Composite-Fermion Edge States in Fractional Quantum Hall Systems

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We describe the edge states of fractional quantum Hall systems with alternating compressible and incompressible strips using a composite-fermion picture. The current carried by composite fermions in a compressible region depends on the difference between the electron filling factors in the two adjacent incompressible regions, consistent with the results of the interacting-electron picture given by Beenakker [Phys. Rev. Lett. **64**, 216 (1990)] and tested by recent experiments. This result allows the application of the Landauer-Büttiker formula for the composite fermion transport. [S0031-9007(98)08210-6]

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In the two-dimensional electron gas (2DEG) system, the concept of edge states combined with the Landauer-Büttiker formula has been very useful in describing the magnetotransport behavior of integer quantum Hall systems [1]. Several theoretical attempts have been made to extend the edge state picture to explain the results of transport experiments [2,3] in the fractional quantum Hall regime; for a slowly varying confinement potential, the formation of alternating compressible and incompressible strips was suggested using an interacting-electron picture [4,5].

The composite-fermion (CF) theory [6] has been successful in explaining the fractional quantum Hall effect and leads to the phenomenological similarity between the integer and fractional quantum Hall effects. The composite fermions result from a singular gauge transformation [7,8] and consist of electrons bound to an even number of fictitious magnetic flux quanta. From its successful description in a bulk region, it is natural to extend the CF approach to the edge states in the fractional quantum Hall regime [9,10]. In previous work, the CF energy levels near edges were calculated using the Hartree approximation [9]. Including the many-body effect such as the divergence of the CF effective mass m^*_{CF} [8,11] near the filling factor $\nu = 1/m$ with even number m, other theoretical work proposed that the propagating direction of the CF edge states is the same as that of electron drift motions in a slowly varying confinement potential [10]. However, to our knowledge, the CF approach in describing the electron transport near edges has not been well established and not compared with the interacting-electron picture. For example, for a system with incompressible regions having the filling factors $\nu_b = 2/5$ and $\nu_e = 1/3$, as shown in Fig. 1, one may expect that the CF edge states in compressible regions carry the same current, because in each compressible region one CF energy level intersects with the effective CF chemical potential μ_{eff}, in analogy with the edge state theory in the integer quantum Hall regime. However, this argument contradicts the results of the interacting-electron approach [4]. Thus, it is not clear how the edge channels are defined within the CF approach and whether the Landauer-Büttiker formula [1] can be similarly used for the CF systems.

In this paper, we investigate the nature of CF edge states in the fractional quantum Hall system which consists of alternating compressible and incompressible strips. We demonstrate the importance of the effective CF potential, which varies with current distribution, in determining the change of the effective CF chemical potential $\Delta\mu_{\mathrm{eff}}$ in each incompressible region due to a small variation of the electron chemical potential $\Delta\mu$, where μ_{eff} is the energy cost to add one CF into the system. We find the change of current carried by CF's in a compressible strip to be $\Delta I = (-e/h)\Delta\nu\Delta\mu$, where $\Delta\nu$ is the difference of the electron filling factors between the two adjacent incompressible regions. This result is consistent with previous theoretical work [4] and recent experimental data [2,3]. Thus, we suggest that the CF edge channels are well defined, so that the Landauer-Büttiker formula can be applied for CF systems within our approach. We also discuss the scattering rate

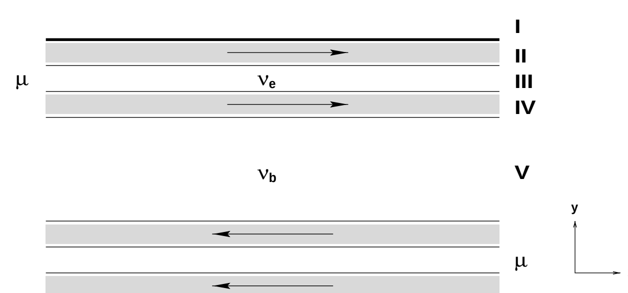

FIG. 1. A schematic diagram for a two-dimensional conductor connected to the left and right reservoirs with the chemical potential μ. Compressible (shaded) and incompressible (white) regions are labeled. The arrows indicate additional current flows induced by $\Delta\mu$.

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between the CF edge states in different compressible regions.

