Unusual Doppler Effect in the *B* Phase of Superfluid ^{3}He

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We calculate the unusual Doppler effect in the *B* phase of superfluid ^{3}He in the Ginzburg-Landau regime and at low temperatures. We analyze the nontrivial dependence of the sound velocities and of the internal Doppler-shift coefficients on the superfluid velocity in the Ginzburg-Landau regime. The value of the fourth-sound Doppler-shift coefficient is most remarkable: it exceeds 2.2 (the intuitive order of magnitude is 1). [S0031-9007(98)07981-2]

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The unusual Doppler shift in superfluid He was predicted by Khalatnikov [1]. Further investigation of the phenomenon in ^{4}He and in the ^{4}He-^{3}He mixture was made in Refs. [2–5]. Experimental measurements were made by Rudnick *et al.* [6] and Kojima *et al.* [7,8].

In this work we investigate the Doppler phenomenon in the *B* phase of superfluid ^{3}He both in the Ginzburg-Landau (G-L) regime and at low temperatures. The G-L temperature range is more interesting because in this regime ρ_s (superfluid density) is a nontrivial function of relative velocity w ($\vec{w} \equiv \vec{v}_n - \vec{v}_s$, where v_s and v_n are superfluid and normal fluid velocities, respectively) [9,10]

$$\rho_s^T \equiv \rho_s^T(T, v_s) = \rho_s(T)(1 - \beta w^2), \qquad \beta \equiv \frac{b^2}{1 - \frac{T}{T_c}}, \qquad \frac{1}{b} \approx 13.2 \text{ cm/sec}. \tag{1}$$

The dependence of the superfluid density on the relative velocity w is also present in ^{4}He, but it can be, as a rule, neglected there. Critical velocities (v_c) cannot be approached in ^{4}He because of the vorticity phenomenon [11]. In ^{3}He, especially in the G-L regime, v_c is rather small (<1 cm/sec), and superfluid velocity should be taken into account.

The quantitative description of the Doppler effect for the ith sound is made by means of Doppler coefficients γ_i and parameters Γ_i [4,5]. The γ_i are defined by (henceforth all velocities are in one direction)

$$\Delta u_i - v = \gamma_i w, \qquad \vec{v} = \frac{1}{\rho}(\rho_s \vec{v}_s + \rho_n \vec{v}_n), \tag{2}$$

where v_n and u_i are, respectively, the normal component and the sound velocity in the laboratory frame, and Δu_i is the velocity of the center of spreading ith sound sphere [2–5]. The definition of Γ_i depends on which component (normal or superfluid) is dominant in the structure of the particular sound for this particular temperature. The normal component is dominant for the first sound at T close to T_c and the second sound at $T \ll T_c$. The superfluid component is dominant in the structure of the second and fourth sounds for T close to T_c and the first and fourth sounds at $T \ll T_c$. Thus for $v = 0$ one can write

$$\begin{aligned} &\Delta u_i = \Gamma_i v_n, \qquad \Gamma_i = \gamma_i \Big/ \frac{\rho_s}{\rho} \\ &\quad (i = 1 \quad \text{at } T \approx T_c \quad \text{or} \quad i = 2 \quad \text{at } T \ll T_c), \\ &\Delta u_i = \Gamma_i v_s, \qquad \Gamma_i = -\gamma_i \Big/ \frac{\rho_n}{\rho} \\ &\quad (i = 2,4 \quad \text{at } T \approx T_c \quad \text{or} \quad i = 1,4 \quad \text{at } T \ll T_c). \end{aligned} \tag{3}$$

Γ_i describes deviation of the center of spreading sound velocity Δu from its zero value in the rest frame of the liquid. For the classical liquid $\Gamma_i \equiv 0$. The Doppler effect is termed "normal" if Γ_i is in the range $0 \le \Gamma_i \le 1$. The case $\Gamma_i < 0$ when the direction of Δu is opposite to the direction of the dominant component is called backentrainment effect (BEF). The case $\Gamma_i > 1$ in which Δu is Γ_i times larger than the velocity of the dominant component is called the outstripping effect (OEF) [4].

