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Stochastic and *Bona Fide* Resonance: An Experimental Investigation

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The experimental evidence of stochastic resonance in the polarized emission of vertical cavity surface emitting lasers is given. We report for the first time in an experimental work a complete characterization of the phenomenon based on the residence times probability density. We also give evidence of the *bona fide* resonance, clarifying this recently debated subject. [S0031-9007(98)08285-4]

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Noise is usually considered a limiting factor in the good performance of a device. There are, however, nonlinear systems where increasing the noise up to an optimum value can give rise to a large amplitude output signal. This effect is commonly called stochastic resonance (SR). The phenomenon was first proposed in 1981 by Benzi *et al.* [1] and Nicolis *et al.* [2] to explain the periodicity of the ice age in the quaternary climate. It was, however, only during the past decade that SR became the subject of extended investigation, mainly devoted to theoretical studies or numerical and analog simulations. This phenomenon appears in a large variety of physical systems and it has, in fact, been used in different fields, from neurophysiology, in the study of neuronal processes, to solid-state physics, to shed light on the behavior of tunnel diodes and SQUID's (for a review, see, e.g., [3]).

The rise of interest for SR first of all coincides with the experimental observation in a bistable ring laser, performed in 1988 by McNamara *et al.* [4]. However, as of today, experimental work is far from permitting a complete and satisfactory comparison with the rich amount of theoretical results.

In this paper, we describe the observation and the investigation of SR in vertical cavity surface emitting lasers (VCSELs). With respect to previous experimental works, our system exhibits some peculiar characteristics; in particular, it allows one to change over a large range the input parameters, such as the modulation frequency and amplitude, noise intensity, and bandwidth. These features, together with the high stability of the system, allow an unprecedented comparison with theoretical works; for example, the study of SR as a function of modulation frequency and a complete characterization of the SR by means of probability distributions are reported here for the first time in an experimental work. Thanks also to an accurate choice of the monitored indicator we clarify the possibility of observing SR as a true resonance, i.e., as a function of the modulation frequency. As a consequence, the so-called *bona fide* resonance is experimentally shown.

We employ a VCSEL lasing at 850 nm, thermally stabilized (better than 1 mK), and with a carefully controlled pump current. The overall stability allows for long time measurements, even in the presence of critical behaviors. Two linear polarization directions are defined in the laser emission and can be selected using a polarizer and a half-wave plate. The laser intensity is monitored by an avalanche detector and the signal is recorded by a digital scope. An optical isolator prevents optical feedback effects. The signals from a 10-MHz-bandwidth white-noise generator and a sinusoidal oscillator are summed and coupled into the laser by means of a bias tee.

In a previous paper [5], we observed that in some narrow regions of pump current the laser emission is characterized by noise-driven polarization flips, marking the transition between two different transverse mode configurations with corresponding different polarized intensities. We have described this feature by means of a Langevin equation with a phenomenological quasipotential $V(q)$, inferred from the histogram of fluctuations of the intensity "q" in one polarization [6]. In the critical regions, $V(q)$ has a two-well shape, with the abscissa of the minima corresponding to the two involved intensity levels (upper and lower). The laser spends, on the average, a (noise-dependent) time T_K (Kramers' time) in each well, before undergoing a noise-driven jump [7]. Changing the pump current in the critical region, the shape of the quasipotential is modified; we have analyzed a situation for which the quasipotential depth ΔV is the same for the two wells. In fact, $V(q)$ is, in general, asymmetric, yielding different permanence times in the two levels. The Kramers' times, for the experimental parameters used in our work, are some tenths of μs. We point out that, for different laser samples and current levels, we have observed times ranging from a few ns to a few seconds.

A modulation of the current produces a slightly modulated intensity, superimposed to the random jumps between the two above-mentioned levels (Fig. 1). Increasing the amount of noise (summed to the pump current), the jumps between the two states tend to synchronize with the applied modulation. In this case, the output modulation is much stronger than without the input noise. Increasing the noise further, the synchronization is lost due to frequent jumps, yielding quite a noisy output. This is the typical signature of stochastic resonance.

We emphasize that we do not observe any effect in the total laser emission, i.e., without selecting the polarization.

For a more quantitative analysis, the response of the system to an input modulation at frequency ω can be described as a variation of the polarized output intensity of the kind $q(t) = q_0 \cos(\omega t + \phi)$. In Fig. 2a we report the observed amplitude q_0, obtained from the frequency spectrum of q, as a function of the noise. The calibration of the input noise diffusion constant D in units of the activation energy ΔV, useful for comparison with the theoretical studies, has been obtained from the behavior of the intensity histograms as a function of the noise. The values of q_0 are normalized to half the difference between the two levels; in this way, a square wave would yield an amplitude of $4/\pi$, a value effectively approached by the experimental data.