For an ideal 2DEG system, which is connected to two electron reservoirs of an electron chemical potential μ, electrons are confined to move in the xy plane in the presence of an external uniform magnetic field $\vec{B} = B\hat{z}$ and a slowly varying electrostatic confinement potential $U(y)$, as shown in Fig. 1. The electron filling factor in the incompressible bulk region V is $\nu_b = p_b/(m_b p_b + 1)$, where p_b is the CF filling factor and m_b is the number of flux quanta bound to each electron. Although there may be many incompressible strips near edges, we focus on a simple case with one incompressible region III sandwiched between two compressible regions II and IV. This incompressible strip has the filling factor $\nu_e = p_e/(m_e p_e + 1) < \nu_b$, where p_e and m_e are the CF filling factor and the number of flux quanta, respectively. Because of the excitation energy gap [4,5], the electron densities n_{e} in the incompressible regions III and V are uniformly distributed, while in the compressible strips, they decrease as going to the region I, where $\nu = 0$ or $p = 0$. The effective CF chemical potential in an incompressible region R_i is denoted by μ_{eff,R_i}, where $R_i \in \{\mathrm{I}, \mathrm{III}, \mathrm{V}\}$; however, μ_{eff,R_i}'s will be shown to be the same over the incompressible regions.

We first consider the case of $m_e = m_b = m$. In the mean field theory, the effective electric and magnetic fields which interact with CF's are written as

$$\vec{E}_{\mathrm{eff}} = \nabla_r U/e + \langle \vec{v} \rangle \times m\phi_0 n_{\mathrm{e}} \hat{z}\,, \tag{1}$$

$$\vec{B}_{\mathrm{eff}} = \vec{B} - m\phi_0 n_{\mathrm{e}} \hat{z}\,, \tag{2}$$

where e is the absolute value of electron charge, $\phi_0 = h/e$ is the magnetic flux quantum, and $\langle \vec{v} \rangle$ is the average drift velocity. The second terms in Eqs. (1) and (2) represent the fields induced by the magnetic flux bound to each CF [12]. From Eq. (1), we obtain the effective CF potential U_{eff} such as

$$U_{\mathrm{eff}}(y) = U(y) + m\phi_0 \int^y dI\,, \tag{3}$$

where I denotes a current. In the incompressible regions, since B_{eff} is constant, the noninteracting CF energies can be written as [8]

$$E_p = (p + 1/2)\hbar e|B_{\mathrm{eff}}|/m^*_{\mathrm{CF}} + U_{\mathrm{eff}}\,, \tag{4}$$

where $p = 0, 1, \ldots$. In the compressible strips with nonuniform electron densities, although we need self-consistent calculations to obtain the exact CF energies, we may guess the same expression as that of Eq. (4) for a sufficiently slowly varying potential U. Depending on the signs of p_b and p_e, there are several possible energy configurations, as shown in Figs. 2(a)–2(c). If the sign of B_{eff} does not change in the compressible regions, the energy levels in the sandwiched incompressible region will be smoothly connected to those in the neighboring regions [see Fig. 2(a)]. However, if there exists locally a position with a filling factor $\nu(y_{1/m})[= n_{\mathrm{e}}(y_{1/m})\phi_0/B] = 1/m$ in

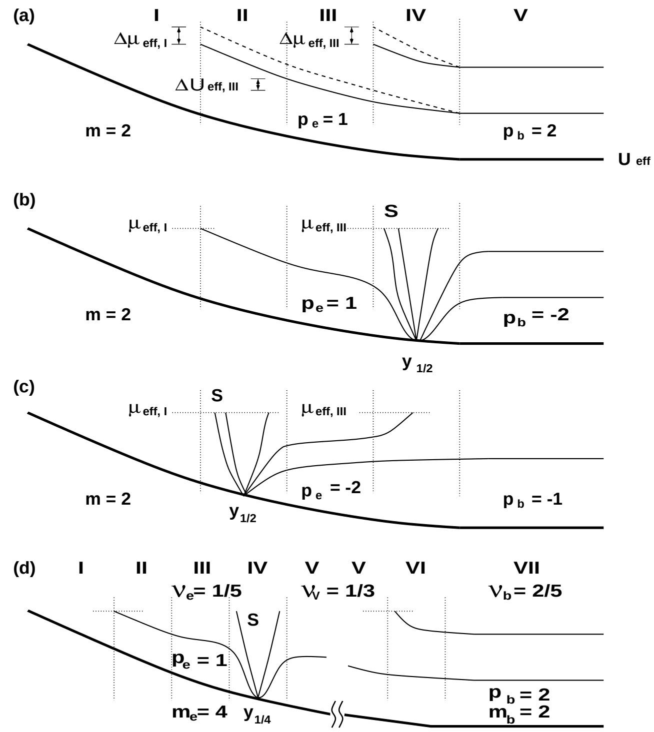

FIG. 2. Schematic diagrams for the CF energy levels (solid lines) for $m_b = m_e = 2$; (a) $\nu_b = 2/5$, $\nu_e = 1/3$, (b) $\nu_b = 2/3$, $\nu_e = 1/3$, and (c) $\nu_b = 1$, $\nu_e = 2/3$. The labeled regions are the same as those in Fig. 1. The heavy lines indicate the effective CF potentials and the dashed lines in (a) represent the energy levels changed by ΔU_{eff}. The energy levels for $m_b \neq m_e$ are drawn in (d).