The basic two-fluid hydrodynamics equations are

$$\begin{aligned} &\dot{\rho} + \nabla \vec{j} = 0, \qquad \vec{j} = \rho_n \vec{v}_n + \rho_s \vec{v}_s, \\ &\frac{\partial(\rho\sigma)}{\partial t} + \nabla(\rho\sigma\vec{v}_n) = 0, \\ &\dot{j}_i + \nabla_k(\rho v_{ni} v_{nk} + \rho_s v_{si} v_{sk} + P\delta_{ik}) = 0, \\ &\dot{\vec{v}}_s + \nabla\left(\mu - \frac{1}{2} v_s^2\right) = 0, \\ &d\mu = \frac{1}{\rho} dP - \sigma dT - \frac{\rho_n}{\rho}(v_n - v_s) d(v_n - v_s) \end{aligned} \tag{4}$$

(where σ is the specific entropy, P the pressure, and μ the chemical potential). The condition for the existence of solutions in the linear approximation (see [3,4,12]) gives the velocities of the first and the second sounds in the following form ($\vec{U} \equiv \vec{u} - \vec{v}$):

$$U_{1,2} = U_{1,2}^0 + \Delta U = U_{1,2}^0 + \gamma_{1,2} w, \tag{5}$$

where

$$(U_1^0)^2 = \frac{\partial P}{\partial \rho}, \qquad (U_2^0)^2 = \frac{\rho_s \sigma^2}{\rho_n \frac{\partial \sigma}{\partial T}} \frac{(1 - 3\beta w^2)}{(1 + 3\frac{\rho_s}{\rho_n}\beta w^2)}, \tag{6}$$

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and

$$\gamma_2 = \frac{\left(2\,\frac{\rho_s^T}{\rho} + \frac{\rho_s^2}{\rho\rho_n}\,\beta w^2(3 - 3.5\beta w^2)\right) + \frac{\rho}{\rho_n}\,\frac{\partial T}{\partial \sigma}\,\sigma\,\frac{\partial(\rho_s^T/\rho)}{\partial T}}{1 + 3\,\frac{\rho_s}{\rho_n}\,\beta w^2},$$

$$\begin{aligned}\gamma_1 = {} & \frac{\frac{1}{\rho^2}\left(\frac{\partial\rho}{\partial T}\right)^2}{\frac{\partial\rho}{\partial P}}\,\frac{\left(2\,\frac{\rho_s^T}{\rho} + \frac{\rho_s^2}{\rho\rho_n}\,\beta w^2(3 - 3.5\beta w^2)\right)\frac{\partial\sigma}{\partial T} + 2\,\frac{\rho}{\rho_n}\,\sigma\,\frac{\partial(\rho_s^T/\rho)}{\partial T}}{\left(\frac{\partial\sigma}{\partial T} - \frac{\rho_s}{\rho_n}\,\sigma^2\,\frac{\partial\rho}{\partial P}\,\frac{(1 - 3\beta w^2)}{(1 + 3\frac{\rho_s}{\rho_n}\beta w^2)}\right)^2\left(1 + 3\,\frac{\rho_s}{\rho_n}\,\beta w^2\right)} \\ & - \frac{\left(2\,\frac{\rho_s^T}{\rho} + \frac{\rho_s^2}{\rho\rho_n}\,\beta w^2(3 - 3.5\beta w^2)\right)\frac{1}{\rho^2}\left(\frac{\partial\rho}{\partial T}\right)^2\frac{\partial P}{\partial\rho} + \frac{\rho}{\rho_n}\,\sigma\,\frac{\partial P}{\partial\rho}\,\frac{\partial\rho}{\partial T}\,\frac{\partial(\rho_s^T/\rho)}{\partial P}}{\frac{\partial\sigma}{\partial T}\left(1 + 3\,\frac{\rho_s}{\rho_n}\,\beta w^2\right) - \frac{\rho_s}{\rho_n}\,\sigma^2\,\frac{\partial\rho}{\partial P}\,(1 - 3\beta w^2)} \\ & + \frac{\frac{\sigma}{\rho^2}\,\frac{\partial\rho}{\partial T}\left(4\rho_s - 4\rho_s\beta w^2 + \frac{\rho_s^2}{\rho_n}\,\beta w^2(3 - 4.5\beta w^2)\right)}{\frac{\partial\sigma}{\partial T}\left(1 + 3\,\frac{\rho_s}{\rho_n}\,\beta w^2\right) - \frac{\rho_s}{\rho_n}\,\sigma^2\,\frac{\partial\rho}{\partial P}\,(1 - 3\beta w^2)}\,. \end{aligned} \tag{7}$$

