Degenerate and Quasidegenerate Majorana Neutrinos

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We study mixing and *CP* violation of three left-handed Majorana neutrinos in the limit of exactly degenerate masses, identify the weak-basis invariant relevant for *CP* violation, and show that the leptonic mixing matrix is parametrized only by two angles and one phase. After the lifting of the degeneracy, this parametrization accommodates the present data on atmospheric and solar neutrinos, as well as double β decay. Some of the leptonic mixing *Ansätze* suggested in the literature correspond to special cases of this parametrization. [S0031-9007(98)08204-0]

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The Super-Kamiokande Collaboration [1] has recently provided evidence confirming the atmospheric neutrino anomaly, as well as the solar neutrino deficit. The interpretation of these experimental results within the framework of three left-handed neutrinos, without sterile neutrinos, together with the assumption that relic neutrinos constitute the hot dark matter of the Universe [2], inescapably leads to highly degenerate neutrinos [3].

In this Letter, we analyze in detail neutrino mixing and *CP* violation in the case of three Majorana neutrinos with exactly degenerate masses and then consider the case of quasidegenerate masses. We identify the weak-basis invariant, which controls the strength of *CP* violation in the limit of exact mass degeneracy and point out that, in this limit, the neutrino mixing matrix is in general parametrized by two angles and one phase. We then show that a two-angle parametrization suggested by the exact degeneracy limit can fit all the present atmospheric and solar neutrino data and complies with the bound imposed by neutrinoless beta decay. Furthermore, we point out that various of the recently suggested neutrino mixing schemes, such as the bimaximal mixing [4], the democratic mixing [5], as well as the scheme suggested by Georgi and Glashow [6], correspond to specific cases of our two-angle parametrization.

The limit of exact degeneracy.—Let us consider three left-handed neutrinos and introduce a generic Majorana mass term,

$$\mathcal{L}_{\rm mass} = -(\nu_{L_\alpha})^T C^{-1} m_{\alpha\beta} \nu_{L_\beta} + \text{H.c.}, \qquad (1)$$

where $m = (m_{\alpha\beta})$ is a 3×3 complex symmetric mass matrix, and ν_{L_α} denote the left-handed weak eigenstates. We shall work in the weak basis (WB) where the charged lepton mass matrix is diagonal, real, and positive. The neutrino mass matrix can be diagonalized by the transformation [7]

$$U^T \cdot m \cdot U = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}). \qquad (2)$$

The weak eigenstates, ν_{L_α}, are related to the mass eigenstates, ν_{L_i}, by $\nu_{L_\alpha} = U_{\alpha i}\nu_{L_i}$, so that the charged current of the lepton weak interactions is given by

$$\mathcal{L}_W = \frac{g}{2}(\overline{e}, \overline{\mu}, \overline{\tau})_L \gamma_\mu U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L W^\mu + \text{H.c.} \qquad (3)$$

It is well known that for the nondegenerate case the neutrino diagonalization matrix U can be parametrized by three angles and three phases that are *CP* violating. In the limit of exact degeneracy, we shall show here that, in general, U cannot be rotated away, and its parametrization requires two angles and one *CP* violating phase. Furthermore, we shall see that only if the theory is *CP* invariant and the three degenerate neutrinos have the same *CP* parity can the matrix U be rotated away. This is to be contrasted to the case of Dirac neutrinos, where there is no mixing or *CP* violation in the exact degeneracy limit.

Let us consider the limit of exact degeneracy with μ the common neutrino mass. It is useful to define the dimensionless matrix $Z_0 = m/\mu$ which from Eq. (2) can be written as

$$Z_0 = U_0^\star \cdot U_0^\dagger, \qquad (4)$$

where U_0 denotes the mixing matrix in the exact degeneracy limit. It follows from Eq. (4) that Z_0 is a unitary symmetric matrix. By making a WB transformation, under which $Z_0 \to K \cdot Z_0 \cdot K$, with K a diagonal unitary matrix, it is possible to choose the first line and the first column of Z_0 real, while keeping the charged lepton mass matrix diagonal real and positive. Without loss of generality, the matrix Z_0 can then be written as

$$Z_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\phi & s_\phi \\ 0 & s_\phi & -c_\phi \end{pmatrix} \cdot \begin{pmatrix} c_\theta & s_\theta & 0 \\ s_\theta & z_{22} & z_{23} \\ 0 & z_{23} & z_{33} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\phi & s_\phi \\ 0 & s_\phi & -c_\phi \end{pmatrix}. \qquad (5)$$

