Quasi-Two-Dimensional MHD Turbulence in Three-Dimensional Flows

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We simulate how nearly two-dimensional magnetohydrodynamic turbulence can be initiated and how it evolves with distance in the third dimension. Expanding flows lead to a suppression of the nonlinear cascades, mainly due to the changing transverse length scales. Velocity fluctuation levels are determined more from the 2D dynamics than from the expansion except when the velocity fluctuations are initially dominant. With expansion, magnetic fluctuations always dominate velocity fluctuations at late times. [S0031-9007(98)08244-1]

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Two-dimensional studies of the nonlinear dynamics of magnetized plasmas, in which both the dependence of all quantities and the fluctuation vectors are confined to a plane, were initially carried out in part because they were numerically possible on then-current computers. The approach provided a rich set of insights [1], including the suppression of cascades along a mean magnetic field which leads naturally to a two-dimensional turbulence state [2]. Two-dimensional fluctuations have recently been intensively studied as part of various models for the evolution of the expanding solar wind. In the latter case, the spherical expansion leads to additional complications that have so far mainly been dealt with using multiple-scale expansions of the basic equations for small-scale fluctuations in a slowly varying background followed by further approximations to make the equations tractable [3]. Faster computers now make it possible to solve numerically the full set of compressible magnetohydrodynamic (MHD) equations in three dimensions (3D) in either Cartesian or spherical coordinates, and this is the subject of this Letter. We present the first simulations of the initiation and nonlinear spatial evolution of nearly two-dimensional (quasi-2D) fluctuations as the plane of the fluctuations propagates in a third direction. We consider nonexpanding and expanding geometries, and magnetically dominated, Alfvénic (equipartitioned and correlated magnetic and velocity fluctuations), and kinetically dominated cases. We show that while the nonexpanding cases are very like their purely 2D counterparts, the expanding cases are fundamentally different in important respects from the Cartesian expectations. The simulations provide insights on the nonlinear behavior of the system that will complement and enhance the accuracy of multiscale models.

Most previous simulations of 2D turbulence have to be performed using Fourier-method codes that yield high-order accuracy but impose periodic boundary conditions in all directions [4]. To relax the boundary conditions, we have developed a flux-corrected-transport (FCT) code to solve the MHD equations in spherical geometry. The code is a natural extension of those used previously [5]. We have tested the code for conservation of flux invariants, the proper results for known solutions such as Alfvén waves, and the preservation of a zero divergence of the magnetic field (to round off accuracy), and have performed the usual tests of varying the resolution and precision of the calculation, all with good results. We solve the ideal continuity, momentum, energy, and induction equations for the velocity $\mathbf{u}$, magnetic field $\mathbf{B}$, density ρ, and total energy e. The pressure is found from the ideal gas relationship $p = (\gamma - 1)e_{\text{int}}$ with $\gamma = 5/3$ and e_{int} the internal energy. Artificial dissipation occurs isotropically at scales smaller than the input wave spectrum due to the FCT algorithm but is otherwise negligible.

We impose a flow and fluctuations on one end of the 3D simulation domain (in x or r) and let the flow evolve supersonically and superAlfvénically with distance down the box. We have implemented a variety of boundary conditions with essentially the same results; for this study we used periodic conditions transverse to the flow, and simple outflow (linear gradient preservation) for the far end of the domain.

Sufficient inflow conditions to initiate the evolution consisted of a uniform (radial) flow and mean magnetic field that initially filled the box uniformly in x or $\propto 1/r^2$, and static fluctuations defined by the four lowest Fourier modes (of equal amplitude) for the transverse B fluctuations such that $\mathbf{B}_\perp = B_y(z)\mathbf{e}_y + B_z(y)\mathbf{e}_z$, and analogously in angle variables for the expanding case. The fluctuations in $\mathbf{u}$ are proportional to the magnetic fluctuations but with a multiplying factor that varied the cases from magnetically dominated to “Alfvénic” to