a compressible strip, the mean field approximation is no longer valid near $y_{1/m}$ because B_{eff} is very small and the gauge fluctuation cannot be ignored. This situation leads to the strongly diverging m^*_{CF} [8,11]; then, the energy levels near $y_{1/m}$ can be expressed as $E_p(y) = U_{\mathrm{eff}}(y)$ [see region IV in Fig. 2(b)] [10]. We address that although the exact expressions for the energy levels and U_{eff} are unknown, our following theory is independent of the exact form. In analogy with the previous theory [10], for the energy levels in Fig. 2, the direction of CF current is the same as that of electron current, consistent with recent edge-magnetoplasmon experiments [13]. Notice that $n_{\mathrm{e}}(y_{1/m})[= B/(m\phi_0)]$ is fixed for a given magnetic field B, so that $dU_{\mathrm{eff}}(y_{1/m})$ is related to $d\mu_{\mathrm{eff}}$ such as

$$dU_{\mathrm{eff}}(y_{1/m}) = d\mu_{\mathrm{eff}}\,, \tag{5}$$

because $(\mu_{\mathrm{eff}} - U_{\mathrm{eff}})$ is proportional to n_{e} at $y_{1/m}$ [10]. Then, the so-called *silent modes* marked as S in Fig. 2, which start from $y_{1/m}$ and intersect with μ_{eff} in the compressible region, do not contribute to the current change [10].

If the chemical potential of the left reservoir is disturbed by $\Delta\mu$, the effective CF chemical potential μ_{eff} and the current I in each region will also be changed by $\Delta\mu_{\mathrm{eff}}$ and ΔI, respectively. However, for a sufficiently small $\Delta\mu$, $\Delta I = 0$ in the incompressible regions, because of the uniform electron densities due to the energy gap $\hbar e|B_{\rm eff}|/m_{\rm CF}^*$. Then, from Eq. (3), $\Delta U_{{\rm eff},R_i}$ in an incompressible region R_i is simply written as

$$\Delta U_{{\rm eff},R_i} = \phi_0 \sum_{R'=R_i}^{\rm V} m\Delta I_{R'}\,, \qquad (6)$$

i.e., $\Delta U_{\rm eff,I} = \phi_0 m(\Delta I_{\rm II} + \Delta I_{\rm IV})$ and $\Delta U_{\rm eff,III} = \phi_0 m \Delta I_{\rm IV}$, and the energy levels E_p in this region are shifted by a constant value $\Delta U_{{\rm eff},R_i}$, as shown in Fig. 2(a). Because the energy shift $\Delta U_{{\rm eff},R_i}$ also changes the effective chemical potential, $\Delta\mu_{{\rm eff},R_i}$ satisfies the relation,

$$\Delta\mu_{{\rm eff},R_i} = \frac{e^*_{R_i}}{-e}\,\Delta\mu + \Delta U_{{\rm eff},R_i} \qquad (7)$$

for $R_i \in \{\rm I, III\}$, where $e^*_{R_i}$ is the local CF charge, i.e., $e^*_{\rm I} = -e$ and $e^*_{\rm III} = -e/(m_e p_e + 1)$. In this case, the first term results from the fact that $|mp + 1|$ CF's with a local charge e^* are excited in the incompressible region with $\nu = p/(mp + 1)$, when one electron is added to that region [8,12]. From the continuity of the chemical potential on the boundaries between the compressible and incompressible regions, we find that for $p_b > 0$ and $p_e > 0$, p_e energy levels intersect with $\mu_{\rm eff,I}$ on the edge of region II, while $(p_b - p_e)$ levels intersect with $\mu_{\rm eff,III}$ in the case of region IV [see Fig. 2(a)]. These levels give rise to the current changes $\Delta I_{\rm II}$ and $\Delta I_{\rm IV}$, respectively. The remaining p_e levels below the effective chemical potential in region IV contribute to both $\Delta I_{\rm II}$ and $\Delta I_{\rm IV}$, because of the energy shift $\Delta U_{\rm eff,III}$. As a consequence, the total current changes in regions II and IV, which are derived from $I_p = -(e/h)\int dE_p$, where I_p is the current associated with the energy level p [1], are self-consistently related to $\Delta\mu_{\rm eff}$ and $\Delta U_{\rm eff}$,