Unitarity of Z_0 then implies that either s_θ or z_{23} must vanish. It can be readily verified that the case $s_\theta = 0$ automatically leads to CP invariance. Assuming $s_\theta \neq 0$, then the most general form for the symmetric unitary matrix Z_0 is given by

$$Z_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\phi & s_\phi \\ 0 & s_\phi & -c_\phi \end{pmatrix} \cdot \begin{pmatrix} c_\theta & s_\theta & 0 \\ s_\theta & -c_\theta & 0 \\ 0 & 0 & e^{i\alpha} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\phi & s_\phi \\ 0 & s_\phi & -c_\phi \end{pmatrix}. \quad (6)$$

The parametrization of Z_0 in Eq. (6) does not include the trivial case where CP is a good symmetry and all neutrinos have the same CP parity. In order to show that this is indeed the case, let us assume that the most general Z_0 for degenerate neutrinos given by Eq. (4) is a real matrix, so that CP invariance holds. In that case, Z_0 can be diagonalized by an orthogonal transformation $Z_0 \to O^T \cdot Z_0 \cdot O$, which leaves invariant both $\mathrm{Tr}(Z_0)$ and $\det(Z_0)$. Apart from trivial permutations, the eigenvalues of Z_0 will be $(1,1,1)$, $(1,-1,1)$, or $(1,-1,-1)$. It is well known [8] that the first case corresponds to three neutrinos with the same CP parity, while the other two cases correspond to one of the neutrinos having a CP parity opposite to that of the other two. Now, in the parametrization of Eq. (6), one obtains $\det(Z_0) = -e^{i\alpha}$, $\mathrm{Tr}(Z_0) = e^{i\alpha}$, and therefore the cases $(1,-1,1)$ and $(1,-1,-1)$ can be obtained, corresponding to $\alpha = 0$ and $\alpha = \pi$, respectively. Obviously the case $(1,1,1)$ cannot be obtained by the parametrization of Eq. (6). As we previously mentioned, this case corresponds to a trivial mixing matrix, which can be rotated away. The matrix Z_0 given by Eq. (6) can be diagonalized through the transformation of Eq. (2), with U_0 given by

$$U_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\phi & s_\phi \\ 0 & s_\phi & -c_\phi \end{pmatrix} \cdot \begin{pmatrix} \cos(\frac{\theta}{2}) & \sin(\frac{\theta}{2}) & 0 \\ \sin(\frac{\theta}{2}) & -\cos(\frac{\theta}{2}) & 0 \\ 0 & 0 & e^{-i\alpha/2} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (7)$$

The matrix U_0 is then the mixing matrix appearing in the leptonic charged currents. Given the Majorana character of neutrino masses and the fact that U_0 is not an orthogonal matrix, it is clear that one cannot rotate away U_0 through a redefinition of the neutrino fields. This is the case even in the CP invarance limit, i.e., $\alpha = 0, \pi$.

The strength of CP violation and a WB invariant.—We have seen that CP violation may arise even when the three Majorana neutrinos have identical mass [9]. Now, we present a weak-basis invariant which controls the strength of the CP violation in the limit of exact degeneracy. It can be readily verified that a necessary and sufficient condition for CP invariance, in the degenerate limit, is

$$G \equiv \mathrm{Tr}[(m \cdot h \cdot m^\star), h^\star]^3 = 0\,, \quad (8)$$

where $h = m_\ell \cdot m_\ell^\dagger$, and m_ℓ denotes the charged lepton mass matrix. The nonvanishing of G signals CP violation, while the vanishing of G implies CP invariance in the limit of mass degeneracy. Since G is a WB invariant, it can be expressed in terms of lepton masses and mixings. In the evaluation of G, it is convenient to choose the WB where h is diagonal, i.e., $h = \mathrm{diag}(m_e^2, m_\mu^2, m_\tau^2)$. One obtains

$$\begin{aligned} G &= 6i\Delta_m \,\mathrm{Im}[(Z_0)_{11}(Z_0)_{22}(Z_0)_{12}^\star(Z_0)_{21}^\star] \\ &= \frac{3i}{2}\Delta_m \cos(\theta)\sin^2(\theta)\sin^2(2\phi)\sin(\alpha)\,, \end{aligned} \quad (9)$$

where $\Delta_m = \mu^6(m_\tau^2 - m_\mu^2)^2(m_\tau^2 - m_e^2)^2(m_\mu^2 - m_e^2)^2$ is a multiplicative factor which contains the different masses of the charged leptons and the common neutrino mass μ. In Ref. [9] various examples of CP-odd WB invariants were constructed, but all of those invariants automatically vanish in the limit of exact degeneracy. The special feature of the WB invariant of Eq. (8) is the fact that, in general, it does not vanish, even in the limit of exact degeneracy of the three Majorana neutrino masses.