The physical origin of SR can be found in a time-scale coincidence between the modulation period and the typical

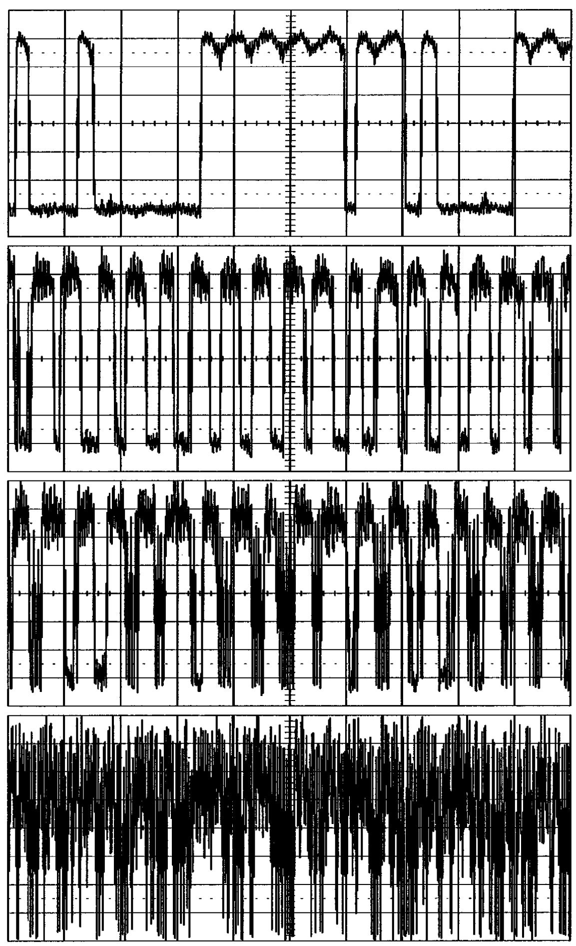

FIG. 1. Temporal behavior of the polarized laser intensity in the presence of a current modulation, increasing the input noise from top to bottom. The time scale is 10 μs/division.

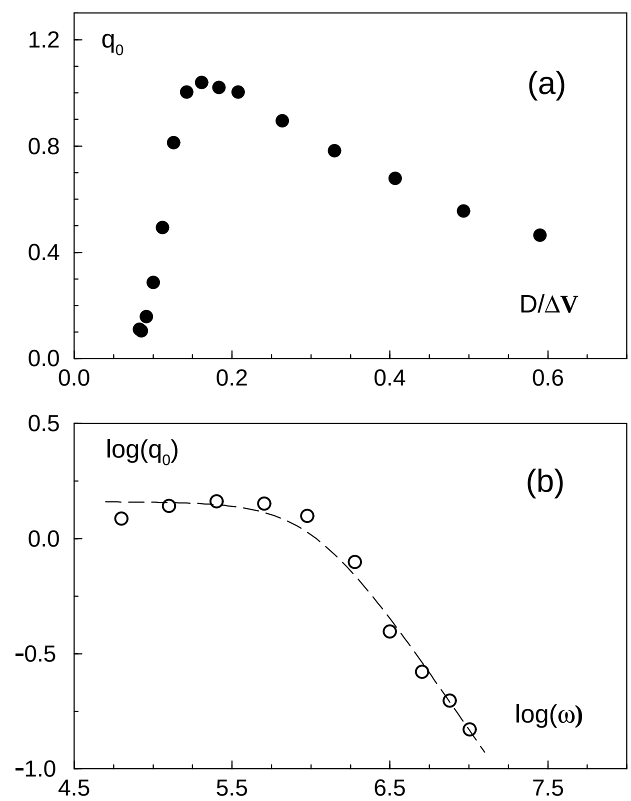

FIG. 2. Amplitude of the output signal q_0. (a) q_0 vs input noise at a modulation frequency $\omega/2\pi = 150$ kHz; the predicted value for a perfect synchronization is $4/\pi$. (b) The dashed line is the fitting with a single-pole transfer function, which yields a cutoff frequency of $\omega_0/2\pi = 165$ kHz. The calibration of the noise parameter D in units of ΔV is obtained from the experimental intensity histograms.

time constant (Kramers' time) of the system, which monotonically decreases as the noise increases. According to this heuristic argument, one would also expect a resonance behavior by changing the modulation period. However, calculations based on a simple system (a symmetric, quartic potential) [3] forecast a response characterized only by a single pole at $\omega_0 = 2r_K$, where $r_K = 1/T_K$ is the Kramers' rate. This prediction is confirmed for the first time by our experiment, as shown in Fig. 2b. The plotted fit of the experimental data actually gives a single-pole behavior, with a cutoff frequency $\omega_0/2\pi = 165$ kHz. For a comparison with the theory, we can consider the average permanence times in the two levels, measured without modulation and with the same amount of noise. The measured values are $\tau_{\mathrm{upper}} = 2.2\ \mu$s and $\tau_{\mathrm{lower}} = 2.3\ \mu$s, giving the cutoff frequency $\omega_0/2\pi = 2/\pi \times (\tau_{\mathrm{upper}} + \tau_{\mathrm{lower}})^{-1} = 140$ kHz. The agreement is quite good, but a better theoretical and experimental investigation for the case of asymmetric potential would certainly be of interest.

