Measurement of the Plasma Density Using the Intensification of z-Mode Waves at the Electron Plasma Frequency

Iver H. Cairns*
Department of Physics and Astronomy, University of Iowa, Iowa City, Iowa 52242
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A new method is proposed for measuring the number density of a plasma using spectral measurements of plasma waves. The method is based on predictions that the ratio of electric to magnetic fields and the electric intensity is significantly enhanced in a narrow frequency band about the electron plasma frequency for z-mode waves propagating at small angles to the background magnetic field in cold relatively strongly magnetized plasmas. The method is applied to emissions observed in space over Earth's polar cap, allowing identification of the plasma frequency and the wave mode consistent with the data available. This method should be useful for other cold, strongly magnetized plasmas in space and in the laboratory. [S0031-9007(98)08182-4]

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The number density n of ionized particles is a fundamental and basic parameter of a plasma that is often difficult to measure directly. Common problems in space and laboratory applications are that particle detectors perturb the plasma or that at least one ion or electron component in the plasma is too cold or too extended in velocity space to be measured accurately [1–4]. Accordingly, it is often very attractive to measure the plasma density using the natural resonance frequencies of a plasma, i.e., using plasma waves [1–4]. Existing methods for measuring n using waves near the (angular) electron plasma frequency $\omega_p = (ne^2/m_e\epsilon_0)^{1/2} = 56\sqrt{n}\ \mathrm{s}^{-1}$, where n is measured in m^{-3}, are not always applicable. The goals of this Letter are threefold: First, to propose a new method for identifying ω_p using z-mode waves and standard measurements of electric and magnetic wave intensity spectra; second, to present strong theoretical arguments that the ratio of electric to magnetic wave fields and the electric intensity should increase in a narrow frequency bandwidth centered on ω_p for z-mode waves propagating at small to moderate angles to the magnetic field direction; third, to apply this method to a class of emissions observed in space above Earth's polar caps and to show that the resulting identifications of ω_p and the wave mode are consistent with the available data. The new method and theoretical wave properties are believed appropriate to cold, strongly magnetized plasmas in which ω_p is smaller than the electron cyclotron frequency Ω_{ce} and relevant plasma thermal speeds are much smaller than the speed of light c. This situation is typical of laboratory plasma devices and of many space plasmas, including the inner magnetospheres of Earth and Jupiter where the international spacecrafts Polar and Galileo are active, but not the solar wind and outer magnetospheres.

Magnetoionic theory provides a simple, generally well-known, theoretical description of waves in a cold magnetized plasma with frequencies greater than relevant ion cyclotron frequencies and with phase and group speeds that are much larger than relevant plasma thermal speeds [5–8]. Three of the four magnetoionic modes are widely familiar: the o and x modes correspond to radiation (light) in free space and the whistler mode is named for "whistling atmospherics." This Letter focuses on the lesser-known z mode. The magnetoionic dispersion equation is derived from Maxwell's equations and Newton's law of motion for a cold, electron fluid and takes the form $AN^4 + BN^2 + C = 0$. Here, $N = kc/\omega$ is the refractive index of a wave with wave number k and frequency ω, and the coefficients A, B, and C are functions of ω_p, Ω_{ce}, and the angle θ between the wave vector $\mathbf{k}$ and the magnetic field direction. Figure 1 shows how N^2 varies with ω/ω_p for a plasma with $\Omega_{\mathrm{ce}} = 4\omega_p$ and $\theta = 30°$ and $5°$, parameters appropriate to the application described below. The x and o modes are the solutions with $\omega \sim kc$ (i.e., $N^2 \sim 1$) at large ω that move to cutoffs ($N^2 = 0$) near $\omega = \Omega_{\mathrm{ce}} + \omega_p^2/\Omega_{\mathrm{ce}}$ and at ω_p, respectively, and are evanescent at frequencies below their cutoffs. The whistler mode has a resonance ($N^2 \to \infty$) near ω_p (that depends on θ) and another at low frequencies. The z mode is visible in Fig. 1 as the mode stretching from a resonance near Ω_{ce} to a cutoff well below ω_p, with a significant variation in N^2 near ω_p for $\theta = 30°$ and, particularly, $5°$.