Since in the limit of exact degeneracy there is only one independent WB invariant controlling the strength of CP violation, it is meaningful to ask when is CP violation maximal. From Eq. (9), it follows that G assumes its maximal value for $\phi = \pi/4$, $\alpha = \pi/2$ and $\sin(\theta) = \sqrt{2}/\sqrt{3}$, $\cos(\theta) = 1/\sqrt{3}$. For these values of ϕ, θ, α the matrix Z_0 assumes a very special form:

$$Z_0 = K \cdot 1/\sqrt{3}\begin{pmatrix} \omega & 1 & 1 \\ 1 & \omega & 1 \\ 1 & 1 & \omega \end{pmatrix} \cdot K \quad (10)$$

with $\omega = e^{-i2\pi/3}$ and $K = \mathrm{diag}(e^{i\pi/3}, e^{-i\pi/3}, e^{-i\pi/3})$. Thus the imposition of maximal CP violation leads to a structure of the Majorana neutrino mass of the type that one obtains in the framework of universal strength for Yukawa couplings [10].

Lifting the degeneracy.—We have seen that, in the limit of exact degeneracy, the leptonic mixing matrix can be parametrized by two angles θ, ϕ, and one phase α. Obviously, the physically interesting case corresponds to quasidegenerate neutrinos. The degeneracy is lifted through a small perturbation:

$$Z = Z_0 + \varepsilon Q\,, \quad (11)$$

where ε is a small parameter and Q is a symmetric complex matrix of order one. At this stage, it is worth recalling that in the exact degeneracy limit, the neutrino mixing matrix U_0 is defined only up to an arbitrary orthogonal transformation $U_0 \to U_0 \cdot O$. In the presence of a small perturbation εQ, the full matrix Z will be diagonalized by a matrix $U = (U_0 \cdot O) \cdot W$, where W is a unitary matrix close to the identity. In first order we have

$$W = \mathbb{1} + i\varepsilon P \quad (12)$$

with P a Hermitian matrix. In view of the above, it is useful to diagonalize Z in two steps. First, we make the transformation

$$Z \to Z' \equiv U_0^T \cdot Z \cdot U_0 = \mathbb{1} + \varepsilon Q', \qquad (13)$$

where we have used the fact that $U_0^T \cdot Z_0 \cdot U_0 = \mathbb{1}$ and have defined $Q' = U_0^T \cdot Q \cdot U_0$. The matrix Z' is then diagonalized by

$$Z' \to (OW)^T \cdot Z' \cdot (OW) = \mathbb{1} + \varepsilon d, \qquad (14)$$

where d is diagonal and real. Using Eqs. (12), (13), and (14) one obtains in leading order of the perturbation

$$O^T \cdot A \cdot O = d; \qquad P + P^T = -O^T \cdot B \cdot O, \quad (15)$$

where A, B are real symmetric matrices defined by $A = \mathrm{Re}(Q')$, $B = \mathrm{Im}(Q')$. Equations (15) have a simple interpretation. In the presence of a small perturbation around the degeneracy limit, the mixing matrix becomes, to leading order, $U_0 \cdot O$, where O is no longer arbitrary, being the orthogonal matrix which diagonalizes the symmetric real matrix A. We have, of course, assumed that the degeneracy is lifted in first order of perturbation. From the above discussion it is clear that for quasidegenerate neutrinos, in leading order, only one CP violating phase appears in the leptonic mixing matrix, namely, the phase α present in U_0.

Phenomenological implications.—At this stage, one may ask whether, after the lifting of the degeneracy, the two-angle parametrization given by Eq. (7) can still accommodate the present experimental data on atmospheric and solar neutrinos, as well as the constraints on double beta decay. It will be shown that this is indeed the case and, in fact, some of the *Ansätze* suggested in the literature are special cases of this parametrization.

