Incompressibility of Nuclear Matter from the Giant Monopole Resonance

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$E0$ strength distributions in ^{90}Zr, ^{116}Sn, ^{144}Sm, and ^{208}Pb have been measured with inelastic scattering of 240-MeV α particles between $0° \leq \theta_{\rm lab} \leq 6°$ to greater precision than previously available. In Sn, Sm, and Pb, $E0$ strength was concentrated in approximately symmetric peaks, whereas in ^{90}Zr it had a significant high energy tail. Comparing with microscopic calculations using the Gogny interaction, these and our previously reported results for ^{40}Ca are consistent with a nuclear matter incompressibility of 231 ± 5 MeV. Previous data gave an average of 215 MeV and the value for different nuclei disagreed by up to 40 MeV. [S0031-9007(98)08291-X]

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The compression modulus of nuclear matter, $K_{\rm nm}$, is important in the description of properties of nuclei, supernovae explosions, neutron stars, and heavy ion collisions. The value of $K_{\rm nm}$ can be obtained directly from the energies of the isoscalar giant monopole resonance (GMR) in nuclei [1]. In 1980 a comparison of GMR energies to microscopic calculations [1] suggested $K_{\rm nm} = 210 \pm 30$ MeV [1]. The large error was due both to the experimental error in GMR position and a significant mass dependence. As more precise experimental GMR energies became available, this mass dependence remained and was one of the motivations for a revival of interest [2] in extracting $K_{\rm nm}$ by fitting GMR energies with the Leptodermous expansion [1] which suggested $K_{\rm nm}$ might be as high as 325 MeV [2]. However, the data were not of sufficient quality to obtain all the parameters of the expansion from the fit [3], and some model assumptions were required which could have a large effect on the value of $K_{\rm nm}$ obtained [4]. In 1995, Blaizot *et al.* [5] addressed the mass dependence by taking into account pairing and anharmonicity corrections in microscopic calculations for ^{40}Ca, ^{90}Zr, ^{116}Sn, and ^{144}Sm. They argued that such corrections are negligible for ^{208}Pb. The GMR location in ^{40}Ca was not known, but a substantially smaller $K_{\rm nm}$ was required to reproduce the experimental GMR energy in ^{90}Zr than in the heavier nuclei. They concluded that the GMR in ^{208}Pb leads to $194 < K_{\rm nm} < 240$ MeV, and in ^{116}Sn to $207 < K_{\rm nm} < 225$ MeV. Since then other authors have reported a variety of calculations relating the GMR energy to nuclear incompressibility [6–10], but they did not include pairing and anharmonicity corrections.

We have measured the GMR strength distribution in ^{90}Zr, ^{116}Sn, ^{144}Sm, and ^{208}Pb using inelastic scattering of 240 MeV α particles, where excellent peak to continuum ratios had been obtained in lighter nuclei [11,12], and where competing reactions are well above the region where GMR strength is expected. We reported [12] the possible location of $92 \pm 15\%$ of the $E0$ energy weighted sum rule (EWSR) in ^{40}Ca in 1997. The experimental technique is described in Refs. [11] and [12]. Spectra obtained for ^{90}Zr and ^{208}Pb at two angles are shown in Fig. 1. For the runs at 0°, ^{24}Mg spectra were taken before and after the run with each target to provide a check on the calibration. The 13.85 ± 0.02 MeV $L = 0$ state [13] of ^{24}Mg is strong and very close in energy to the Pb GMR.

Distorted-wave Born approximation (DWBA) calculations were carried out using the deformed potential model. The methods of calculation, form factors, and sum rules used in this work are described in Ref. [12]. Optical model parameters obtained for 240 MeV α particle scattering from ^{116}Sn [14] were used for ^{90}Zr, ^{116}Sn, and ^{144}Sm, while parameters from ^{197}Au [15] were used for ^{208}Pb. Calculations of the contributions of the isovector