Existing techniques for identifying ω_p from plasma wave data include two based on magnetoionic theory and two based on unmagnetized plasma theory. First, identify ω_p as the low-frequency limit of free-space electromagnetic radiation, assumed to be due to reflection and absorption of o-mode radiation at its cutoff [3,4,9]. This technique requires the presence of natural broadband radiation and the absence of higher density plasma between the radiation's source and the observer, conditions which are often not met. Second, identify ω_p as the high-frequency limit of whistler-mode signals, assumed due to the resonance at ω_p for whistlers with very small θ [10]. Often, however, no whistler waves are present or else their

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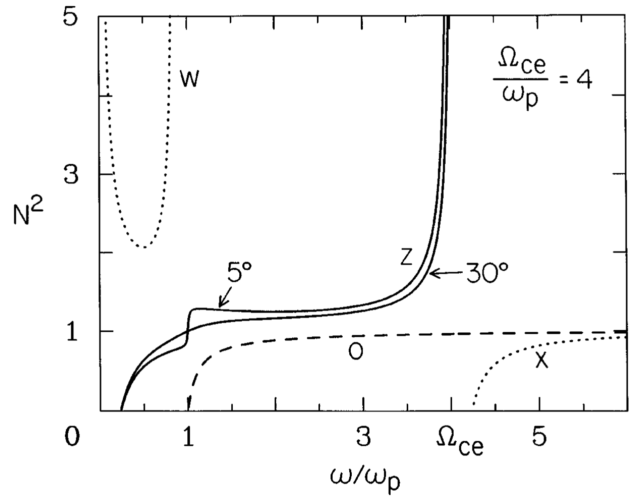

FIG. 1. Plot of N^2 versus ω/ω_p for $\Omega_{\rm ce} = 4\omega_p$ and $\theta = 30°$ showing the o, x, whistler, and z modes. The z mode for $\theta = 5°$ is also shown.

wave vector directions are unknown. Third, identify ω_p (and the electron temperature) using the characteristically peaked electric spectrum of thermal plasma noise in an essentially unmagnetized plasma [2]. However, available plasma wave instruments may have inadequate frequency resolution or the plasma may be too strongly magnetized for existing theory to justify using the technique. Fourth, electrostatic Langmuir waves typically have frequencies near ω_p, thereby often reliably marking ω_p, and are easily driven unstable by electron beams. However, Langmuir waves need not be present in a given situation and driving electron beams can substantially move the wave frequency away from ω_p [11].

The first evidence that z-mode waves can be used to identify ω_p comes from the strikingly abrupt variation of N^2 with frequency near ω_p for $\theta = 5°$ shown in Fig. 1: while the mode goes from $N^2 > 1$ at $\omega/\omega_p > 1$ to $N^2 < 1$ at ω/ω_p for all $\theta \neq 0$ (with $N^2 = 1$ at $\omega = \omega_p$), the range of ω/ω_p for this transition decreases rapidly with θ for $\theta \lesssim 30°$. Observationally, however, variations in N^2 are extremely difficult to measure directly, since they require simultaneous measurement of the wave number and frequency. The method proposed here is much simpler: use standard measurements of the waves' electric and magnetic intensity spectra to identify ω_p as the frequency where the electric intensity and the ratio of wave electric to magnetic fields increase significantly in a narrow bandwidth for z-mode waves with small to moderate values of θ. The simplicity of this method means that it may be widely useful in space and laboratory plasmas for even basic plasma wave instruments. Two fundamental characteristics of the z mode underlie this identification of ω_p: first, an increase in the ratio of wave electric to magnetic fields, and second, a marked depression in the group speed of the waves. These characteristics are now described in detail.