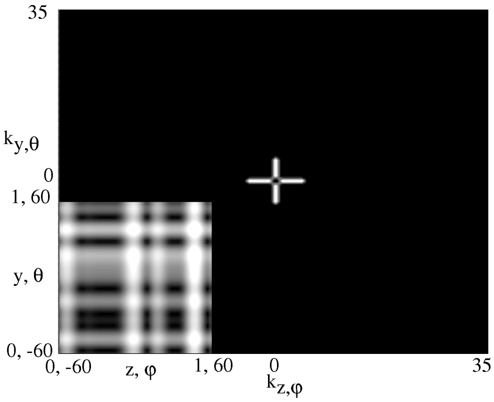

FIG. 1. Initial conditions for Cartesian and spherical runs for B at the inflow end of the box (inset) and for the log of the trace of the power spectrum of the inflow magnetic fluctuations. The dimensions are degrees for angles, arbitrary code units for distances, and mode number for spectra. The greyscale proceeds black (minimum) to white (maximum).

velocity dominated. (Alignment of the fields is only significant in the Alfvénic case.) The transverse rms magnetic fluctuation amplitude at the inflow, δB, is about 0.7 of the mean field except in the velocity dominated case in which the initial fluctuation energy (comparable to that in the magnetically dominated runs) is kinetic and very little is magnetic. The inflow sonic and Alfvénic Mach numbers were usually 3 and 5, respectively, yielding a plasma β of about 3. (Based on a few runs of other cases, the main results do not seem sensitively dependent on β.) The grids were $200 \times 70 \times 70$ for most runs, with the 200 in the flow direction. Since the boundaries were time independent, once the transients had moved through the box the flow became steady with each parcel of plasma undergoing identical temporal evolution. Thus successive cuts transverse to the flow direction in the final state provide a record of the time evolution of the initial 2D fluctuations.

The conditions at the inflow are shown in Fourier space (logarithmically spaced filled contours of the trace of the transverse magnetic spectral matrix) and in real space (the magnitude of the transverse magnetic field in the inset) in Fig. 1. The Cartesian run at the end of the box (Fig. 2) shows evolution like that seen in pure 2D codes at similar times (here, about two eddy-turn-over times) [4] in which the original fluctuations are distorted and become confined to thinner structures. The evolution continues to near isotropization if the box is longer. The dynamics always remain quasi-2D with little structure parallel to the flow as shown by cuts along the flow (Fig. 3), and this explains why compressive effects, such as shock generation, are very weak. The initially low kinetic energy in fluctuations (5% of the total in fluctuations) is driven upward by the magnetic forces, such that it becomes about $1/3$ of the total near the end of the box and remains at that level if the box is extended. The value of $\delta B/B_x$ decreases slowly as the energy is transferred, such that it is about 0.6

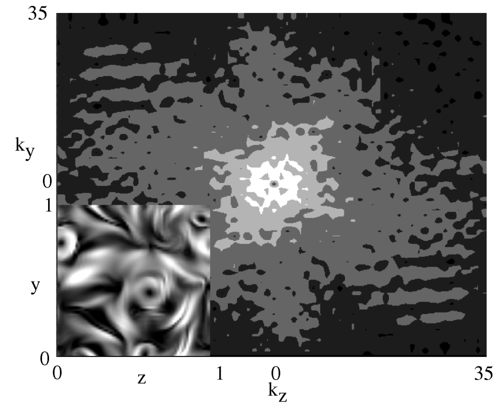

FIG. 2. Same as Fig. 1 for the outflow end of the box in a Cartesian run.

at the time of Fig. 2. The above results are consistent with previous 2D simulations and theoretical considerations, and confirm that spatial evolution is nearly equivalent to time evolution in the purely 2D case, as often had been assumed.

The expanding case with 120° opening angles and the same conditions at the inflow as in Fig. 1 evolves very differently, as shown in Fig. 4, which corresponds in distance down the box to Fig. 2. Very little evolution has occurred, despite the increase of $\delta B/B_r$ to over 3. This is evident in both the morphology of the spatial fluctuations, which is both smooth and uncontorted, and the relative permanence of the initial mode spectrum. The initial spatial scales were arranged to be the same as in the Cartesian case, but the box expands by a factor of 5 as the flow proceeds. Extending the box leads to very little further evolution, but the evolution in the first part of the box, when the transverse scales are similar, is comparable to the Cartesian case. The evolution that does occur is mainly in the 2D plane, as before.