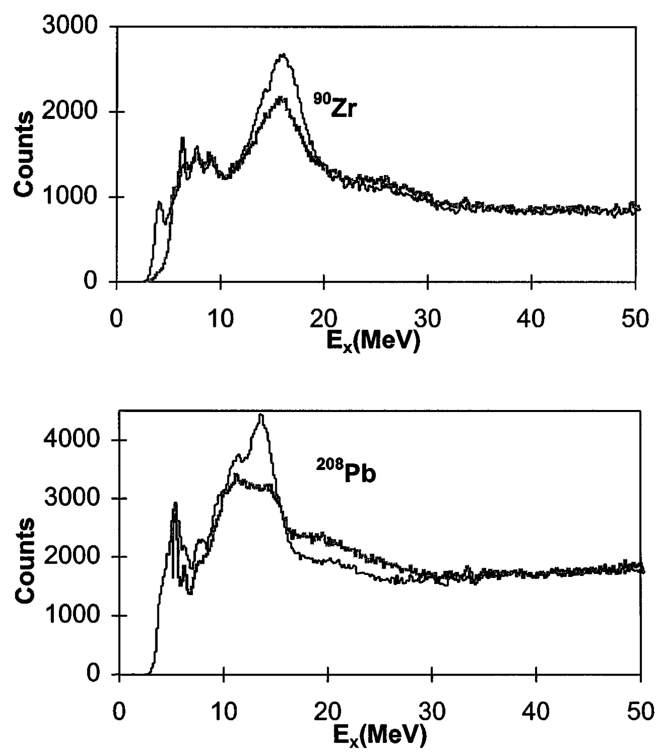

FIG. 1. Inelastic α spectra taken with the spectrometer at $\theta_{\rm spec} = 0°$ for the nuclei indicated. The black line is the spectrum for the most forward angle bin while the gray line is the spectrum for the largest angle bin.

giant dipole resonance (IVGDR) followed the prescription of Satchler [16] using neutron and proton rms radii from Hartree Fock random phase approximation (HF-RPA) calculations [17].

Since the GMR cross section is strongest at 0° and decreases rapidly with angle, whereas for other multipoles it is about either constant or slight increases over this angle range, monopole strength distributions were obtained by subtracting [11,12] a spectrum taken at a larger angle ($\theta_{avg} \approx 1.8°$) from a spectrum taken at a smaller angle ($\theta_{avg} \approx 1.1°$). This enhances the GMR relative to $E2$ and isovector $E1$ strength. The simplest possible straight line continuum adjustment was made to bring the spectrum to zero at the edges of the $E2, E0$ peak, resulting in the spectra shown in Fig. 2. A ^{24}Mg spectrum is also shown to illustrate the proximity of the 13.85 MeV state used to check the calibration. The spectra shown in Fig. 2 were then each fit with two Gaussians, one for the giant quadrupole resonance (GQR) and one for the GMR. Typical fits are shown along with the $E0$ EWSR percentages obtained. The GQR strengths shown in Fig. 2 agree nicely with DWBA for Pb but are somewhat larger than predicted by DWBA for Zr, Sn, and Sm. A relatively small change in continuum shape would bring these in agreement also. The expected IVGDR contribution is shown except for Pb, where it is essentially zero.

As the primary goal of these experiments was to measure the GMR energy, the effects of the continuum were assessed by making five different adjustments (all smooth and monotonic with E_x) ranging from unreasonable with a bias toward low excitation through the simple straight one used for Fig. 2 to unreasonable with a bias toward high excitation. For Pb the maximum difference in the GMR centroid was 80 keV with a standard deviation of 33 keV. For Zr the maximum difference was 45 keV and the standard deviation was 22 keV. Results for the other nuclei fell between these. For each nucleus, eight statistically independent data sets were obtained (one of which is shown in Fig. 2). The weighted average excitation energies obtained from the data are given in the first column of Table I. The errors quoted are the larger of the standard deviations among the eight sets of data (^{144}Sm and ^{208}Pb) or the errors obtained from the peak fitting (^{90}Zr and ^{116}Sn) and include calibration errors and those due to the continuum adjustments discussed above. The energies obtained are compared to those in the literature [3] with the smallest quoted error in Table I. Our results are in excellent agreement with these previous measurements. However, what is needed to compare with theory are the moments [1] of the $E0$ strength distribution rather than the moments of the cross section. As the cross section is a strong function of excitation energy for a constant strength [11,12], the moments of the strength distributions are shifted significantly from those of the cross section. For ^{208}Pb the centroid of the cross section and the centroid of the strength (m_1/m_0) differ by 330 keV, well outside the errors of the present measurements. In previous experimental works, the moments of the cross section were reported rather than the moments of the $E0$ strength.

