Measurement of Electron-Beam Bunch Length and Emittance Using Shot-Noise-Driven Fluctuations in Incoherent Radiation

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Longitudinal and transverse phase space information has been obtained from a statistical analysis of fluctuations in the radiation spectrum of an electron bunch. Uncorrelated shot noise fluctuations in longitudinal beam density result in incoherent radiation with a spectrum that consists of spikes, with width inversely proportional to the bunch length. Measurements were performed at $\lambda = 620$ nm on a 1–5 ps long, 44 MeV bunch propagating through a wiggler. Bunch length and emittance obtained with this single shot technique agree with independent measurements. [S0031-9007(99)09493-4]

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It has long been recognized that noise can be used to infer properties of physical systems. Noise by nature carries information without disturbing its system, in contrast to externally applied diagnostics, which often modify the very parameter they attempt to measure. Since the pioneering experiments in the early decades of the century, in which measurements of noise provided the standard for fundamental quantities—electric charge, the Boltzmann constant, and temperature—fluctuation-based diagnostic techniques have been pursued in diverse applications [1]. The range of disciplines, from astrophysics, circuits, acoustics, thermodynamics, material science, industrial engineering, biology, to optics, across which researchers have isolated the signature of essential parameters in noise spectra is reviewed in Ref. [2].

In this Letter, we report first measurements of the full single shot spectra of wiggler spontaneous emission with the resolution required to demonstrate the predicted 100% fluctuation of the spectral intensity and to investigate its detailed properties. The fluctuational characteristics of the wiggler spontaneous emission spectra are studied and utilized to extract electron beam longitudinal and transverse phase space information. The calibrations obtained from the fluctuation measurements rely on a comparison with optical diffraction limited spot size, which provides an absolute calibration standard.

It has been recently proposed that properties of a bunch of charged particles can be obtained through measurement of the fluctuations of incoherent emissions from the bunch [3]. Emission can be produced through interaction of the particle with external electromagnetic fields (i.e., wiggler, undulator) or media (i.e., transition radiation, Cerenkov, Smith-Purcell, etc.). The Fourier component, $E(\omega)$, of the electric field of the spontaneous radiation from charged particle beams is given by

$$E(\omega) = e(\omega)\sum_j e^{i\omega t_j}, \qquad (1)$$

where $e(\omega)$ is the radiation amplitude of an individual particle, and t_j is the time of arrival of the jth particle at, for example, the entrance of a wiggler. Equation (1) can contain coherent and incoherent contributions to the radiated field. Coherent radiation arises in the presence of microbunching on the electron beam or for very short bunch envelopes, i.e., $\omega\tau_b \ll 1$, where τ_b is the bunch length. It has a cutoff frequency determined by the inverse of τ_b and amplitude proportional to the number of particles in the bunch, N_b. Incoherent radiation occurs when $\omega\tau_b \gg 1$: the large random argument of the exponent in the phase factor causes the real part of each term in the sum in Eq. (1) to randomly fluctuate between -1 and $+1$. The expected value of $E(\omega)$ is zero, while the amplitude of the fluctuations in $E(\omega)$ is proportional to $\pm\sqrt{N_b}$, which leads to 100% intensity fluctuations in the incoherent radiation spectrum. Shot noise, which follows Poisson statistics, appears in the incoherent emission spectrum as spikes of width $\sim 1/\tau_b$. The disparity in frequency scales allows coherent and incoherent contributions to be separated. In order to treat the radiation classically, the degeneracy parameter of the radiation must be greater than one, and in this case, one can neglect quantum noise in the spectrum.

The experimental setup is shown in Fig. 1. The measurements [4] were conducted at the Accelerator Test Facility at Brookhaven National Laboratory [5]. Single bunches, variable from 1–5 ps with corresponding charge of 100–400 pC, were produced by an rf photocathode gun [6] and accelerated to 44 MeV by an S-band linac. Nominal normalized emittance was a few π mm-mrad and energy spread was $\sim 0.7\%$ full width. Bunch length was independently monitored with a time-integrated technique [7] in the front end of the accelerator.

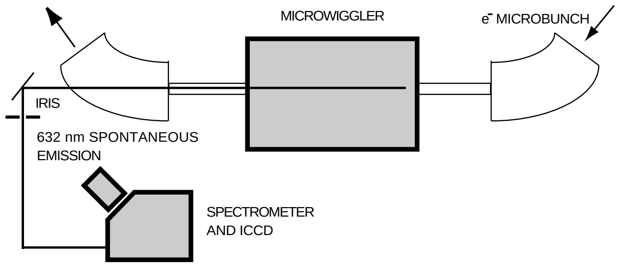

FIG. 1. Experimental setup.

