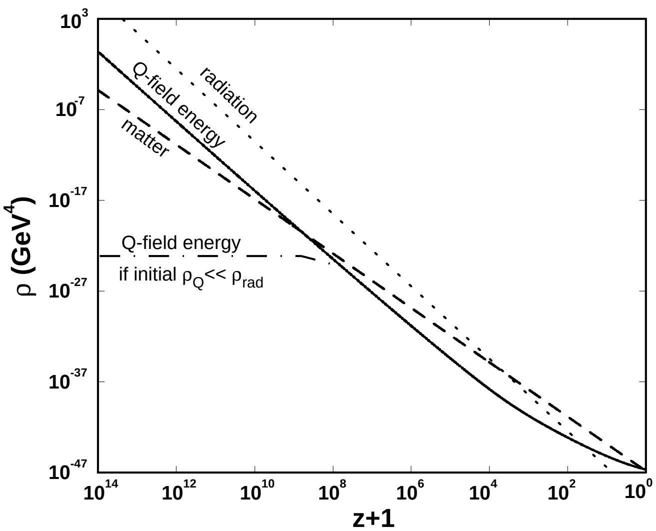

FIG. 1. The evolution of the energy densities for a quintessence component with $V(Q) = M^4[\exp(M_p/Q) - 1]$ potential. The solid line is where ρ_Q is initially comparable to the radiation density and immediately evolves according to tracker solution. The dot-dashed curve is if, for some reason, ρ_Q begins at a much smaller value. The field is frozen and ρ_Q is constant until the dot-dashed curve runs into tracker solution, leading to the same cosmology today: $\Omega_m = 0.4$ and $w_Q = -0.65$.

kinetic energy $\frac{1}{2}\dot{Q}^2$ dominates over the potential energy density $V(Q)$. The kinetic energy density redshifts as $1/a^6$ and eventually Q comes nearly to a stop at $Q \approx 0.5[\rho_Q(z_i)/\rho_B(z_i)]^{1/2}M_p$. By this point, Q has fallen below the tracker solution, $\bar{\rho}_Q$. Now, Q remains nearly frozen and H decreases until Eq. (3) is satisfied. Then, Q tracks the same solution as before. Hence, any initial ρ_Q less than the initial background radiation density, including equipartition initial conditions, leads to the same tracker solution and the same cosmology.

The only troublesome case is if Q dominates over the radiation density initially, $\rho_Q \gg \rho_B$. In this case, Q grows to a value greater than M_p before it slows down; this overshoots the tracker solution to such an extent that the tracker is not reached by the present epoch and ρ_Q is insignificant today. On the other hand, the initial condition $\rho_Q \gg \rho_B$ seems unlikely.

An interesting aspect of the pure inverse power-law potential is that, whether in the radiation- or matter-dominated era, the energy density in the Q component decays as a constant power of the scale factor, $\rho_Q \propto a^{-3(1+w_Q)}$ and

$$w_Q \approx \frac{\frac{\alpha}{2}w_B - 1}{1 + \frac{\alpha}{2}}, \qquad (4)$$

where this approximation is valid when $\rho_B \gg \rho_Q$. The variable w_B is the equation of state of the background: $w_B = 0$ in the matter-dominated epoch and $w_B = 1/3$ in the radiation-dominated epoch. That is, the Q component acts as a fluid with constant equation of state, but its value of w_Q depends both on its effective potential $V(Q)$ and on the background. The effect of the background is through the $3H\dot{Q}$ in the equation-of-motion for Q, Eq. (1); when w_B changes, H also changes which, in turn, changes the rate at which the tracker field evolves down the potential.

The second remarkable feature of the tracker solutions is that w_Q automatically decreases to a negative value as the Universe transforms from radiation to matter dominated, whether w_Q is positive ($\alpha > 6$) or negative ($\alpha < 6$) in the radiation-dominated epoch. This means that ρ_Q decreases at a slower rate than the matter density. Consequently, the matter-dominated era cannot last forever. Eventually, perhaps close to the present epoch, the Q component overtakes the matter density.

The third remarkable feature is that, once the Q component begins to dominate, its behavior changes again: The Q field slows to nearly a stop causing the equation of state w_Q to decrease towards -1. Hence, the Universe begins a period of accelerated expansion.

If $\Omega_m \geq 0.2$ today, then the Q component has dominated for only a short time and w_Q has not had time to reach -1 today. For $\alpha \gg 1$, w_Q is nearly $1/3$ during the radiation epoch, nearly zero in the matter dominated epoch, and has fallen to a value $\gtrsim -1/3$ today. The predicted current value is larger than recent supernovae results suggest [16]. As α is made smaller, w_Q is smaller at each stage along the tracker solution, including today. For $\alpha \leq 6$, for example, we find $w_Q \gtrsim -0.8$ for $\Omega_m \geq 0.2$, in closer accord with recent supernovae results [16].