The magnetic fluctuations decrease somewhat more rapidly than $1/r$, consistent with simple flux conservation (as well as more involved arguments) modified by the

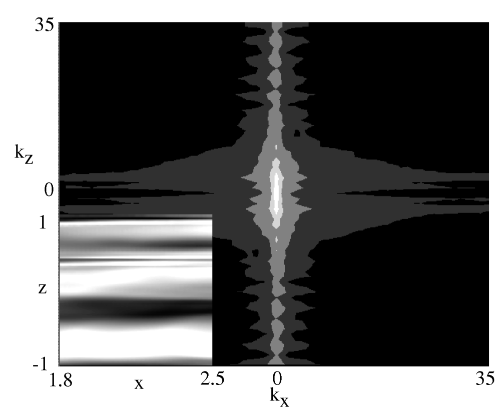

FIG. 3. A cut in the x-z plane for the case in Fig. 2. Very low power horizontal streaks in the spectrum are due to nonperiodicity in the radial direction.

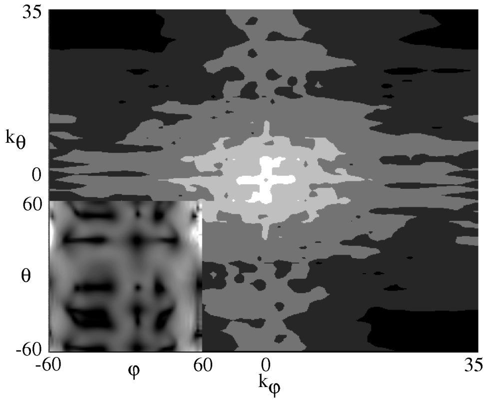

FIG. 4. Same as Fig. 2 for a rapidly expanding case with the same input.

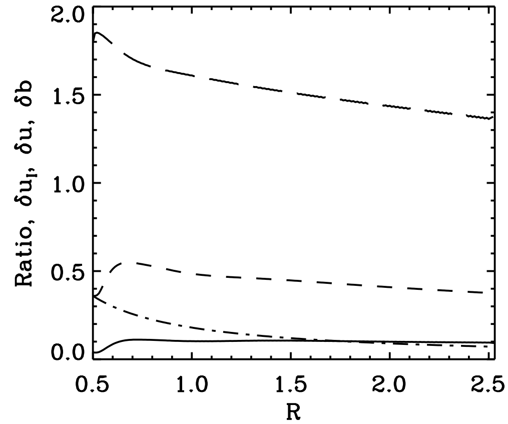

FIG. 5. (Top to bottom) Amplitude of the magnetic fluctuations $[\delta B/(4\pi\rho)^{1/2}]$ (——), amplitude of the velocity fluctuations (– –), "ideal" velocity fluctuation amplitude for the expanding case (—·—), and the ratio of kinetic to magnetic energy (—) in the fluctuations as a function of r for the case of Fig. 4.

2D dynamics that transfer energy to the velocity field. Theoretical predictions for the "ideal" evolution (ignoring nonlinear interactions) of the rms velocity fluctuation, δu, in an expanding box [3] are that it should also be $\propto 1/r$. However, the quasi-2D dynamics increases the kinetic energy of the fluctuations to 12% of the magnetic (see Fig. 5), and the subsequent evolution nearly maintains this level. Since the kinetic energy is proportional to density, which decreases as roughly $1/r^2$, this implies a nearly constant Su, as shown in the figure; the ideal velocity fluctuation evolution is also shown. Evolution that is intermediate between that shown in Figs. 1 and 3 can be generated by increasing the transverse scales in the Cartesian case or by decreasing the initial scales or the opening angles for the expanding case. The expansion always eventually increases the scales to the point where little evolution occurs. This is reminiscent of the conclusions of Grappin *et al.*, who found that expansion inhibited nonlinear cascades, but their case involved a parallel propagating mode; we will treat the latter case in another publication. On the other hand, the effect we see is not primarily due to the transverse velocities resulting from the projection of the radial flow on a Cartesian plane transverse to a radial vector, and in general we find a similar evolution for expanding and nonexpanding cases when the transverse scales (L) and thus the nonlinear times $L/\delta b$ are similar. [Note that $\delta b = \delta B/(4\pi\rho)^{1/2}$ is the relevant speed determined from the magnetic fluctuations.]