Double beta decay.—Let us first consider the constraints arising from neutrinoless double beta decay, which can only occur if neutrinos are of Majorana type, irrespective of whether or not there is CP violation or nontrivial neutrino mixing. The amplitude for neutrinoless double beta decay is proportional to $\langle m \rangle$, an average neutrino mass, given in standard notation by

$$\langle m \rangle = \sum_i U_{ei}^2 m_{\nu_i} = m_{ee}^\star, \qquad (16)$$

where the U_{ei} denote the elements of the first row of the mixing matrix U, and m_{ee} is the $(1,1)$ element of the mass matrix m. The experimental upper bound on $\langle m \rangle$ depends on the model that is used for the nuclear matrix elements. At present, the strongest bound is $|\langle m \rangle| = |m_{ee}| < 0.46$ eV [11]. In the limit of exact degeneracy, we have $m_{ee} = \mu \cos(\theta)$, where we have used the parametrization of Eq. (6). If we fix $\mu = 2$ eV, then neutrino masses are equal to a precision sufficient to neglect their differences, and the experimental bound on m_{ee} immediately translates into a single bound on the parameter θ, namely, $|\cos(\theta)| < 0.23$.

Atmospheric and solar neutrino data.—The atmospheric neutrino data support the existence of oscillations of atmospheric neutrinos to tau neutrinos or to a sterile neutrino, with a large mixing angle satisfying the bound $\sin^2(2\theta_{\mathrm{atm}}) > 0.82$, and the neutrino mass square difference in the range $5 \times 10^{-4}\ \mathrm{eV}^2 < \Delta m_{\mathrm{atm}}^2 < 6 \times 10^{-3}\ \mathrm{eV}^2$. Recent data from the CHOOZ Collaboration [12] provides on the other hand some evidence against the possibility that atmospheric muon neutrinos oscillate into electron neutrinos, although in some special scenarios this possibility might still be open [13].

In the context of three left-handed neutrinos, the probability for a neutrino ν_α to oscillate to other neutrinos is

$$1 - P(\nu_\alpha \to \nu_\alpha) = 4 \sum_{i<j} U_{\alpha i} U_{\alpha i}^\star U_{\alpha j}^\star U_{\alpha j} \times \sin^2\left[\frac{\Delta m_{ji}^2}{4} \frac{L}{E}\right], \qquad (17)$$

where $\Delta m_{ji}^2 = |m_j^2 - m_i^2|$, E is the neutrino energy, and L denotes the distance traveled by the neutrino between the source and the detector. Since in the range L/E that is relevant for atmospheric neutrinos the term in $\sin^2[(\Delta m_{21}^2/4)(L/E)]$ can be disregarded, we may identify $\sin^2(2\theta_{\mathrm{atm}})$ with $4(U_{21}U_{21}^\star U_{23}^\star U_{23} + U_{22}U_{22}^\star U_{23}^\star U_{23})$. In the framework of our two-angle parametrization of Eq. (7), the above combination of matrix elements has a simple form and one obtains $\sin^2(2\theta_{\mathrm{atm}}) = \sin^2(2\phi)$, i.e., θ_{atm} can be identified with the angle ϕ and thus the atmospheric neutrino data lead to the constraint $\sin^2(2\phi) > 0.82$.

The discrepancy between the observed and the calculated [14] solar neutrino fluxes also requires neutrino oscillations, although at this stage various schemes are still possible, namely, within the framework of the Mikheyev-Smirnov-Wolfenstein mechanism [15] there is a small angle solution $\sin^2(2\theta_{\mathrm{sol}}) \approx 7 \times 10^{-3}$ with $\Delta m_{\mathrm{sol}}^2 \approx 6 \times 10^{-6}\ \mathrm{eV}^2$, and a large angle solution $\sin^2(2\theta_{\mathrm{sol}}) \sim 0.6$–$0.8$ with $\Delta m_{\mathrm{sol}}^2 \approx 9 \times 10^{-6}\ \mathrm{eV}^2$. Another solution could be vacuum oscillations with $\sin^2(2\theta_{\mathrm{sol}}) \approx 0.9$ and $\Delta m_{\mathrm{sol}}^2 \approx 10^{-10}\ \mathrm{eV}^2$. Since in our two-angle parametrization one has $U_{13} = 0$ we obtain $\sin^2(2\theta_{\mathrm{sol}}) = 4U_{11}U_{11}^\star U_{12}^\star U_{12}$ leading to $\sin^2(2\theta_{\mathrm{sol}}) = \sin^2(\theta)$, i.e., in our parametrization $2\theta_{\mathrm{sol}} = \theta$.

