High Spectral Coherence in Long-Pulse and Continuous Free-Electron Laser: Measurements and Theoretical Limitations

A. Abramovich, M. Canter, A. Gover, J. Sokolowski, and Y. M. Yakover
Department of Electrical Engineering-Physical Electronics, Faculty of Engineering, Tel-Aviv University, Israel

Y. Pinhasi
Judea and Samaria College, Ariel, Israel

I. Schnitzer and J. Shiloh
Rafael, Haifa 31021, Israel
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Single mode operation was exhibited on the Israeli tandem free-electron laser (FEL). This enabled judicious measurements of narrow spectral linewidth, frequency chirp, and relaxation-oscillation effects. Exact 3D simulations of the FEL oscillator showed good agreement with the measurements, and permitted an estimation of the fundamental Schawlow-Towns linewidth limit of the FEL (including "α effect") as well as technical noise linewidth limits. We estimate that with voltage-controlled stabilization high-power (10 kW) tunable (over 60% bandwidth) quasi-cw coherent $[(\Delta\nu/\nu)_{\text{rms}} \approx 10^{-10}]$ mm-FIR (far infrared) radiation is attainable in the tandem FEL. [S0031-9007(99)09370-9]

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Electrostatic accelerator free-electron lasers (EA-FEL) are different from other types of FELs primarily in their capability to operate with long electron beam pulses. Though none has yet operated continuously, their pulse duration is tens of microseconds long, much longer than the cavity recirculation time. Consequently, their Fourier-transform-limited linewidth is narrower than the intermode spacing contrary to rf accelerator FELs, which are inherently multimode. Since the FEL is an homogeneously broadened laser [1], the nonlinear mode competition process, taking place at the saturation regime, normally drives the EA-FEL oscillator to single mode operation [2–5].

Once single mode operation is attained in an EA-FEL oscillator, the spectral linewidth is fundamentally limited by finite time Fourier transform broadening, and at the cw operation—by the "Schawlow-Towns" expression for the FEL spectral linewidth [6]. These give extremely narrow linewidth limits, and a prospective for attaining a highly coherent and bright spectroscopic source, tunable over a wide spectral region. Of course, as in other lasers and other electron beam oscillators (e.g., gyrotrons) [7], the FEL linewidth will be limited by technical noise before it reaches its fundamental limits. Until now there has been no experimental or computational estimates of this limit.

Recently, lasing has been demonstrated by us, in a new kind of EA-FEL based on an "internal resonator configuration" [8]. This preferred EA-FEL configuration has the potential to operate in a high average power continuous wave (cw) mode. We report here first measurements of the spectral characteristics and of the spectrum evolution processes of this promising EA-FEL configuration. These were also evaluated by us numerically with the aid of an exact nonlinear 3D simulation code (FEL3D [9]) showing good agreement with the experimental measurements and predicting record high spectroscopic qualities of this EA-FEL configuration.

Figure 1 is a schematic of the EA-FEL oscillator in the internal resonator configuration. It would exhibit stable e-beam energy and a cw operation if the beam transport from the cathode to the collector is perfect. If the e-beam current is intercepted in the terminal at a rate larger than the charging current, then the terminal voltage would drop at a rate determined by the capacitance between the terminal and the grounded shell. This limits the laser pulse duration, because when the e-beam energy drops, the FEL gain curve shifts to lower frequency, and the gain of the built-up resonator mode falls down. When it drops below the resonator loss, lasing stops, and a lower frequency mode gets excited and grows up (mode hopping effect [8]).

The nominal parameters of the Israeli Tandem FEL are e-beam voltage and current: $V = 1.4$ MV, $I = 1.5$ A; linear wiggler field, period, and periods number:

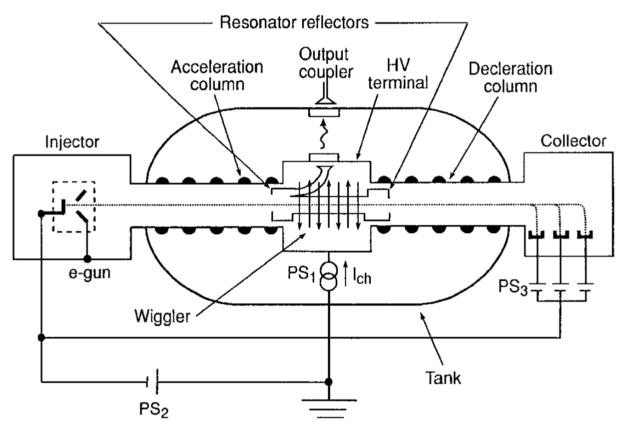

FIG. 1. Schematic of EA-FEL with internal cavity.

