Measurement of the Intrinsic Dissipation of a Macroscopic System in the Quantum Regime

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We report on the first measurements of the intrinsic dissipation in a macroscopic system cooled at very low temperature (35 mK) and operating in the quantum regime. The system under study is an rf SQUID with a high quality Josephson junction. Below 50 mK the tunneling probability of escape from a metastable well vs applied flux presents a series of maxima due to energy level quantization. From the shape of the tunneling probability we can evaluate the intrinsic dissipation related to the overall system as well as the coherence time related to the Rabi oscillations in a future macroscopic quantum coherence experiment. [S0031-9007(99)09476-4]

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The success of quantum mechanics (QM) in predicting the behavior of the microscopic world is undoubtedly one of the most complete in the history of physics. Since the very beginning of QM, however, there has been a debate on the interpretation of QM predictions versus the macrorealism (MR) predictions. The most famous example is the paper by Einstein, Podolski, and Rosen [1]. This problem, for the specific case of a macroscopic object, was discussed by Leggett and co-workers [2,3] who proposed an experiment to test Bell inequalities in a system performing tunneling oscillation between two macroscopically distinct states. The proposed system is an rf SQUID that, biased by a half flux quantum, is subjected to a double well potential with two degenerate energy levels in the left and right wells. The variable describing the system dynamics is the magnetic flux Φ linked to the SQUID, related to the collective motion of a macroscopic number of particles. In the past decade there was a big effort in testing the behavior of Josephson junctions and rf SQUIDs in the quantum regime ([4–8], and references therein). For the macroscopic variables describing these systems, both tunneling to a continuum and resonant tunneling between distinct quantum levels have been observed; however, a high degree of dissipation was also observed, either because of the low quality of the Josephson junction [7] or because of a strong coupling with the environment [8]. Very low dissipation, instead, is a mandatory requirement in any experiment aiming either to detect the superposition of macroscopically distinct states [namely, a macroscopic quantum coherence (MQC) experiment, where also an *ad hoc* setup is required [3,9,10]] or to perform tests on quantum computing by means of Josephson devices [11].

The quantitative analysis of the dissipation problem for an rf SQUID in the quantum regime has been analyzed by many authors [12–15]. If applied to the case of an rf SQUID biased by a half flux quantum we can write the conditional probability of measuring the flux at a time t in one of the two wells, once the system has been prepared in the same well at a time $t = 0$ [13]: $P(t) \cong \cos(\omega t - \varphi)\exp(-t/\tau)/\cos(\varphi)$, where ω is the tunneling frequency, φ is the phase, and τ is the decoherence time. In the limit of low dissipation, expressed by the condition on Leggett's parameter $\alpha_L \ll 1$ [3] and low temperature ($T < T^* = \hbar\omega/\pi k_B \alpha_L$), the decoherence time can be written as [13]

$$\tau = \frac{T^*}{\omega T} \cong (1.5\ \mu\text{s})\left[\frac{R}{1\ \text{M}\Omega}\right]\left[\frac{1\ \text{mK}}{T}\right]. \qquad (1)$$

Now, setting the operating temperature at the base temperature of a dilution refrigerator that is ~mK, the decoherence time is determined essentially by the value of the overall dissipation R.

In this Letter we report on the first measurements of the intrinsic dissipation of an rf SQUID operating in the quantum regime, and we show that the measured values are in the range that allows one to perform, with success, experiments involving macroscopic quantum coherence of Cooper pairs.

The rf SQUID can be described by a single dynamical variable, the total magnetic flux Φ linked to the ring, which is subjected to a potential $U = (\Phi - \Phi_x)^2/2L - (i_c\Phi_0/2\pi)\cos(2\pi\Phi/\Phi_0)$, where Φ_x is the applied flux, L is the SQUID inductance, i_c is the junction critical current, and $\Phi_0 = h/2e$ is the flux quantum [16]. The system equation is the same as a particle of mass C (the junction capacitance) with friction coefficient $1/R$, subjected to the same potential. If the parameter $\beta_L = 2\pi L i_c/\Phi_0$ is greater than one, this potential is a succession of wells, corresponding to metastable flux states of the SQUID, superposed to a quadratic term. If β_L is large, more than one metastable state is available to the system, once the energy is given. In this paper we study the process of escape from a metastable well. By sweeping the externally applied flux, the potential barrier on one side of the well is decreased, until, at a critical value of Φ_{xc}, the particle can overcome the barrier by thermal fluctuations or quantum tunneling, and it rolls down along the potential until it gets trapped in a nearby well. For the rf SQUID, the characteristic Φ (i.e., the rf SQUID response, read out through a nearby dc SQUID) versus Φ_x (i.e., the sweeping external flux) is recorded. To measure the statistical distribution of the switching values Φ_{xc}, we used a well-known technique [17]: The sudden jump in the characteristics, which corresponds to the critical applied flux Φ_{xc} that produces a transition between metastable states (minima in the potential well), triggers the data acquisition system, so that the corresponding value of the sweeping current is recorded and converted into a flux value. The Φ_x sweep cycle is repeated $\sim 10^4$ times; the values are collected in a histogram, to estimate the switching probability $P(\Phi_x)$, from which the escape rate out of the well and any other quantities related to $P(\Phi_x)$ are calculated.