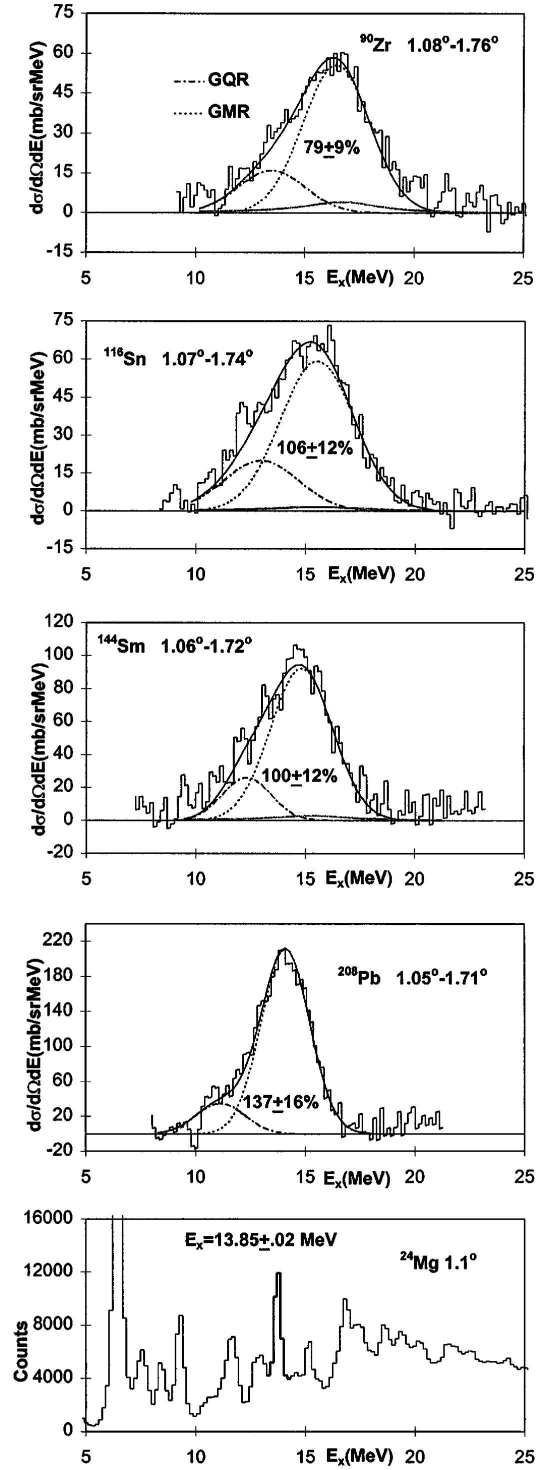

FIG. 2. Difference spectra obtained as described in the text are shown by the histograms. Best fits for the GMR (dashed line) and GQR (short-long dashed line) are shown and the expected (from DWBA) strength of the IVGDR is indicated by the broad gray line. The fifth panel shows a ^{24}Mg spectrum for $\theta_{avg} = 1.1°$.

The centroids of the $E0$ strength (m_1/m_0) obtained from the Gaussian cross section distributions are given in the third column of Table I.

The data were also analyzed by subtracting a continuum and "slicing" the remaining peak into 1 MeV wide bins following the technique described in Ref. [11]. The angular distribution for each of these bins was fit by a sum of $T = 0$ $E0$, $E1$, $E2$, and $E3$ strength. The known strength distribution for the isovector giant dipole resonance was included. The $E2$ distributions obtained agreed with those from the literature [18]. The broad peak seen in Fig. 1 above the GQR/GMR peak in Pb (and in Sn and Sm) was found to consist entirely of $T = 0$ $E1$ and $E3$ strength while in Zr it also contained $E0$ strength. The $E0$ distributions obtained for the four nuclei are shown in Fig. 3 and are nearly symmetric in Sn, Sm, and Pb with m_1/m_0 in excellent agreement with that obtained from the Gaussian fits. These are listed in the fourth column of Table I. For ^{90}Zr, the slice analysis revealed monopole strength extending up to $E_x \approx 25$ MeV, well above the Gaussian previously associated with the GMR in Zr. Thus m_1/m_0 obtained for ^{90}Zr from the slice analysis is considerably larger than that obtained from a Gaussian fit just to the peak. With this additional strength, 102% of the $E0$ EWSR is accounted for in ^{90}Zr. An RPA calculation for ^{90}Zr using SkM* by Hamamoto *et al.* [8] also shows $E0$ strength tailing to higher energy, and the shape of the distribution is quite similar to that obtained from the slice analysis. The $E0$ distribution was found to be relatively insensitive to the choice of continuum, because the continuum was not forward peaked. The errors listed in Table I include an estimate of the effects of different choices of continua.