For the first and second sounds calculations are made in the rest frame of the liquid. Under fourth sound conditions ($v_n \equiv 0$) it is more convenient to perform calculations in the laboratory frame. For that reason the symbol u_4 is used instead of U_4,

$$\begin{aligned} (u_4^0)^2 &= \frac{\rho_s}{\rho}\left(\frac{\partial P}{\partial\rho} + \sigma^2\,\frac{\partial T}{\partial\sigma}\right)(1 - 3\beta v_s^2)\,, \\ -\gamma_4 &= \frac{\rho_s^T}{\rho} + \frac{\partial(\rho_s^T/\rho)}{\partial P}\,\rho\,\frac{\partial P}{\partial\rho} - \frac{\partial(\rho_s^T/\rho)}{\partial T}\left(\sigma\,\frac{\partial T}{\partial\sigma} + \frac{1}{\rho}\,\frac{\partial T}{\partial\sigma}\,\frac{\partial P}{\partial\rho}\,\frac{\partial\rho}{\partial T}\right). \end{aligned} \tag{8}$$

Using the expression for the free energy [9]

$$F = F_0 - \frac{3}{2}\left(\frac{\rho}{m}\right)^{1/3}[C_2T^2 + C_3'(T_c - T)^2]V + \frac{3}{5}\,\rho\left(1 - \frac{T}{T_c}\right)\frac{m}{m^*}\,v_s^2(1 - \eta v_s^2)V\,, \tag{9}$$

where

$$\begin{aligned} C_2 &= \frac{1}{3}\left(\frac{\pi}{3}\right)^{2/3}\frac{m^*}{\hbar^2}\,k_B^2\,, \\ C_3 &= \frac{m^*}{2(3\pi^2)^{2/3}\hbar^2}\left[\frac{8\pi^2}{7\zeta(3)}\right]k_B^2\,, \\ C_3' &= C_3\left(1 - \frac{\eta}{3}\,v_s^2\right), \qquad \eta \approx \frac{1}{(10.3\text{ cm/sec})^2}\left(1 - \frac{T}{T_c}\right), \end{aligned} \tag{10}$$

m^* is the effective mass, and V is the volume, one finds

$$\begin{aligned} \sigma &\approx \frac{3}{m}\left(\frac{m}{\rho}\right)^{2/3}\left[C_2T - C_3(T_c - T)\left(1 - \frac{\eta}{6}\,v_s^2\right)\right] - \frac{3}{5}\,\frac{1}{T_c}\,\frac{m}{m^*}\,v_s^2, \\ \frac{\partial\sigma}{\partial T} &\approx \frac{3}{m}\left(\frac{m}{\rho}\right)^{2/3}(C_2 + C_3)\,, \\ \frac{\partial\rho}{\partial T} &= -\frac{2}{c_1^2}\left(\frac{\rho}{m}\right)^{1/3}\left[C_2T - C_3(T_c - T)\left(1 - \frac{\eta}{6}\,v_s^2\right)\right] - \frac{3}{5}\,\frac{m}{m^*}\,\frac{\rho}{T_c}\,\frac{v_s^2}{c_1^2}\,. \end{aligned} \tag{11}$$

It is well known that

$$\frac{\partial P}{\partial\rho} \approx c_1^2, \tag{12}$$

where c_1 is the first-sound velocity.

The expressions for $\rho_s(T)$ and its derivatives in the Ginzburg-Landau regime are [9]

$$\begin{aligned} \rho_s &\approx 2\rho \frac{m}{m^*} \frac{T_c}{T} \left(\frac{T_c}{T} - 1\right)(1 - \beta w^2), \\ \frac{\partial(\rho_s^T/\rho)}{\partial T} &\approx -\frac{2}{T_c} \frac{m}{m^*} (1 - 2bw^2), \\ \frac{\partial(\rho_s^T/\rho)}{\partial P} &\approx \frac{\partial T_c}{\partial P} \frac{2}{T_c} \frac{m}{m^*} (1 - 2bw^2). \end{aligned} \tag{13}$$

From the experimental data [13] $\partial T_c/\partial P$ is found to be

$$\frac{\partial T_c}{\partial P} \approx 0.11 T_c \frac{1}{Bar} = 1.1 \times 10^{-6} T_c \frac{1}{Pa}. \tag{14}$$