$B_w = 2$ kG, $\lambda_w = 4.44$ cm, $N_w = 20$; a curved parallel plates resonator of length 1.308 m and quality factor $Q = 30\,000$ operating at the TE_{01} mode. Radiation pulses up to 30 μsec long were measured, limited by the high voltage (HV) terminal droop. Pulsed power was 10 kW at $\nu \approx 100$ GHz. Very wide range laser tuning (70–110 GHz) was measured, controlled by a wide range variation of the HV—terminal energy (1.1–1.5 MeV).

The spectral characteristics of the FEL radiation and its temporal dependence were measured by mixing a sample of the radiation signal with a stable W-band (70–110 GHz) tunable local oscillator wave in a diode mixer [8]. The intermediate frequency (IF) signal produced by the mixer was recorded using a wide band digitizing scope (4×10^9 samples/sec sampling rate, 1 GHz analog bandwidth, 128 μs recording time). This enables high resolution recording of the frequency shifted radiation signal *field* (amplitude and phase) as a function of time. The recorded digitized signal was then processes and analyzed by an off-line computer. To resolve the spectral evolution of the FEL, we employed a "running-window" fast Fourier transform (FFT) on the IF signal. This time-frequency analysis, also known as Gabor transformation or spectrogram, performs a localized FFT on a windowed section of the signal with spectral resolution of $\sim 1/T_w$, where T_w is the (effective) window width (a Hanning window has been used). In the analysis shown, we "scan" the IF signal with a constant window, $T_w = 1$ μs, which limits our spectral resolution to $\Delta\nu_w = 1$ MHz.

Figure 2 displays a typical spectrogram of the FEL radiation signal. The spectrogram amplitude is displayed on a time-frequency phase plane in a logarithmic scale. Since the mixer-IF output signal shows no distinction between positive and negative beat frequency of the signal (ν_m) and local oscillator (ν_{LO}) waves—$\nu_{\mathrm{IF}} = |\nu_m - \nu_{\mathrm{LO}}|$—one must carefully interpret the discrete frequencies depicted in Fig. 2: $\nu_m = \nu_{\mathrm{LO}} \pm \nu_{\mathrm{IF}}$. Given the free spectral range (FSR) of the resonator longitudinal mode frequencies: $\Delta\nu_{\mathrm{FSR}} \cong v_g/2L_c = 112$ MHz and the local oscillator frequency $\nu_{\mathrm{LO}} = 100\,500 \pm 5$ MHz, we identified in Fig. 2 the dominant mode $\nu_{\mathrm{mo}} = 99\,982 \pm 5$ MHz, the main satellite mode $\nu_{\mathrm{mo}+9} = 100\,983 \pm 5$ MHz (which keeps for 15 μsec at intensity 20 dB below the dominant mode), and five other modes (out of the ≈ 30 possible), which decay within 2–3 μsec after the laser saturation.

Figure 3 displays a high frequency resolution expanded spectrogram of the dominant mode ν_{mo} ($T_w = 1$ μs, $\Delta\nu_w = 1$ MHz). The terminal voltage droop in this experiment was measured to be $\Delta V = -20$ kV in 20 μs, i.e., $\Delta V/\Delta t = -1 \times 10^9$ V/sec. The spectrogram displays a distinct transient of frequency instability (about 3 MHz) as the oscillator enters the saturation region. This is interpreted as a relaxation-oscillation transient effect [10], taking place before the laser arrives at the steady state operation. After the transient, the frequency stabilizes to a linewidth narrower than the spectrogram resolution $\Delta\nu < 1/T_w = 1$ MHz. The temporal resolution of the 1 μsec window suffices to reveal also a distinct negative chirp effect in the pulse: $\Delta\nu/\Delta t = -75 \pm 40$ kHz/μsec. The chirp rate observed in various shots was correlated to the measured terminal voltage droop.