The energy moments $(m_1/m_{-1})^{1/2}$ adopted for the GMR in these nuclei using the Gaussian analysis for Sn, Sm, and Pb and the slice analysis for Zr are listed in the last column in Table I. Using also our previous ^{40}Ca results [12], we calculated $K_{\rm nm}$ by comparing the GMR energies in five nuclei to those calculated by Blaizot *et al.* [5]. For ^{116}Sn and ^{208}Pb Blaizot *et al.* reported monopole energies calculated with several interactions, so $K_{\rm nm}$ was obtained from Sn and Pb by fitting the $D1S$, $D1$, and $D250$ results. For ^{40}Ca, ^{90}Zr, and ^{144}Sm they report only the result with $D1S$, so $K_{\rm nm}$ was obtained from the $D1S$ result by assuming $K_{\rm nm} \propto E_x^2$. The values obtained are shown in Fig. 4, where the weighted average ($K_{\rm nm} = 231 \pm 5$ MeV) is also shown. The error given for the average is the standard deviation of the five points which is larger than that calculated from the errors on the individual points. The errors shown on the individual points are those due to the uncertainty in position of the $E0$ strength, although the theoretical uncertainties (those discussed by Blaizot *et al.* and those of extracting $K_{\rm nm}$ from the calculated point) are now likely to be greater than the experimental error. $K_{\rm nm}$ obtained using previous "best" values are also shown for comparison.

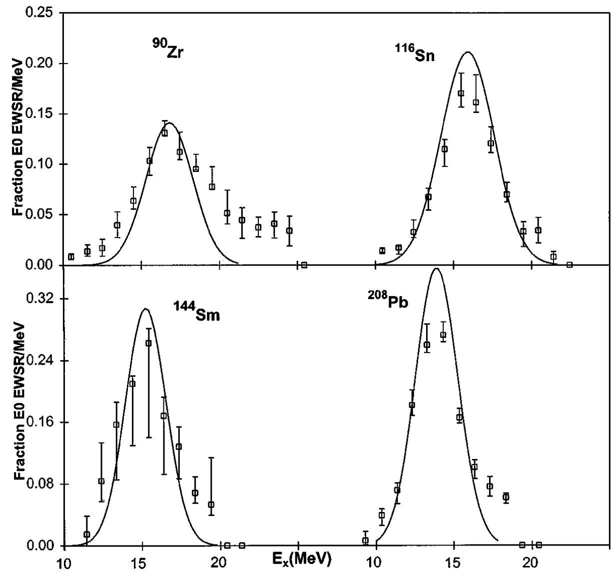

FIG. 3. The $E0$ strength distributions obtained from the slice analysis (squares) are plotted versus excitation energy. The errors represent the extent of $E0$ variation when the best χ^2 is allowed to double. The line is the strength distribution corresponding to the Gaussian fits shown in Fig. 2.

Farine *et al.* [6] fit the (then accepted) breathing mode energies by modifying a generalized Skyrme force which they required to fit the masses and radii. Our results for Zr, Sn, Sm, and Pb are in excellent agreement with their calculation for SkK240 which corresponds to $K_{\rm nm} = 240$ MeV. An analysis by Chossy and Stocker [7] used relativistic mean field parameters to calculate the terms in the leptodermous expansion and predict breathing mode

TABLE I. GMR energies and errors in MeV.