Maxwell's equations give the ratio E/cB of wave electric (E) to magnetic (B) fields as

$$\frac{E}{cB} = \frac{1}{N}\frac{1}{\sin\alpha}, \tag{1}$$

where α is the angle between $\mathbf{k}$ and the electric polarization vector $\mathbf{e} = \mathbf{E}/E$. Electrostatic waves have $\alpha \sim 0°$ while transverse, electromagnetic waves have $\alpha \sim 90°$. Standard methods of calculation [7,8] then yield N, $\mathbf{e}$, and α for the z and whistler modes, as well as the ratio E/cB shown in Fig. 2. An increase in the ratio E/cB near ω_p is clearly visible for the z mode at small $\theta \lesssim 5°$. Since $N^2 = 1$ for the z mode at $\omega = \omega_p$, this increase in Ec/B is due to the z mode becoming increasingly electrostatic as θ decreases. Using the quasilongitudinal approximation [8], an analytic prediction for the enhancement at ω_p is

$$\frac{E}{cB} = \frac{\omega_p}{\Omega_{\rm ce}}\frac{1}{\sin\theta}. \tag{2}$$

Comparisons with multiple numerical solutions (e.g., Fig. 2) show that Eq. (2)'s prediction is very accurate for $\sin\theta \lesssim \omega_p/\Omega_{\rm ce}$. Equation (2) predicts that $E/cB \to \infty$ as $\theta \to 0$. This behavior predicted for the ratio E/cB at ω_p offers a powerful and practical means to identify ω_p, provided that both electric and magnetic spectra are available.

Defining the ratio R_E of wave electric field energy to total energy (fields plus induced particle kinetic energy) by Eq. (2.69) of Ref. [8], direct calculations (Fig. 3) show that R_E remains $\sim 1/2$ at ω_p for the z mode. Accordingly, while the z mode becomes closely electrostatic near ω_p for small θ, the fraction of the total wave energy carried by electric fields remains essentially constant; the energy formerly carried by magnetic fields is now carried by induced motions of the plasma particles. In contrast,

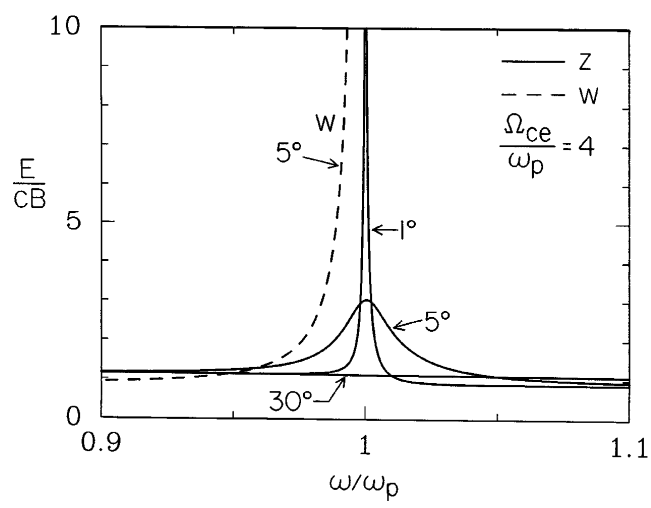

FIG. 2. The ratio E/cB versus ω/ω_p for the z and whistler modes at various θ and $\Omega_{\rm ce} = 4\omega_p$. Full lines denote the z mode and dashed lines the whistler mode.

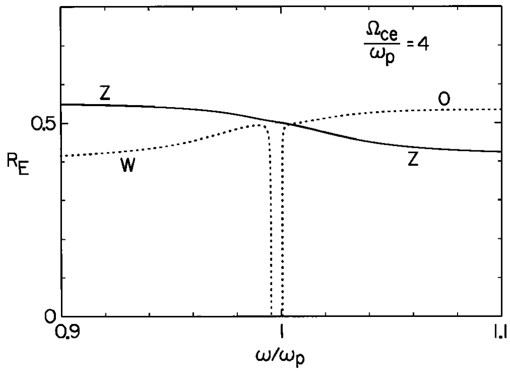

FIG. 3. The quantity R_E versus ω/ω_p for the z, o, and whistler modes with $\Omega_{\rm ce} = 4\omega_p$ and $\theta = 5°$.