The electron bunch was transported to the Stoner-Bekefi microwiggler [8], which has an on-axis peak field strength of 0.42 T with an 8.8 mm period, corresponding to a wiggler parameter, a_w, of 0.34 over a total length of $L_w = 50$ cm ($N_w = 60$ periods). Wiggler radiation at 620 nm was apertured and imaged onto the entrance slit of a spectrometer (model Spex 270M, with focal length 0.27 m and a 1200 grooves per mm grating). At the output, the spectrum was imaged onto a Gen IV image intensifier with 35%–40% quantum efficiency in the visible region. In the images acquired by the frame grabber, 19 pixels corresponded to 1 nm.

Representative measurements of the single shot spectrum of electron bunches with bunch lengths of 1.5 and 4.5 ps (determined from the time-integrated technique), are shown in Figs. 2a and 2b, respectively. The spectra are composed of spikes of random amplitude and frequency which have a characteristic width $\Delta\omega \sim 1/\tau_b$. By comparing Figs. 2a and 2b it is clear that for the longer bunch (Fig. 2b) the spikes are narrower than for the shorter bunch (Fig. 2a). The morphology of the individual spectra changes randomly from shot to shot, but the average of consecutive shots approaches the familiar wiggler spectral envelope of width $1/N_w$.

In the measured spectra, the level of modulation depends distinctly on beam bunch length, with nearly 100% modulation at the shortest bunch (Fig. 2a) and a pedestal for the longer bunches (Fig. 2b). The pedestal in these long bunch spectra can be mainly attributed to (a) finite resolution of spectral measurement, and (b) finite transverse size of the radiation source. For long bunches, finite resolution results in the addition of intensities of closely spaced neighboring spectral spikes, giving rise to the pedestal. Longer bunches, which contain more charge are known to have a larger transverse emittance due to space charge effects [9]. This results in an electron beam transverse emittance, ϵ_b, comparable to the transverse phase space area of the radiation of one electron. The different transversely coherent portions of the source radiate independently, also contributing to the pedestal.

For comparison with the experimental data, numerical data for the spectral intensity $|E(\omega)|^2$ was generated by using Eq. (1). After convolution with instrumental broadening, the presence of multiple transverse modes was included by mixing independent spectra. Using this simple model, the artificial spectra (Figs. 2c and 2d) were found to reproduce all features of the experimental data: the measured characteristic spike width, the systematic reduction in spike width with increasing bunch length, the mean number of spikes contained within the wiggler spectrum envelope, the scaling of fluctuation level with frequency bin size and bunch length, and the scaling of the intensity distribution with bunch length.

The spectral intensity distribution provides a functional tool for extracting diagnostic information from the fluctuations. One can expect that the distribution will follow the gamma distribution [10], as it describes the sum of k independent Poisson processes:

$$f(x;\lambda,k) = \frac{x^{k-1}\lambda^k}{\Gamma(k)}\,e^{-\lambda x}. \tag{2}$$

For our application, f is related to the number of measurement bins which have intensity around I and the parameter k is the number of independent modes which a measurement bin collects. If we define x as the normalized intensity, $x = I/\langle I\rangle$, then $\lambda = k$ and the distribution can be written as a function of a single parameter:

$$f(x,k) = \frac{x^{k-1}k^k}{\Gamma(k)}\,e^{-kx}. \tag{3}$$

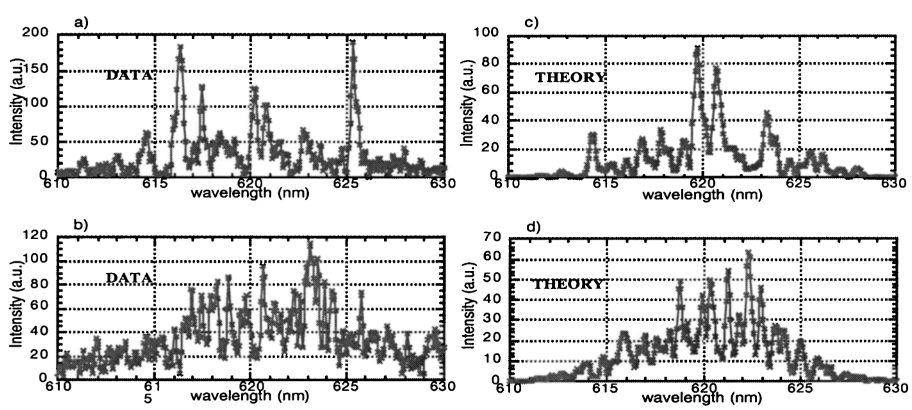

FIG. 2. Comparison of experimental data and simulations. (a) and (b) show typical single shot spectra for a single electron bunch passing through the wiggler for bunch lengths of 1.5 and 4.5 ps, respectively. Numerically generated spectra at the same bunch lengths (c) and (d) reproduce the salient features of the data: level of modulation and characteristic spike width.