The SR has been introduced as a behavior of the output modulation amplitude, or of the signal-to-noise ratio, vs noise. More recently, Gammaitoni *et al.* [8] and Zhou *et al.* [9] proposed the observation of SR from the probability density of the residence times (RTPD) in the two states. This type of characterization is more appropriate in different fields, in particular, for biological systems, in order to uncover a SR effect. By plotting the histogram of the time periods spent by the system in one state, between two jumps, one obtains a decreasing background due to random jumps (in general, with a negative exponential behavior; see [6]), superimposed to a peak structure due to the modulation. The first peak corresponds to about half the modulation period $T_{\mathrm{mod}} = 2\pi/\omega$, while the other peaks are separated by T_{mod}.

If the noise is weak, several peaks are observed: The system stands for some oscillation periods before jumping. The synchronization is then revealed by the growth of the first peak, while for higher noise the background masks the modulation and thus the peak structure. This behavior is well described in Fig. 3, where we report the normalized histograms of the time periods spent in the lower level, for different values of the noise strength. The SR is clearly observed by plotting the area P_1 of the first peak as a function of the input noise, as shown in the inset.

This way of monitoring SR is not trivial. As recently observed in [10], there is a region of D (around the resonance) where the background underlying the peaks can lead to an erroneous evaluation of P_1. In fact, the intensity of the background at $T_{\mathrm{mod}}/2$ displays a resonant behaviour vs D also without modulation. Thus, all of the previous theoretical predictions should be reconsidered.

To avoid this problem, we subtract the background obtained from the fitting of the RTPDs. The function used in such a procedure is an exponential decay plus a Gaussian peak. P_1 is defined as the area of the Gaussian.

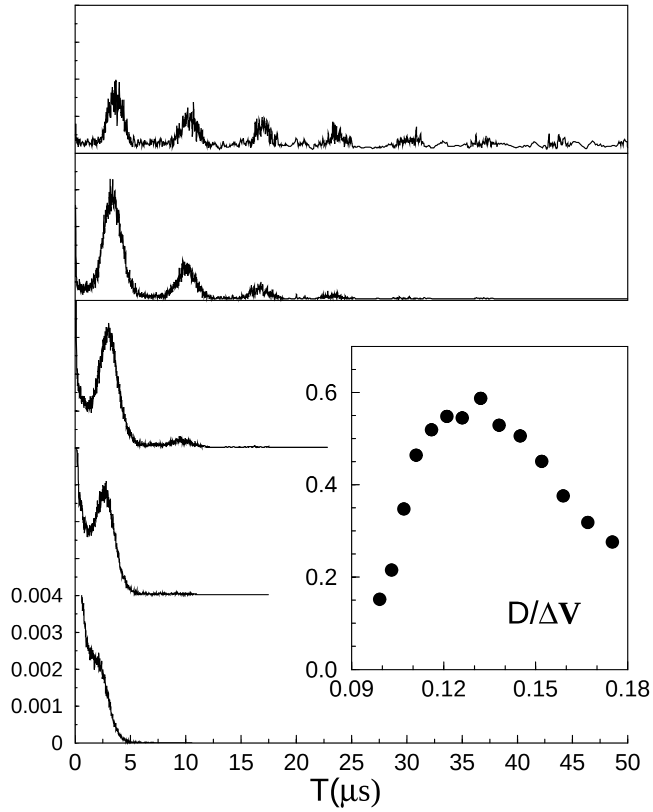

FIG. 3. Normalized histograms of the time spent by the system in the lower level, between two successive jumps, with an input modulation at $\omega/2\pi = 150$ kHz. From top to bottom, $D/\Delta V = 0.10$, 0.12, 0.14, 0.15, and 0.17. In the inset the first peak area P_1 is plotted as a function of the input noise. Before the calculation, the background has been subtracted (see text). The histograms are evaluated from a series of 2×10^7 data points, sampling at 10^8 samples/s.

In this way, we can show in Fig. 3 the first unquestionable experimental evidence of SR in the RTPD.

An important, recently debated, point is whether in such a system a true resonance can exist (i.e., as a function of the modulation frequency). This feature of SR (*bona fide* resonance), proposed by Gammaitoni *et al.* [11], is of particular relevance for the investigation of SR in real systems, as the analysis of the frequency response is, in general, straightforward.