A schematic of the experimental setup is shown in Fig. 1. The main chip, realized at the Istituto di Elettronica dello Stato Solido-CNR using Nb/AlO$_x$/Nb technology [18], is inserted into the mixing chamber of a dilution refrigerator. The rf SQUID is a gradiometric double loop device having an inductance $L = 235$ pH, a critical current $i_c = 12$ μA, and a capacitance $C = 0.45$ pF (measured by means of Fiske steps in a separate experiment). Separate devices fabricated in the same wafer as the test chip have allowed the measurement of the junction critical current and quality and of the inductance (this has been obtained by the critical current depth modulation of a dc SQUID with identical geometry as the rf). In some cases, we also interrupted the rf SQUID loop after the measurement and contacted the junction, to check the critical current value. Actually, once the flux characteristic of the rf SQUID is acquired, the measurement of period and hysteresis gives a direct evaluation of the parameter β_L: the reduction of the hysteresis due to fluctuations in the value of Φ_{xc} can be taken into account by the subsequent data analysis and corrected with a recursive procedure. The overall accuracy on the measured SQUID parameters can be estimated to 5%. The flux from the rf SQUID is directly coupled, without any flux transformer, to a nearby dc SQUID amplifier, built on the same chip, having inductance $L_{\mathrm{dc}} = 235$ pH, shunt resistance $R_{\mathrm{dc}} = 2$ Ω, and critical current $I_{\mathrm{dc}} = 12$ μA. A flux jump of the rf SQUID couples $15m\Phi_0$ into the dc SQUID, which, in turn, is read by a second dc SQUID. The voltage output from the second dc SQUID, hence proportional to the rf SQUID flux response, is sent to a discriminator set at a reference level between the voltage corresponding to two subsequent flux states of the rf SQUID. The pulse from the discriminator provides the trigger for a 16 bit analog-to-digital converter that reads the current producing the rf SQUID external flux Φ_x.

The dilution refrigerator is inserted into an aluminum dewar surrounded by three μ-metal and two aluminum shields to provide both low and high frequency shielding. All of the wires are coaxial cables with CuNi shield and series CRC filters to provide high frequency attenuation. In the case of the rf SQUID, however, the attenuation requirements are less severe with respect to quantum effect measurements in Josephson junctions, since there are no direct connections with the external world. The problem could arise for the electromagnetic coupling with a very near conducting shield [8] (in our case, however, we have a superconducting lead box). For our device the typical dimension is of the order of less than 1 mm; the SQUID can then be considered as a concentrated parameter device and we can neglect the radiative processes. The small coils coupling the SQUID with the external bias or the dc SQUID have a very low coupling ($\sim$4 pH) and in practice do not load the rf SQUID with their losses. As regards the first dc SQUID magnetometer, the absence of peaks in the experimental histograms shows that it does not affect the rf SQUID with rf interference. This is due to the smooth characteristics of the magnetometer and to the fact that its working frequency is very low (of the order of a few GHz) with respect to the energy level spacing (hundreds of GHz). The normal resistance of the magnetometer, referred to the input and hence to the rf SQUID, following the calculation by Hilbert and Clarke [19], gives an upper limit for the dissipation of about 2 GΩ. The conclusion is that reasonably the system will be limited only by the intrinsic dissipation of the device itself.

On this apparatus we made measurements of escape from the metastable well to the nearest lower well. We

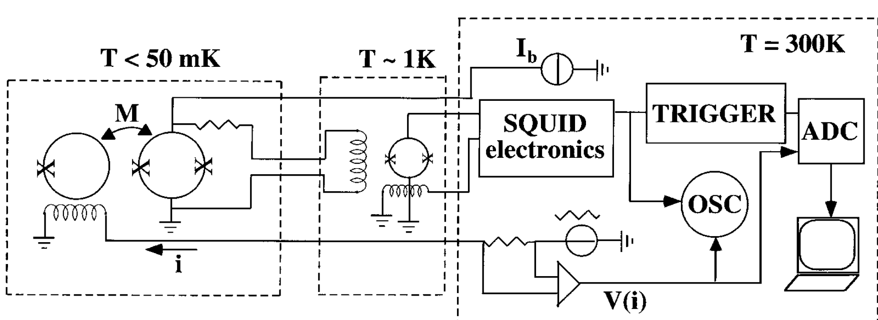

FIG. 1. Schematic of the experimental setup.

first measured the histogram of the switching fluxes in the function of temperature between 4.2 and 1 K finding the expected behavior for a system described by a simple Kramers model in the thermal regime [14,20]. After this test, we made the same measurements below 50 mK: At these temperatures we clearly observed the effects of energy level quantization in the SQUID wells, showing up as the appearance of peaks in the histogram of the switching fluxes.