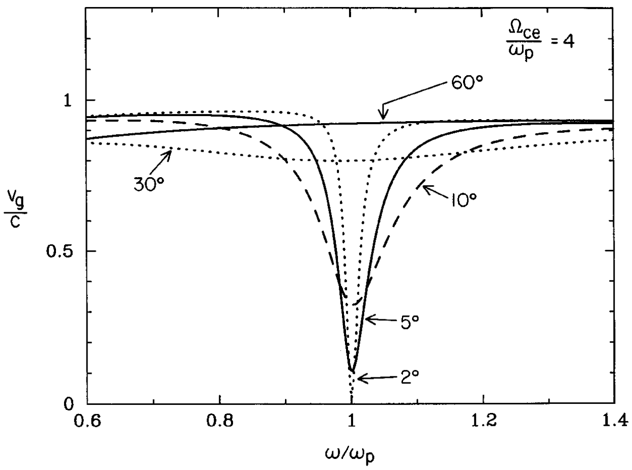

FIG. 4. The ratio v_g/c versus ω/ω_p for the z mode with $\Omega_{\rm ce} = 4\omega_p$ and $\theta = 60°$, 30°, 10°, 5°, and 2°.

Fig. 3 shows that the electric fields of o- and whistler-mode waves become vanishingly small as $\omega \rightarrow \omega_p$. It is argued next that instabilities and variations in the wave group speed provide a strong reason for electric intensification of z-mode waves near ω_p; instabilities will also likely make it unreliable experimentally to identify ω_p using the decrease in magnetic intensity at ω_p predicted by Eq. (2) and the result $R_E \sim 1/2$.

Growth of waves with small group velocities $\mathbf{v}_g = \partial\omega/\partial\mathbf{k}$ is often favored in plasmas. One general reason is related to spatial inhomogeneity, whether in the free energy source driving wave growth or the spatial locations in the plasma where wave growth can proceed effectively or both: Waves with small v_g can remain longer in a growth region of spatial extent L ($T \sim L/v_g$) and so experience a larger amplification factor,

$$e^{\gamma T} = e^{(\gamma L/v_g)}, \tag{3}$$

than waves with larger v_g but similar temporal growth rates γ. Another reason is that waves with small rather than large v_g are typically destabilized more easily by nonrelativistic sources of free energy (such as beams) typical of most laboratory and space plasmas. A final reason unrelated to instabilities is that waves tend to pile up and achieve detectable fields where $v_g \sim 0$.

Figure 4 demonstrates that the z mode has a pronounced depression in v_g near ω_p for a wide range of θ. This depression is centered on, but extends over a larger range than, the transition region in N^2 shown in Fig. 1. The decrease in v_g is most severe and localized for small θ but extends noticeably to at least $\theta \sim 30°$ for $\Omega_{\rm ce}/\omega_p = 4$. Using the quasilongitudinal approximation again, the minimum value predicted for v_g at ω_p is

$$\frac{v_g}{c} = \left(1 + \frac{\omega_p^2}{\Omega_{\rm ce}^2 \sin^2\theta}\right)^{-1} \approx \sin^2\theta \, \frac{\Omega_{\rm ce}^2}{\omega_p^2}, \tag{4}$$

where the last form assumes $\sin^2\theta \ll \omega_p^2/\Omega_{\rm ce}^2$. This prediction agrees very well with the numerical solutions for a wide range of θ ($\lesssim 30°$ for $\Omega_{\rm ce} = 4\omega_p$). Noting that the depressions in v_g shown in Fig. 4 are large, and that Eq. (4) predicts that $v_g \rightarrow 0$ as $\theta \rightarrow 0$, Eqs. (3) and (4) argue strongly that major intensifications of z-mode waves with small to moderate θ should be observable at ω_p.

These ideas are next found to be consistent with plasma frequency (PF) emissions recently discovered in the polar cap and auroral regions of Earth's magnetosphere [12,13]. Observed at heights of order 1–3 Earth radii, PF emissions occur in a very cold plasma ($V_e \lesssim 10^{-3}c$ and $V_i \lesssim 10^{-5}c$) for which magnetoionic theory is plausible. Identified originally as narrowband ($\Delta\omega/\omega \sim 0.2$) enhancements in wave intensity at frequencies of order expected values of ω_p, these emissions are electromagnetic with ratios $E/cB \sim 20$ and occur where $\Omega_{\rm ce} \sim (2\text{–}4)\omega_{pe}$ [12]. Additionally, although z-mode and whistler emissions are likely often superposed in PF events, at least 70% of PF events with well-defined polarizations are polarized in the sense of the z mode and not the whistler mode [13]. Even so, in the absence of direct measurements of n or independent estimates of ω_p, these analyses can provide only qualitative support for PF emissions being z-mode waves with small θ near ω_p [12,13].