A particularly interesting initial condition is one in which $\mathbf{u}_\perp = \pm\mathbf{b}_\perp \equiv \pm\mathbf{B}_\perp/(4\pi\rho)^{1/2}$, implying parallel and energetically equipartioned magnetic and velocity fluctuations. This condition, which is termed Alfvénic although here no waves are propagating because all wave vectors are transverse, leads to no incompressive nonlinear couplings, and a Cartesian run with this initial state shows almost no evolution. However, in the expanding geometry the tendency for the kinetic energy to decrease more rapidly than the magnetic leads to a rapid violation of the Alfvénic condition, and the evolution proceeds nearly the same as before. The relative energy in the velocity fluctuations (see Fig. 6), while remaining below the magnetic, is maintained at levels much above that for an "ideal" $1/r$ decrease in δu due to a continuous pumping from the magnetic field. The latter thus decreases slightly faster than $1/r$, as shown by the slow decrease in δb in the figure and as occurred in the previous expansion runs.

In the Cartesian geometry there is a general similarity between magnetically and kinetically dominated initial fluctuations. Although the specific pattern changes, the state corresponding to Fig. 2 when the same modes have their magnetic and velocity amplitudes reversed is generally similar, with very similar spectral evolution. The expanding case is entirely different, however, because now there is no magnetic field energy to preserve the fluctuation energy. The velocity fluctuations now drop nearly as $1/r$, unlike the cases above (see Fig. 7). Heuristically, the reason for this is that the angular momentum of any "elementary swirl" is preserved in

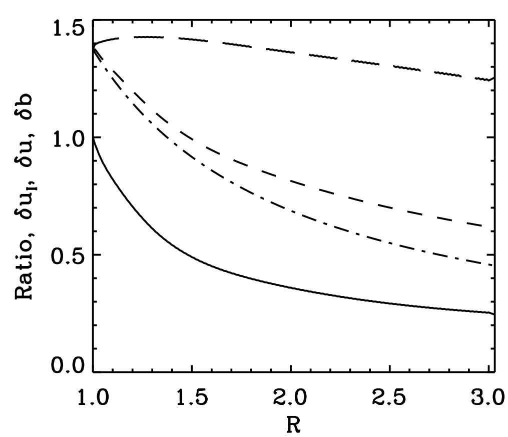

FIG. 6. Quantities as a function of r for the "Alfvénic" case. Symbols are as in Fig. 5.

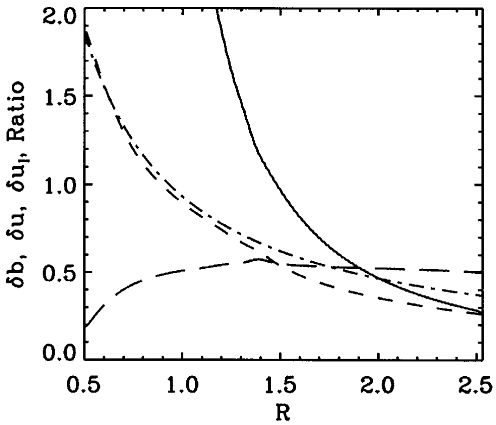

FIG. 7. Same as Fig. 6 for the initially velocity dominated case. The ratio of kinetic to magnetic energy starts near 100.

the near absence of magnetic forces, making the velocity times a local transverse distance from the swirl's center a constant. As the expansion occurs, the transverse distance is proportional to r, implying $\delta u \propto 1/r$. The plasma performs some extra work on its surroundings, leading to a rapid loss of fluctuation energy with $\rho \delta u^2 \propto r^{-4}$, neglecting small corrections due to the acceleration of the plasma by pressure gradients. The same scalings are found in multiscale arguments. The velocity fluctuations induce magnetic fluctuations, as Fig. 7 shows, leading to an extra decrease in δu until the magnetic energy becomes dominant at a low level near $r = 1.5$ and the evolution becomes more like that of the magnetically dominated case in Fig. 5.

This first simulation study of quasi-2D fluctuations in an expanding medium thus shows that a (quasi)static condition, such as might exist on the Sun at some scales, leads to evolution similar to simple 2D cases. Substantial differences in the evolution exist between expanding and nonexpanding cases, mainly associated with the changing transverse scales and the tendency for rapid decrease of the velocity (but not magnetic) fluctuations in the expanding medium. Quasistatic inflow conditions are in sharp contrast to the energetic, active processes near the Sun that would tend to produce fluctuations propagating along the mean magnetic field. The relative roles of quasi-2D and propagating, mostly Alfvénic fluctuations is a subject of much recent debate [6] and will be studied as a natural extension of the present work along with the related question of the extent to which quasi-2D fluctuations can be generated by dynamical processes in an expanding medium.

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