While q_0 is a monotonous function of ω, a different behavior can be deduced from the analysis of the RTPDs, which is reported in Fig. 4 for different values of the modulation frequency. The histograms show an oscillatory behavior with an exponentially decaying peak amplitude, as predicted in [9,12]. According to Ref. [11], a resonance is found by plotting the area under the peaks as a function of frequency, denoting the presence of SR. This interpretation is, however, criticized by Choi *et al.* [10], who observe that such an indicator leads to ambiguous results due to the contribution of the background,

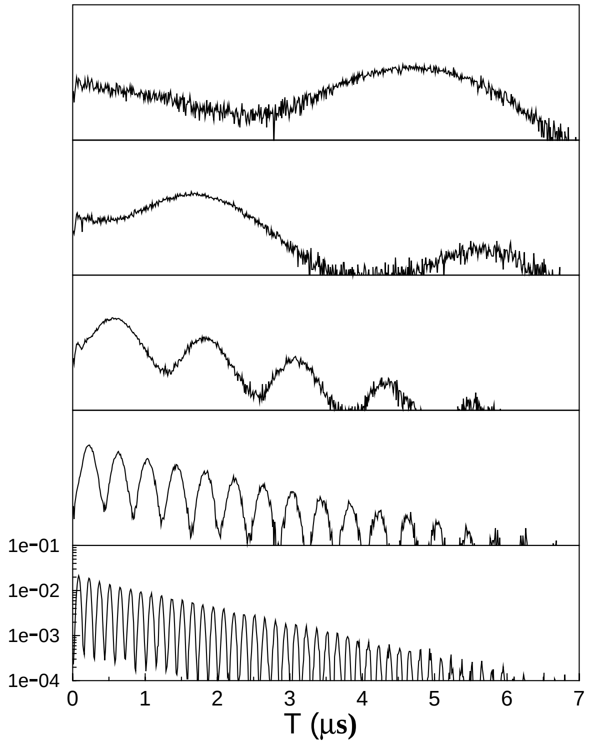

FIG. 4. The same as in Fig. 3 but with a fixed input noise ($D/\Delta V = 0.11$) and for different modulation frequencies. From top to bottom, $\omega/2\pi = 100$, 250, 800, 2500, and 7000 kHz.

as already explained. They instead suggest, as a quantifier of SR based on the RTPD, the difference between the height of the first peak and that of the background. This procedure would yield a monotonous function of ω. Their conclusion was that the claim to detect SR as a function of ω appears questionable.

However, we experimentally demonstrate here that the *bona fide* resonance is found by monitoring P_1, as shown in Fig. 5, even if the height of the peak is monotonous. The fitting procedure for the background subtraction allows an unambiguous interpretation of the measurements.

We want to stress that the indicator P_1 has a deeper physical meaning than the height of the peak. Indeed, the latter indicator leads to results which depend on the width of the bins chosen for the RTPD. Thus, the use of P_1 is not only correct, but even preferable. Therefore, the evidence of *bona fide* resonance that we report is effective, in disagreement with the conclusions of Choi *et al.*

In conclusion, we have reported the observation of SR in the polarized emission of VCSELs. Our system opens the way to an unprecedented experimental analysis of SR over a large range of parameters; this will stimulate further work on different subjects, such as the effect of the shape of the quasipotential and of different upper- and lower-level Kramers' times, or the introduction of colored noise. Thanks to a correct definition of the indicator, a complete experimental analysis of SR in the residence times probability density, showing also the evidence of *bona fide* resonance, is reported for the first time.

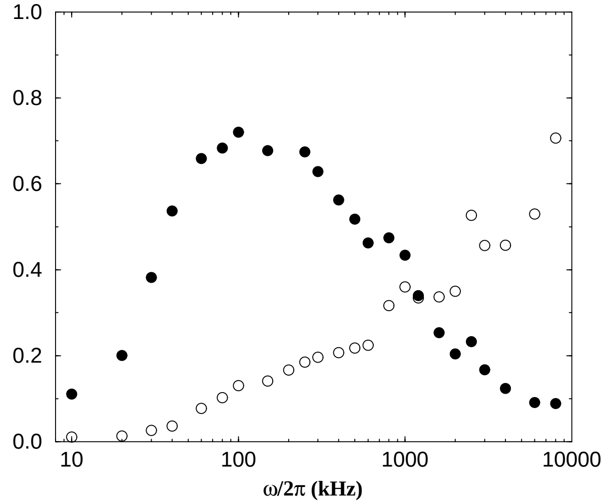

FIG. 5. Area P_1 (filled dots) and amplitude (empty dots) of the first peak, as a function of $\omega/2\pi$. Both of the indicators are evaluated after subtraction of the background (see text).

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