	TAMU 1998 Gaussian Cross Section Centroid MeV	TAMU 1998 Gaussian Cross Section error MeV	Previous Work Gaussian Cross Section Centroid MeV	Previous Work Gaussian Cross Section error MeV	TAMU 1998 Gaussian $E0$ Strength m_1/m_0 MeV	TAMU 1998 Slice Analysis $E0$ Strength m_1/m_0 MeV	TAMU 1998 Slice Analysis $E0$ Strength error MeV	TAMU 1998 Adopted Energies $E0$ Strength $(m_1/m_{-1})^{1/2}$ MeV	TAMU 1998 Adopted Energies $E0$ Strength error MeV
^{90}Zr	16.44	0.07	16.10	0.28	16.80	17.89	0.20	17.81	0.35
^{116}Sn	15.77	0.07	15.60	0.16	16.00	16.07	0.12	15.90	0.07
^{144}Sm	15.16	0.11	15.10	0.14	15.31	15.39	0.28	15.25	0.11
^{208}Pb	13.91	0.11	13.90	0.30	14.24	14.17	0.28	14.18	0.11

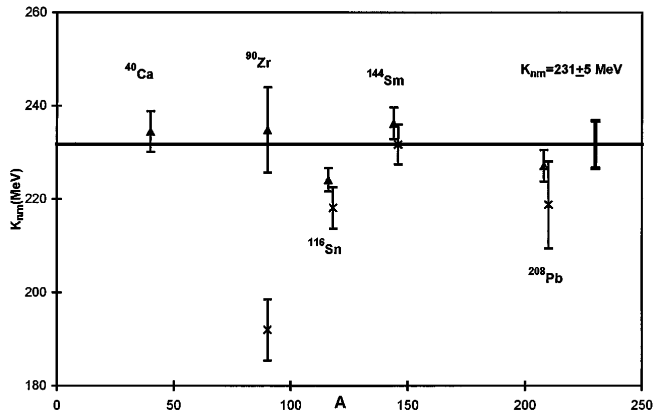

FIG. 4. K_{nm} obtained for each of the five nuclei by comparing the GMR energies to the calculations of Blaizot *et al.* [5]. The triangles are from the present data. The × symbols are from the world data set as of 1993 [4]. The broad line with error bars shows the average. The data points are slightly offset in A for clarity.

energies. Their results for ^{116}Sn and ^{144}Sm using the NLC parameter set with $K_{\mathrm{nm}} = 224.5$ MeV are in good agreement with our data, while our data for ^{90}Zr and ^{208}Pb require K_{nm} about 235 and 255 MeV, respectively. The average of these is 235 ± 14 MeV. Hamamoto *et al.* [8] used various Skyrme interactions to calculate the GMR distributions in ^{40}Ca, ^{90}Zr, and ^{208}Pb. $(m_3/m_1)^{1/2}$ obtained for these nuclei for the SkM* interaction ($K_{\mathrm{nm}} =$ 217 MeV) are slightly high for Pb and Zr, and quite high for Ca, implying that K_{nm} is below 217 MeV. There have also been a number of calculations using relativistic mean field theory which generally for the same GMR energies have much higher compressibilities [9,10]. The calculations by Vretenar *et al.* [9] would be generally consistent with our data for $K_{\mathrm{nm}} \approx 299 \pm 22$ MeV for $A \geq 90$, but the mass dependence is not well reproduced, particularly in lighter nuclei.

In this work we have obtained data with much lower continua than previous data resulting in more precise energies for the GMR and identified a previously unknown tail on the GMR strength in ^{90}Zr which raises the centroid by more than 1 MeV, removing a serious discrepancy with RPA calculations for GMR energies. We also noted that the centroids of $E0$ strength distributions obtained from alpha scattering are significantly higher than the centroids of the $E0$ cross section. With these new results, for the first time a consistent value for K_{nm} can be obtained by comparing microscopic calculations to GMR data for nuclei having a wide range of A (40–208). Comparison to calculations with the Gogny interactions [5] leads to $K_{\mathrm{nm}} = 231 \pm 5$ MeV while those using a generalized Skyrme force [6] leads to $K_{\mathrm{nm}} = 240$ MeV. A scaling model analysis with a relativistic mean field parametrization leads [7] to $K_{\mathrm{nm}} = 235 \pm 14$ MeV.

These values are in excellent agreement with the $K_{\mathrm{nm}} =$ 234 MeV obtained by Myers and Świątecki [19] fitting binding energies and diffuseness with the Thomas-Fermi model. They are also consistent with nucleus-nucleus scattering which was well fit [20] using nucleow-nucleow interactions that correspond to $K_{\mathrm{nm}} = 241$, 252, and 270 MeV, but not for $K_{\mathrm{nm}} = 228$ MeV or lower. They are somewhat above the $K_{\mathrm{nm}} = 210$ MeV estimated from the linear momentum change in heavy ion reactions [21].

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