Figure 5 provides quantitative evidence that PF emissions indeed occur near ω_p, using wideband data for the original event used to identify PF emissions: note the PF emissions at frequencies of 5–8 kHz and a clear but previously unappreciated low-frequency limit for other plasma waves near 2 kHz. It is natural to identify this limit as the low-frequency cutoff of z-mode waves (e.g., Fig. 1), which occurs theoretically at $\omega_z = (\Omega_{\rm ce}^2 + 4\omega_p^2)^{1/2} - \Omega_{\rm ce}/2$ [7,8]. Then, taking $\Omega_{\rm ce}/2\pi = 22\text{–}27$ kHz from magnetometer data and $\omega_z/2\pi \sim 1.6\text{–}2.2$ kHz from Fig. 5, it is straightforward to infer that $\omega_p/2\pi \approx 6.1\text{–}8.0$ kHz. This range matches well with the PF band, confirming that PF emissions occur close to ω_p. Accordingly, Figs. 1–4 and associated theory appear consistent with the observed characteristics of PF emissions (including the narrow bandwidths, quantitative consistency between wave frequencies and

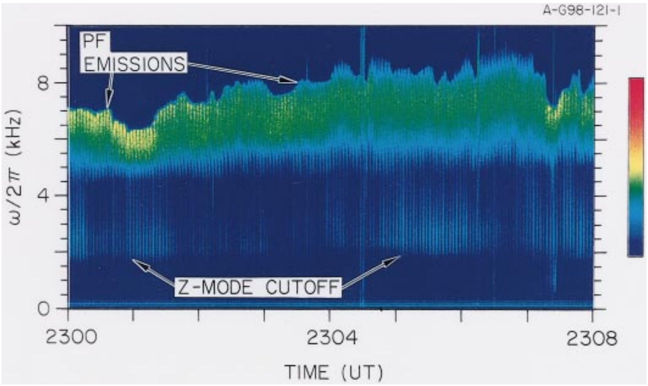

FIG. 5(color). Dynamics Explorer 1 observations of the electric wave intensity (in relative units) as a function of frequency over Earth's northern polar cap on 27 July 1983. PF emissions are present at 5–8 kHz and the clear low-frequency limit to waves near 2 kHz is identified as the z-mode cutoff, thereby allowing independent identification of ω_p.

independent values of ω_p, enhanced ratios E/cB, and predominant z-mode polarization). This argues strongly that PF emissions are indeed z-mode waves with small θ near ω_p which exemplify the intensification phenomenon and technique for identifying ω_p proposed in this paper.

This technique may be useful in other contexts in space and the laboratory. Of course, since the method requires that natural z-mode waves be present it will not always be applicable, a similar weakness to other wave methods. The theoretical arguments above do not address Landau damping and other kinetic effects, quasithermal plasma noise, or an instability to drive the waves, all of which require some consideration. Thermal Landau damping depends on the wave phase speed $v_\phi = c/N$ being resonant with thermal particles; since N remains ~ 1, thermal damping should be relatively weak for the z mode near ω_p in most nonrelativistic plasmas. The standard argument [7,8] that natural modes (such as the z mode) are modified only slightly in weakly unstable situations suggests the technique identified here should be quite robust. It is known that electron temperature anisotropy instabilities can produce z-mode waves above ω_p (and whistler-mode waves below ω_p) [14,15], while mode coupling of whistler waves can also produce z-mode waves near ω_p. Nevertheless, since strong sources of free energy can greatly modify the natural modes or else drive new modes directly dependent on the unstable particles, further investigation into the thermal levels and instabilities for z-mode waves with small θ near ω_p appears warranted.

In conclusion, strong theoretical arguments predict that z-mode waves with small θ should naturally show an increase in the ratio of electric to magnetic fields and an electric intensification in a relatively narrow frequency band around ω_p in cold plasmas with $\Omega_{ce} \gtrsim \omega_p$. Identifying these predicted signatures with observed emissions therefore provides a new technique to identify the plasma density and/or the wave mode. This method and theoretical ideas have been applied and found to be consistent with the available data for one class of natural emissions in Earth's inner magnetosphere.

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*Current address: School of Physics, University of Sydney, Sydney, Australia.

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