For the ideal case of a zero-emittance beam and a diagnostic with sufficient spectral resolution, shot noise results in an exponential distribution (i.e., $k = 1$) since there is only one independent process. 100% fluctuation of the spectral intensity will occur. However, when the beam emittance is sufficiently large or the detector spectral resolution is poor, the spectrum will be similar to a spectrum emitted from several independent sources. The fluctuation level will be reduced, and the spectrum acquires a pedestal as in Fig. 2d.

The parameter k, the product of the number of longitudinal modes, p, and the number of transverse modes, q, can be written explicitly in terms of electron beam bunch length and emittance. The number of longitudinal modes, or the number of independent spikes expected within a frequency measurement bin, is equal to the ratio, $p = (1/\tau_{\rm inst})/(1/\tau_b)$, when $1/\tau_{\rm inst} \gg 1/\tau_b$. The number of transverse modes in the case where the collection angle is held fixed at the single electron radiation opening angle, $1/(\sqrt{N_w}\,\gamma)$, and where the local focal length of the electron beam optics is matched to the radiation Rayleigh range [corresponding to the opening angle of $1/(\sqrt{N_w}\,\gamma)$], is $q = 1 + \frac{\epsilon_b}{\epsilon_r}$ [11]. Here, ϵ_b is the electron beam emittance and $\epsilon_r = \lambda/4\pi$ is the radiation emittance. Thus, in the limit $1/\tau_{\rm inst} \gg 1/\tau_b$,

$$k = pq = \frac{1/\tau_{\rm inst}}{1/\tau_b}\left[1 + \frac{\epsilon_b}{\epsilon_r}\right]. \qquad (4)$$

In the limit $\epsilon_b \ll \epsilon_r$, $q \to 1$ and the fluctuations contain only bunch length information.

The distributions of the intensity fluctuations for two different bunch lengths were analyzed and are shown in Fig. 3(a). Thirty spectra each were used at each bunch length. Good agreement was obtained by fitting a gamma distribution for both the short (1.5 ps) and long (4.5 ps) bunch length using $k = 2.0$ and 3.6, respectively. These values for k are consistent with those expected for the beam emittance and resolution of the spectrometer used in the experiment.

To extract the bunch length and emittance from the spectral data, the correlation of spectral intensity, $C_{\rm meas}(n)$, was computed as a function of n:

$$C_{\rm meas}(n) = \langle I(\omega_i)I(\omega_{i+n})\rangle/\langle I(\omega_i)^2\rangle, \qquad (5)$$

where $I(\omega_i)$ is the spectral intensity in the ith frequency bin and n is the shift in frequency bin. The Fourier transform of the current distribution, $\rho(\omega)$ can be related to $C_{\rm fit}(n)$ by

$$C_{\rm fit}(n) = (1 - a)\frac{[\rho(\omega_n)^2 * S(\omega_n)]}{[\rho(\omega_0)^2 * S(\omega_0)]} + a. \qquad (6)$$

Instrumental broadening of the spectrum was included by convolving $\rho^2(\omega)$ with the instrumental impulse response, $S(\omega)$. For our experimental setup, $S(\omega)$ was determined by measuring the spectrum of a cw HeNe laser operating at 633 nm and was found to be image intensifier limited,

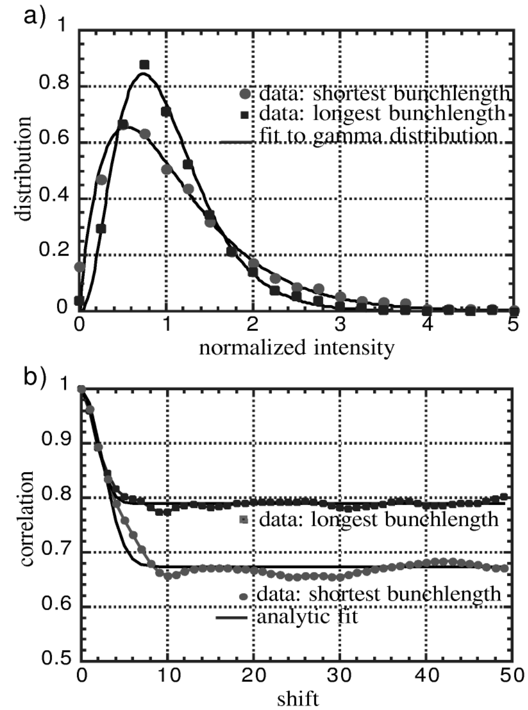

FIG. 3. (a) Measured distribution of intensity fluctuations for shortest (1.5 ps) and longest (4.5 ps) bunch lengths are plotted along with the gamma distribution fit. (b) Average frequency correlation [Eq. (5)] of measured single shot spectra and fit [Eq. (6)] for the shortest and longest bunch lengths.

and of Gaussian form with an rms width of 0.07 nm. The value of the asymptotic level, a, which depends on the number of longitudinal and transverse modes, is given by

$$a = \frac{1}{1 + \frac{1}{k}}, \qquad (7)$$

with k defined as in Eqs. (3) and (4). The correlation function, $C_{\rm fit}(n)$, for small n depends only on beam temporal shape and the known value of instrumental resolution, $\tau_{\rm inst}$, and determines p. The asymptote of $C_{\rm fit}(n)$ at large shift n depends on the product pq. As a result, by performing a fit to $C_{\rm meas}(n)$ over all n, it is possible to extract both τ_b and ϵ_b.