In the weak friction limit the energy levels are sharp and well separated inside the potential well. The dynamics of the escape process can be described by the master equation for the probabilities ρ_j of finding the "particle" at the jth level [21]

$$\begin{aligned} \partial\rho_j/\partial t &= \sum(W_{j,i}\rho_i - W_{i,j}\rho_{j,i}) - \gamma_j\rho_i\,, \\ &\text{with } j = 0,\ldots,N\,, \end{aligned} \tag{2}$$

where $W_{i,j}$ is the transition probability from the ith into the jth level due to the interaction with the thermal bath, and γ_j is the tunneling probability through the barrier.

The complete solution of this equation is not trivial; however, we were not interested in the full description of the escape process, but only in deriving an expression to evaluate the system dissipation. Roughly speaking, the appearance of a single peak in the histogram of the switching probability is a manifestation of the fact that, at that particular flux bias value, most of the escape process is due to one particular level. Changing the flux bias, the interested level also changes, passing through a region in which two adjacent levels contribute in the same way. A simplified approach, then, can consider the analysis of only two adjacent peaks at a time, reconstructing the whole histogram shape piece by piece.

The distribution of the escape probability $f(X)$, where X is the normalized external flux ($X = \Phi_x/\Phi_0$), is equal to $f(X) = |\dot\rho|/\dot X$, where $\dot\rho = \sum\dot\rho_n$. To have a "peak" in $f(X)$ due to a generic inner level k is necessary so that the population variation $\dot\rho_k$ relative to the k level can be larger than all of the variations of the other levels in a narrow region of X: $|\dot\rho_k| \gg |\dot\rho_j|$ (with $j \neq k$). The range of X where the escape process is relevant is very narrow (a few percent of the quantum flux for our measurements), with respect to the variation of the quantum escape rate γ_k; therefore, we can expand the expressions of the energy barrier height and of the attempt frequency around the operating value of X. We can then analyze the peaks relative to two successive levels $(n, n+1)$ to derive an expression for the corresponding escape rate. In this approximation the upper levels $(n+2,\ldots)$ are empty, and the lower levels $(n-1,\ldots)$ are still inactive, and we can solve the master equation written only for the two levels $(n, n+1)$. For the distance between the maxima of $|\dot\rho_k|$ we have $\Delta X_M = X_M^{n+1} - X_M^n = -2\pi/\eta$. The quantity $W_{n+1,n}$ is given by [21] and is supposed constant in the small region of X considered, while

$$\begin{aligned} W_{n,n+1} &= W_{n+1,n}\exp\left(-\frac{E_{n+1} - E_n}{k_B T}\right) \\ &\approx W_{n+1,n}\exp(-2\pi\theta)\,, \end{aligned} \tag{3}$$

where E_i is the energy of the corresponding level and $\theta = T_c/T_e$. $T_c = \hbar\omega_0/2\pi k_B \cong 230$ mK is the crossover temperature between the classical and quantum regime (ω_0 being the frequency of small oscillation in the well), and T_e is a parameter related to the level population. T_e represents the effective noise temperature of the system only if the levels are populated according to a Boltzmann distribution; however, the local population can also be modified by nonthermal effects, for instance, because of incomplete relaxation during the flux sweep between two biasing cycles. Using these relations we can describe the escape process with a function depending on three dimensionless parameters (η, θ, w_d) related to the physical quantities of the system $(\Delta X, T_e, R)$, where $w_d = W_{n+1,n}/\dot X \approx 1/(RC\dot X)^{-1}$ [21].

In Fig. 2 we show the experimental data for the distribution of probability vs the normalized flux together with the fit for two pairs of subsequent peaks; the best fit gives, as explained below, a value of $T_e \cong 0.5$ K and $R \cong 4$ MΩ. In Fig. 2(a) we show the temperature dependence of the fit; we can see that the effect of T_e is a horizontal shift of the lower part of each peak, a feature that is quite insensitive to the particular value of the resistance. In Fig. 2(b) we show the effect of varying the resistance R, showing that we can reasonably assign