The exponential potential, $V(Q) = M^4[\exp(M_p/Q) - 1]$, is an example of combining inverse power-law models, which introduces yet another generic feature of tracking. The exponential potential can be expanded in inverse powers of Q, where the dominant power α varies from high values to low values as Q evolves towards larger values, causing w_Q to decrease as the Universe ages. As a result, Ω_Q grows more rapidly as the Universe ages, making it more likely that Ω_Q dominates later in the

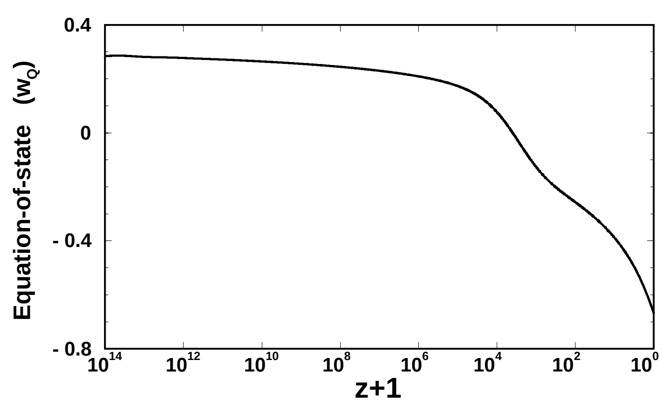

FIG. 2. w_Q vs z for the model in Fig. 1. During the radiation-dominated epoch (large z), $w_Q \approx 1/3$ and the Q energy density tracks the radiation background. During the matter-dominated epoch, w_Q becomes somewhat negative (dipping down to $w_Q \approx -0.2$ beginning at $z = 10^4$) until ρ_Q overtakes the matter density; then, w_Q plummets towards -1 and the Universe begins to accelerate.

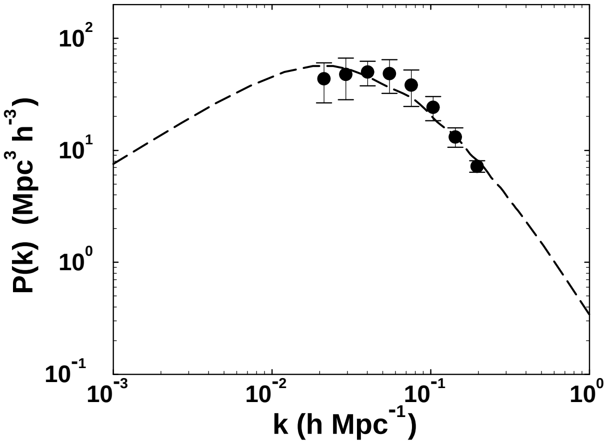

FIG. 3. The linear mass power spectrum for the model in Fig. 1 assuming Hubble parameter $H_0 = 65$ (km/sec)/Mpc, compared to the APM galaxy survey.

history of the Universe rather than earlier. We use this model for the purposes of illustration. In Fig. 1 we show the evolution of ρ_Q relative to the matter and radiation density. We show the case where ρ_Q is comparable to the radiation density at the end of inflation (solid curve) and also the case where ρ_Q is initially much smaller. The latter case produces precisely the same cosmology once the Q field starts rolling. In Figs. 2–4, we illustrate the evolution of w_Q, the comparison of the linear mass power spectrum to recent automatic plate measuring (APM) large-scale structure survey results [17], and the cosmic microwave background temperature anisotropy power spectrum compared to recent data from the COBE, Big Plate, and CAT experiments [9].

An important prediction to emerge from the tracker field models is a relation between Ω_m and w_Q today (for fixed h). For any given potential, the prediction is precise: fixing Ω_m today also fixes the one free parameter, M. Consequently, w_Q is determined, as well. Even without restricting to a particular potential, the trend is

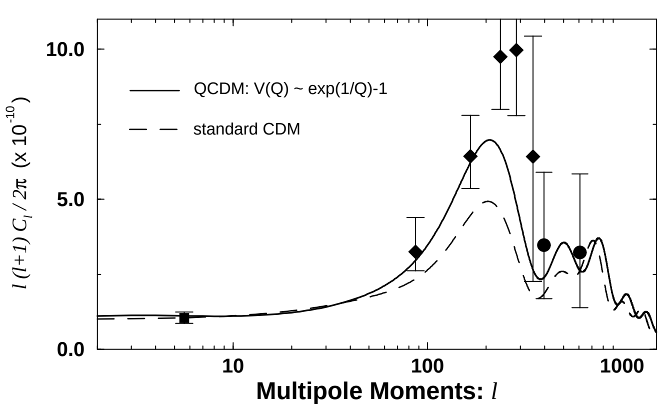

FIG. 4. The cosmic microwave background anisotropy power spectrum for the model in Fig. 1 compared to the standard cold-dark-matter model and recent data [9].

clear: smaller Ω_m means that the tracker field has been dominating longer and w_Q is closer to -1 today. Given that $\Omega_m \geq 0.2$, we have found that it is not possible to obtain $w_Q < -0.8$ without adding artificial complications to the potential. The bound is very weakly h dependent. This bound is significantly different from $w = -1$ for a cosmological constant and one can hope to detect this difference.

One brief word should be added about the future of the Universe: as Q continues to evolve, it slows down and w_Q approaches arbitrarily close to -1. So, the Universe expands as if there is a fixed nonzero cosmological constant, even though the reality is that Q is slowly oozing its way downhill.

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