From the above analysis it follows that an attractive feature of this two-angle parametrization is the fact that each of the experiments considered independently constrains a single parameter: double beta decay and solar neutrino data only constrain θ, while atmospheric neutrino data only put a bound on ϕ.

There have been several attempts to fit solar and atmospheric neutrino data. The form of the matrix U strongly depends on the scheme adopted to explain the solar puzzle, with large or small mixing. It is clear that with small mixing, no strong cancellation in the summation in Eq. (16) can occur, so in this case the double beta decay would forbid quasidegenerate neutrinos with masses in the range of cosmological relevance.

Next, we show that some of the neutrino mixing schemes proposed in the literature correspond to specific cases of the two-angle parametrization suggested by Eq. (7).

(a) *Bimaximal mixing [4].*—In this scheme the lines of the neutrino mixing matrix have the following structure:

$$L_1 = (1/\sqrt{2}, -1/\sqrt{2}, 0); \qquad L_2 = (\tfrac{1}{2}, \tfrac{1}{2}, 1/\sqrt{2}); \qquad L_3 = (\tfrac{-1}{2}, \tfrac{-1}{2}, 1/\sqrt{2}). \tag{18}$$

This pattern of neutrino mixing is obtained within the two-angle parametrization for the following values of θ, ϕ, and α:

$$\alpha = 0; \quad \cos(\theta/2) = -\sin(\theta/2) = -\cos(\phi) = \sin(\phi) = \frac{1}{\sqrt{2}}. \tag{19}$$

(b) *Democratic mixing [5].*—This mixing has been proposed within the framework of a "democratic" structure for the quark and lepton mass matrices. It was pointed out [5] that this neutrino mixing automatically arises if one assumes that in the exact democratic limit, neutrinos have no mass, and acquire mass only through diagonal democracy-breaking terms. In this case the neutrino mixing matrix has, to a very good approximation, the following form:

$$L_1 = (1/\sqrt{2}, -1/\sqrt{2}, 0); \quad L_2 = (1/\sqrt{6}, 1/\sqrt{6}, -2/\sqrt{6}); \quad L_3 = (1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}). \tag{20}$$

Within the two-angle parametrization, one obtains the democratic mixing for the following values of the parameters:

$$\alpha = 0; \qquad \cos(\theta/2) = -\sin(\theta/2) = \frac{1}{\sqrt{2}}; \qquad \cos(\phi) = \frac{1}{\sqrt{2}} \sin(\phi) = \frac{-1}{\sqrt{3}}. \tag{21}$$

In the above analysis, we have not paid attention to the factors "i" appearing in our two-angle parametrization of Eq. (7). As we have previously emphasized, these factors of i have to do with the fact that in the construction of the two-angle parametrization, we have implicitly assumed that in the limit of CP invariance [i.e., $\sin(\alpha) \rightarrow 0$], one of the Majorana neutrinos has relative CP parity opposite the other two. The factors of i do not play any role in the analysis of atmospheric and solar neutrino data but are crucial in the analysis of double beta decay.

(c) *Georgi-Glashow mass matrix [6].*—Using an analysis of the present neutrino data Georgi and Glashow have suggested the following approximate form for the Majorana neutrino mass matrix:

$$(m)_{1i} = \mu(0, 1/\sqrt{2}, 1/\sqrt{2}); \qquad (m)_{2i} = \mu(1/\sqrt{2}, \tfrac{1}{2}, \tfrac{-1}{2}); \qquad (m)_{3i} = \mu(1/\sqrt{2}, \tfrac{-1}{2}, \tfrac{1}{2}). \tag{22}$$

From Eq. (6) it follows that this neutrino mass matrix is obtained, within the two-angle parametrization for the following values of its parameters:

$$\alpha = 0; \qquad \sin(\theta) = 1; \qquad \cos(\phi) = \sin(\phi) = 1/\sqrt{2}. \tag{23}$$

To summarize, we have built a general parametrization for the leptonic mixing matrix in the case of three exactly degenerate Majorana neutrinos, characterized by two angles and one phase and have shown that for quasidegenerate neutrinos, this parametrization accommodates all present neutrino data. A remarkable feature of this parametrization is the fact that each of the relevant experiments considered (solar, atmospheric, and double beta decay) independently constrains a single angle. We have also presented a weak-basis invariant which controls the strength of CP violation in the case of exact degeneracy.

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