$$\Delta I_{\rm II} = -\frac{e}{h}\,(p_e \Delta\mu_{\rm eff,I} - p_e \Delta U_{\rm eff,III})\,, \qquad (8)$$

$$\Delta I_{\rm IV} = -\frac{e}{h}\,[(p_b - p_e)\Delta\mu_{\rm eff,III} + p_e \Delta U_{\rm eff,III}]\,. \qquad (9)$$

Here we point out that the charge carried by each CF in the compressible regions is $-e$, because there is no excitation gap in these regions [4]. From Eqs. (6)–(9), we find that all the incompressible regions have the same change of $\mu_{\rm eff}$,

$$\Delta\mu_{\rm eff} = \frac{\Delta\mu}{m_b p_b + 1}\,, \qquad (10)$$

and the current change ΔI_{R_c} in a compressible region R_c is

$$\Delta I_{R_c} = -\frac{e}{h}\,\Delta\nu_{R_c}\Delta\mu\,, \qquad (11)$$

where $\Delta\nu_{R_c}$ is the difference of the filling factors between the two neighboring incompressible regions; $\Delta\nu_{\rm II} = \nu_e$ and $\Delta\nu_{\rm IV} = \nu_b - \nu_e$. The relations in Eqs. (10) and (11) are satisfied for all possible combinations of p_b and p_e as well as for systems with many alternating incompressible and compressible strips near the edge [14]. For example, if $p_b < 0$ and $p_e > 0$ [see Fig. 2(b)], the first term in Eq. (9) will be changed to $(e/h)(|p_b| + p_e)\Delta\mu_{\rm eff,III}$, because the silent modes do not contribute to the current change and the $(|p_b| + p_e)$ occupied levels are shifted by $\Delta U_{\rm eff}(y_{1/m})$ in Eq. (5), resulting in the same expression as in Eq. (9). From Eq. (10), we note that $\Delta\mu_{\rm eff}$ depends only on m_b and p_b in the bulk region and is the same in all the incompressible regions, although the CF energy levels are shifted by different values. This feature leads to the result that $\mu_{\rm eff}$'s are the same over the incompressible regions under an equilibrium condition.