Substituting Eqs. (11)–(14) into Eqs. (6)–(8) in the limit $T \to T_c$ one finds the expressions for the velocities

$$\begin{aligned} U_1 &= c_1, \\ U_2^2 &\approx \frac{\rho_s}{\rho} \sigma^2 \frac{\partial T}{\partial \sigma} (1 - 3\beta w^2) \approx \frac{C_2}{C_2 + C_3} \frac{6}{m^*} \left(\frac{m}{\rho}\right)^{2/3} C_2 T_c \left(\frac{T_c}{T} - 1\right)(1 - 3\beta w^2), \end{aligned} \tag{15}$$

$$u_4^2 \approx \frac{\rho_s}{\rho} \frac{\partial P}{\partial \rho} \sqrt{1 - 3\beta v_s^2} \approx 2 \frac{m}{m^*} \left(1 - \frac{T}{T_c}\right) \sqrt{1 - 3\beta v_s^2}\, c_1^2, \tag{16}$$

and for the Doppler coefficients

$$\begin{aligned} \gamma_1 &\approx -\left[\frac{8}{3\rho} \left(\frac{C_2}{C_2 + C_3}\right)^2 \frac{C_2 T^2}{c_1^2} + 4C_2 \frac{T}{T_c} \left(\frac{C_2}{C_2 + C_3}\right) \frac{\partial T_c}{\partial P}\right] \left(\frac{\rho}{m}\right)^{1/3} \frac{m}{m^*} (1 - 2bw^2) \\ &\approx -1.06 \times 10^{-7} (1 - 2bw^2), \\ \gamma_2 &= \frac{\rho}{\rho_n} \frac{\partial T}{\partial \sigma} \sigma \frac{\partial(\rho_s^T/\rho)}{\partial T} \approx -\frac{2C_2}{C_2 + C_3} \frac{m}{m^*} (1 - 2bw^2) \approx -0.30(1 - 2bw^2), \end{aligned} \tag{17}$$

$$-\gamma_4 \approx \left(\frac{2\rho}{T_c} \frac{\partial T_c}{\partial P} c_1^2 + \frac{2}{3} \frac{C_2}{C_2 + C_3}\right) \frac{m}{m^*} (1 - 2bv_s^2) \approx 2.24(1 - 2bv_s^2). \tag{18}$$

As can be seen from Eq. (17) there is a weak BEF for the first sound; the corresponding coefficient γ_1 is very small (of the order of -10^{-7}). The normal fluid component is the dominant one in the structure of the first sound near the critical temperature. The first sound propagates by means of density oscillations; hence the most important factor contributing to the corresponding Doppler coefficient has to do with the normal density oscillations. These oscillations are described by the quantity $\partial(\rho_n/\rho)/\partial P$ which appears in Eq. (17) in the form $\partial(\rho_s/\rho)/\partial P = -\partial(\rho_n/\rho)/\partial P$. Thus the main reason for the backentrainment effect is the negative "relative" compressibility of the dominant (normal) component $[\partial(\rho_n/\rho)/\partial P < 0]$.

The first (classical) sound velocity in our approximation does not change with temperature and relative velocity. The complete picture seems to be almost normal except for an anomaly, caused by the phase transition. γ_1 like all the other Doppler coefficients is proportional to the derivatives of the superfluid density which are the second derivatives of the free energy. Consequently there should be a jump in all Doppler coefficients at the critical temperature. For that reason, as $T \to T_c$, γ_1 remains finite and does not go over smoothly into the nonsuperfluid "classical" value: $\gamma_1 = 0$. $\Gamma_1 = \gamma_1 \rho/\rho_s$ diverges at T_c, but the effect itself is still very weak.

The Doppler effect for the second sound is normal, $\Gamma_2 \approx -\gamma_2 \approx 0.30$ [Eq. (3)] at $T \to T_c$. The velocity of the second sound U_2 depends very strongly on w [Eq. (15)]. U_2 vanishes as w approaches the pair-breaking critical velocity $v_{s,c} = (1/3^{3/2} b)[1 - (T/T_c)]^{1/2}$ [9] as it should, because at this velocity the superfluid phase can be considered as destroyed. U_2 also vanishes linearly with the temperature as $T \to T_c$.

There is a strong outstripping effect in the case of the fourth sound: $-\gamma_4 \approx \Gamma_4 \sim 2.2$. It means that the velocity of the center of the spreading sound exceeds by more than 2 times the velocity of the superfluid component, when the normal component is locked. γ_4 is in a sense the biggest Doppler coefficient for all the cases that have been examined till now. In some systems considered earlier [4,5] a big deviation from the normal effect was found, i.e., a large Doppler parameter Γ (up to 43.7 for Γ_1) in [5], but there $\gamma_1 \equiv \Gamma_1 \rho_s/\rho \ll 1$ because of the small superfluid density, and the smallness of γ_1 means the smallness of the Doppler shift Δu_1 in comparison with the relative velocity w [see Eq. (2)]. In our case $-\gamma_4 \sim 2.2$; hence this effect should be much easier to measure. In analogy to the case of BEF for the first sound the main cause of this OEF lies in the value of the relative compressibility of the dominant component, $\partial(\rho_s/\rho)/\partial P$, which is large in comparison with $(1/\rho)(\partial\rho/\partial P)$. Their ratio is the main contribution to γ_4 (the contribution of all the other terms is much weaker) and its numerical value is larger than two.