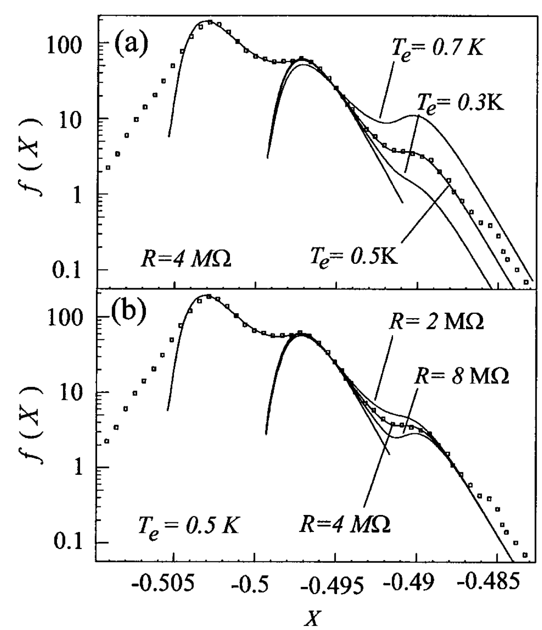

FIG. 2. Experimental data (open squares) of the switching probability $f(X)$ vs the external normalized flux $X = \Phi_x/\Phi_0$. The continuous lines represent the theoretical predictions of a two-peak analysis for a value of $T_e = 0.5$ K and $R = 4$ MΩ. In (a) we keep $R = 4$ MΩ and change the parameter T_e. In (b) we keep $T_e = 0.5$ K and change R.

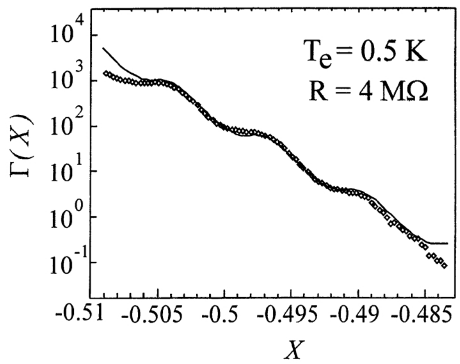

FIG. 3. Experimental data (open diamonds) and theoretical predictions of the escape rate for a multiple-peak analysis.

the quoted value for the equivalent resistance R. We then extended the model to describe all of the peaks of the escape process. In Fig. 3 we show the best fit obtained with this model applied to the data for $T_e = 0.5$ K and $R = 4\ \mathrm{M}\Omega$ on the escape rate $\Gamma(X) = \sum \gamma_n \rho_n / \sum \rho_n$. The determination of the fitting parameters in this case is less precise; however, the best fit with parameters T_e, R agrees well with the experimental data.

From the flux coordinate of the peaks in the switching data we can estimate that approximately 20–30 levels are in the well, the exact number being related to the uncertainty of the experimental SQUID parameters; of these levels, during the measurement the upper 3–4 are not active for the escape process, while the two immediately below contribute most. From the fit of Fig. 2 we can derive the following values for the three parameters: $\eta \cong 900$, $\theta \cong 0.5$, and $w_d \cong 300$ (in our measurements $\dot{\Phi} = 1750\Phi_0\ \mathrm{s}^{-1}$). As a consistency test we can also calculate the quantity η from the definition $\eta = d(\Delta U/\hbar\omega)/dX$. In this case we find a value between 800 and 900 that agrees with the value obtained from the fit of the data. This value also fits well the spacing between the two levels. The value of T_e found from the fit is about 0.5 K, much higher than the thermodynamical temperature; this could be related to the enhanced population of the interested levels due to the incomplete relaxation of the system from the previous switch event: in fact, the typical relaxation times are of the same order of magnitude (1 ms) as the period of the biasing flux sweep.

The value $R = 4\ \mathrm{M}\Omega$, which represents the resistance associated with the overall equivalent dissipation of our system, is the first measurement reported, to our knowledge, for a very low dissipative ($\alpha_L \ll 1$) macroscopic quantum system. We can use Eq. (1) to infer a value of the decoherence time at the operating temperature of a typical MQC experiment (4 mK). Assuming (pessimistically) that the resistance does not increase at all by lowering the temperature from 35 to 4 mK, we calculate $\tau \approx 1.5\ \mu$s, sufficient to detect a coherent oscillation for a tunneling frequency of a few MHz.

In summary, we measured the escape rate out of one of the flux states for an rf SQUID cooled at a temperature of 35 mK, and we saw clear evidence of quantized energy levels in the potential describing the SQUID dynamics. To evaluate the system dissipation, we used a simple model to fit the peaks due to energy level quantization in the escape rate of the SQUID, deriving a value of $R \approx 4\ \mathrm{M}\Omega$ for the intrinsic system resistance. An independent evaluation of the system parameters agrees with those derived from the fit, also describing quite well the level spacing. The obtained value for the system dissipation is very promising for the realization of MQC measurements, and for future tests on quantum computing with Josephson devices.

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