The average of $C_{\rm meas}(n)$ over 30 shots is plotted in Fig. 3(b). In our experiment, the diffraction limited spot size was defined by an iris which limited the collection angle to 0.8 mrad, which was not further limited by the spectrometer. Thus, our collection angle was half $1/\sqrt{N_w}\,\gamma$, increasing the diffraction limited area by a factor of 4, and q is accordingly modified to $q = 1 + \frac{\epsilon_b}{4\epsilon_r}$. The spectrometer permitted several transverse modes to contribute to the spectrum and simultaneous calibrations on bunch length and emittance were performed. In Fig. 4, the reciprocal of bunch length extracted from $C_{\rm fit}(n)$ for each machine setting is plotted against the corresponding time integrated measurement [7] of the reciprocal bunch length. Vertical error bars are statistical for the 30 shots at each setting. In accordance with *a priori* information about bunch

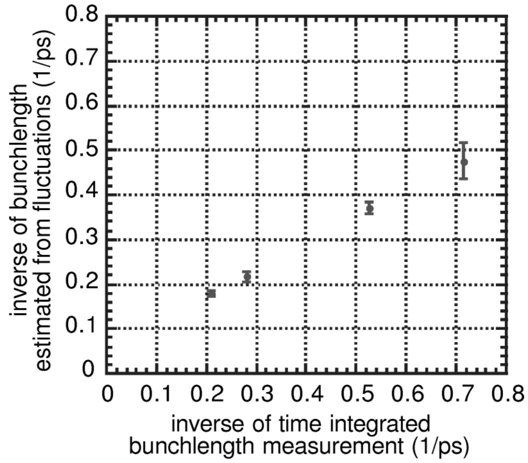

FIG. 4. Reciprocal of the bunch length estimated from fluctuations is compared with time integrated measurements.

shape, bunches of greater than 3 ps were modeled as square in temporal profile; bunches less than 3 ps were assumed to be Gaussian. Although there is a systematic discrepancy between the two bunch length measurements on the order of 35%, given the simple assumptions made on temporal shape, agreement with the independent measurement is surprisingly good. The emittances from the frequency correlation fit of the data were 1.5π mm mrad at 100 pC and 5.5π mm mrad at 400 pC. Emittance estimated from fluctuations increased with bunch length, as expected [9] from space-charge effects.

In general, electron beam diagnosis based on this concept offers a number of desirable features. The technique is nondestructive and favors shorter bunch lengths as the spectrometer resolution requirement is relaxed. This makes it well suited for the diagnosis of sub-ps bunches [12]. The method is applicable over a large range of beam energies, from storage rings to linacs, and can use a wide variety of sources of incoherent radiation. To extend this technique to other regimes of operation, one must choose the detected frequency appropriately to avoid coherent effects (the expected value of $N_b|\rho(\omega)|^2 \ll 1$) and quantum effects (degeneracy parameter $\eta > 1$). For wiggler radiation, the degeneracy parameter can be estimated from

$$\eta \approx \pi \alpha N_b N_w \frac{1}{\omega \tau_b} \frac{a_w^2/2}{1 + a_w^2/2} \frac{1}{q}, \tag{8}$$

where α is the fine structure constant, N_b is the number of electrons in the bunch, N_w is the number of wiggler periods, q is the number of transverse modes, and a_w is the wiggler parameter. If single shot calibrations are desired, the source of incoherent emission and spectrometer resolution should be chosen to maximize the number of recorded spikes in the single-shot spectrum.

In conclusion, the single-shot spectrum of wiggler spontaneous emission has been measured for the first time with the resolution and sensitivity required to study its fluctuational characteristics and to test our understanding of their origin. High brightness microwiggler emissions, at 620 nm from a 44 MeV electron beam, were used to investigate the properties of picosecond bunches with charges as low as 100 pC. Near 100% modulation of the spectrum was demonstrated at the shortest bunch lengths and spikes in the spectrum were observed which depended systematically on beam bunch length. In a proof-of-principle experiment, it is shown that macroscopic properties of an electron beam can be extracted from the fluctuations. Bunch length was extracted from fluctuations and emittance was estimated under the assumption of a matched beam. Agreement was obtained between experiment and theory, and with an independent bunch length measurement technique.

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