u_4 depends very strongly on the temperature and superfluid velocity ($v_s = -w$, because $v_n = 0$). Equations (15) and (16) confirm that U_2 and u_4 vanish as $v_s \to v_{s,c}$ for any temperature and at T_c for any velocity v_s.

For hydrodynamic theory to be valid two conditions should be fulfilled. The first condition is that the relaxation time (τ) has to be much smaller than the period of the oscillations ($2\pi/\omega$) which can be written as $\tau\omega \ll 1$. The second condition is that the mean free path has to be much shorter than the characteristic length of the system (the wavelength of sound or the size of the system). The calculations performed for the low temperature (smaller than $0.3T_c$) regime are valid only for a very large (more than a few meters wide) experimental cell. Such a setup can hardly be regarded as a realistic one. Consequently, the low temperature calculations can serve only as an estimate of the effect as it would be under the realistic conditions.

At low temperatures ($T \ll T_c$), for small enough velocities ($w \ll k_BT/p_F$), the w dependence of ρ_n can be neglected [14]; hence the expressions for the Doppler coefficients and parameters can be found from (6)–(8) by dropping the terms which include β.

The formulas for σ and the derivatives $\partial\sigma/\partial T$ and $\partial\rho/\partial T$ can be derived from the expression for the heat capacity [9].

It should be stressed that all thermodynamic functions in this regime are exponentially small. But, after the substitution into the expressions for the sound velocities and Doppler parameters, all the exponents cancel.

The second sound velocity vanishes linearly for $T \to 0$. However this result does not take into account the phonon contribution to the excitations. These begin to play an important role at temperatures lower than $T \approx 5.6 \cdot 10^{-5}$ K when $U_2 \approx 3\frac{\text{mm}}{\text{sec}}$ but can be neglected until this regime is reached.

The first sound Doppler coefficient Γ_1 is small and vanishes linearly as $T \to 0$ ($\Gamma_1 \approx 0.09T/T_c$). The Doppler effect is normal.

There is a small outstripping effect in the second sound which becomes normal linearly as $T \to 0$ ($\Gamma_2 \approx 1 + k_BT/2\Delta$).

There is a strong outstripping effect for the fourth sound: $\Gamma_4 \equiv -\gamma_4/(\rho_n/\rho) \approx 3.21\Delta/(k_BT)$. Although γ_4 is exponentially small, Γ_4 diverges linearly at $T \to 0$. Unfortunately the effect may be very difficult to measure, because Δu_4 is still exponentially small [in the rest frame of the liquid $v_s \sim \rho_n \sim \exp(-\frac{\Delta}{k_BT})$ (3)].

To sum up, the Doppler coefficients, various parameters, and sound velocities in the B phase of the superfluid ^{3}He both in the G-L regime and at low temperatures were calculated. There is a weak BEF for the first sound near T_c; the corresponding coefficient γ_1 is of the order of 10^{-7}. The Doppler effect for the first sound at low temperatures is found to be normal (i.e., its values are roughly the expected ones). For the second sound, the effect is almost normal (weak outstripping effect at $T \to 0$). However, for the fourth sound we get an outstanding OEF: $\Gamma_4 \approx -\gamma_4 \sim 2.2$ in the G-L regime; i.e., the Doppler shift is 2.2 greater than the expected value. $\Gamma_4 \to \infty$ as $T \to 0$, but $\gamma_4 \to 0$ at the same time. Without taking into account phonon corrections to the excitation spectrum, the second-sound velocity was found to vanish with $T \to 0$.

The results of the calculations in the G-L are amenable to experimental verification. Taking into account the realistic size of the experimental cell, the hydrodynamic approach can be used down to about $0.4T_c$. Below that temperature, in the low temperature regime, the relaxation time and the mean free path of the excitations become too large and the performed calculation can serve only as an estimate of the real values. Near $0.06T_c$ the phonon contribution to the excitations begins to play an important role. Below $0.05T_c$ one can neglect the fermion origin of the system and perform the calculations as for ^{4}He.

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