For $m_b \neq m_e$, we consider a simple case of $\nu_b = 2/5$ and $\nu_e = 1/5$, introducing an additional stable incompressible strip of $\nu_{\rm V} = 1/3$ between the two incompressible regions, as shown in Fig. 2(d). Then, region V can be described either in terms of noninteracting CF's with $m = 2$ or CF's with $m = 4$. If we choose noninteracting CF's with $m = 2$ $(= m_b)$, we can easily calculate $\Delta\mu_{\rm eff,V}$ and $\Delta I_{\rm VI}$ for regions with $\nu \geq 1/3$ [see the right part of Fig. 2(d)], which turn out to be the same as those in Eqs. (10) and (11). However, it is not easy to calculate $\Delta I_{\rm II}$ and $\Delta I_{\rm IV}$, because region III with $\nu = 1/5$ cannot be described by noninteracting CF's with m_b. If noninteracting CF's with $m = 4$ $(= m_e)$ are chosen over the whole region, the regions with $\nu \leq 1/3$, i.e., the left part of Fig. 2(d), can be described by these CF's. In this case, since $\Delta I_{\rm VI}$ is carried out by CF's with $m = m_e$, we can obtain the relation from Eq. (7), $\Delta\mu_{{\rm eff},R_i} = \Delta\mu/(m_e p_b' + 1)$, where $\nu_b = p_b'/(m_e p_b' + 1)$ and $R_i \in \{\rm I, III, V\}$ [14]. This relation verifies the dependence of $\Delta\mu_{\rm eff}$ on m in Eq. (10), and the fractional number p_b' indicates that the bulk region VII requires interacting CF's with m_e. The current changes obtained for the compressible regions II and IV also satisfy the relation in Eq. (11). For all the cases, we find that the current change ΔI_{R_c} in a compressible region R_c satisfies the relation in Eq. (11), consistent with the interacting electron picture by Beenakker [4]; ΔI_{R_c} does not depend on the CF filling factor and the number of flux quanta carried by each CF, but the filling factor difference $\Delta\nu_{R_c}$. Thus, the CF edge channels can be defined as the CF edge states in the compressible regions, and the resulting total current change caused by $\Delta\mu$ is expressed as $(-e/h)\nu_b\Delta\mu$. Furthermore, generalizing the Landauer-Büttiker formula [1], we can write from Eq. (11) the total current change such as $\Delta I = \sum_{R_c} \Delta I_{R_c} T_{R_c}$, assuming that a fraction T_{R_c} of the current change ΔI_{R_c} induced by one reservoir is transmitted to the other. Then, the conductance satisfies the relation $G = (e^2/h)\sum_{R_c} \Delta\nu_{R_c} T_{R_c}$. Our Landauer-Büttiker formula for CF's is more general than that in previous work [9,10], because it provides a simpler and clearer description for complex systems with alternating compressible and incompressible strips. For example, we find that $T_{R_c} = 0$ in the $\nu > \nu_e$ regions in the adiabatic transport regime, while $T_{R_c} = 1$ in the $\nu < \nu_e$ regions, if there exists a ν_e region across a segment of a narrow ν_b 2DEG due to a gate voltage. In this case, the conductance has the simple relation $G = (e^2/h)\nu_e$, in good agreement with experiments [2]. We suggest that recent experimental results for the selective population of edge states [3] can be understood by introducing channel mixings between I_{R_c}'s. This description is different from the previous CF edge theory [10], in which the channel mixings occur between the silent and nonsilent modes. In the noninteracting-electron description, the current change is contributed from the energy levels intersecting with μ; then, ΔI_{R_c} in a compressible region R_c considered here could be written as $\Delta I_{R_c} = (-e/h)n\Delta\mu_{\mathrm{eff}}$, where n is the number of CF energy levels intersecting with μ_{eff}. However, we address that ΔI_{R_c} is contributed from all the energy levels below μ_{eff}, as shown in Eqs. (8) and (9). Thus, the properties of CF edge channels are different from those for noninteracting-electron edge channels. We emphasize that U_{eff} plays an important role in determining μ_{eff} and the current change within the CF edge theory, because U_{eff} and the CF energy levels depend on the distribution of edge currents which flow along compressible strips.

Finally, we suggest that our approach can be used to explain recent experimental results [15] for the scattering rates between resolved fractional edge states. To obtain the scattering rate between the edge states in two neighboring compressible regions R_1 and R_2, we assume that the observed edge states correspond to our CF edge states in compressible regions. If a simple double-well structure is considered for R_1 and R_2, the CF edge states up to μ_{eff} will be filled in each well. The incompressible region R_i with a filling factor ν, which is sandwiched between R_1 and R_2, behaves as an energy barrier of width d_ν and height $V_{R_i}[= (p + 1/2)\hbar e|B_{\mathrm{eff}}|/m^*_{\mathrm{CF}} + U_{\mathrm{eff}}]$, where $V_{R_i} > \mu_{\mathrm{eff}}$ and p is the CF filling factor in this region. The scatterings between the CF edge states in R_1 and R_2 occur only for those near μ_{eff}. Using the WKB approximation, the transmission rate W_ν through the barrier R_i is calculated to be $W_\nu \sim \exp[-d_\nu f(V_{R_\nu}, \mu_{\mathrm{eff}})]$. Although we need the CF energy levels near the edge to obtain d_ν and V_i, we expect that $W_{\nu=1/3}$ is much lower than $W_{\nu=1/5}$ and $W_{\nu=2/5}$ for a sufficiently slowly varying U, because $d_{\nu=1/3}$ is greater than both $d_{\nu=1/5}$ and $d_{\nu=2/5}$ from the hierarchy behavior in the CF approach [6]. This result is consistent with the experimental results for scattering rates [15].

In conclusion we have investigated the edge-state structure in the fractional quantum Hall system using the CF theory. We have derived the relation between the changes of the effective CF chemical potential and the electron chemical potential for all the incompressible regions. In contrast to the noninteracting-electron edge states, we find that the current in a compressible region is not directly proportional to the difference of CF filling factors between the neighboring incompressible regions, which is attributed to the fact that the effective CF potential depends on the distribution of currents near the edge. Our results for the transport properties of the CF edge states and the generalized Landauer-Büttiker formula are consistent with those of the interacting-electron picture by Beenakker [4]. We suggest that the present approach is also applicable for the edge states formed by nonuniform external magnetic fields [16]. Details of the results will be published elsewhere [14].

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