Avoiding in this paper the analysis of the multimode interaction stage [2], we focus our study on the single mode spectral evolution using our single frequency amplifier code FEL3D [9]. It was adopted to simulate an oscillator dynamics by following the round-trip traversal of an initial monochromatic signal: at each traversal the input power of the amplifier $P_{\mathrm{in}}^{(n)}$ is updated by multiplying the output power of the previous traversal $P_{\mathrm{out}}^{(n-1)}$ by the resonator round-trip reflectivity $P_{\mathrm{in}}^{(n)} = RP_{\mathrm{out}}^{(n-1)}$. The code calculates the gain $G^{(n)} = P_{\mathrm{out}}^{(n)}/P_{\mathrm{in}}^{(n)}$ and the interaction region phase shift $\varphi^{(n)}$ in each traversal, and updates the circulating field phase $\Phi^{(n)}$ after each round-trip traversal: $\Phi^{(n)} = \Phi^{(n-1)} + \varphi^{(n)}$. The single path phase shift φ_n is due to the effective index of refraction contributed by any gain medium near the interaction resonance. The

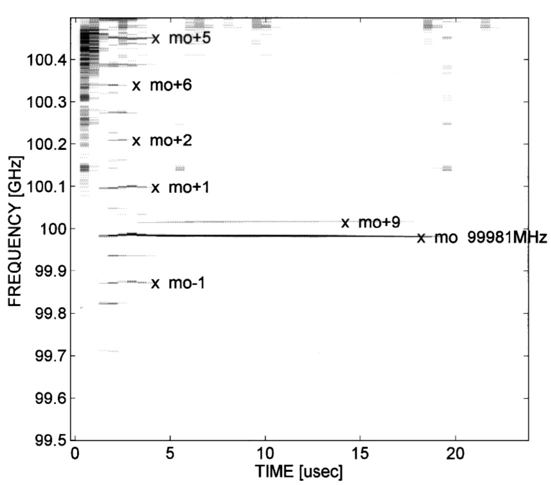

FIG. 2. Spectrogram of the IF signal ($\nu_{\mathrm{LO}} = 100.5$ GHz).

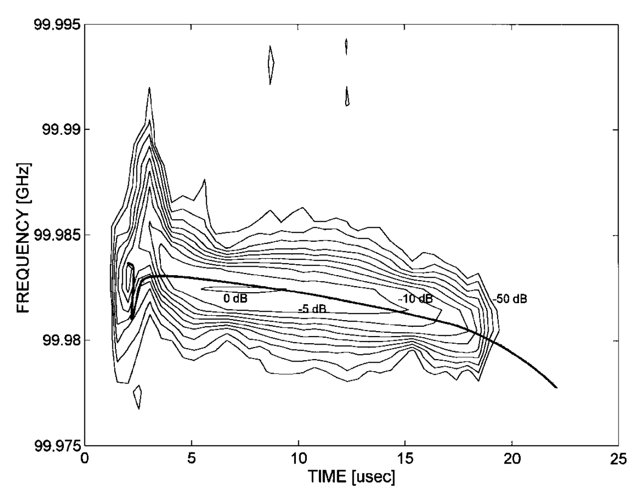

FIG. 3. Expansion of the spectrogram near the dominant mode frequency. Superimposed is the FEL3D simulation data displaying relaxation oscillation and "chirp" effects.

instantaneous frequency shift of the oscillator off the cold resonator mode frequency due to the "loading" of the resonator by the gain medium can be calculated by

$$\Delta\nu^{(n)} = (\Phi^{(n)} - \Phi^{(n-1)})/2\pi t_r = \varphi^{(n)}\Delta\nu_{\mathrm{FSR}}/2\pi\,, \quad (1)$$

where $t_r = 1/\Delta\nu_{\mathrm{FSR}}$ is the round-trip traversal time. This resonator mode frequency shift is equivalent to the frequency pulling effect in lasers [10], and its variation, due to the dependence of φ_n on the wave intensity and other oscillator parameters, is the cause of frequency drift effect, relaxation oscillation, and line broadening.

The instantaneous frequency shift (1) was computed as a function of time for the experimental parameters. The beam voltage droop effect was included by updating the FEL beam voltage after each resonator round-trip. The simulation result is overlayed on the spectrogram of Fig. 3. It displays a "relaxation-oscillation" frequency jump of nearly 2 MHz which takes place exactly as the laser enters saturation, and which is in good qualitative and quantitative agreement with the experimental measurement. Also the chirp rate computed from the slope of the simulated curve $\Delta\nu/\Delta t = -43$ kHz/μs is in reasonable agreement with the experimental measurement $\Delta\nu/\Delta t \cong -75 \pm 40$ kHz/μs. This gives confidence to estimates of important frequency-control parameters of the FEL: $\partial\nu/\partial V = (\Delta\nu/\Delta t)/(\Delta V/\Delta t) =$ 53.7 Hz/V (sim.) = (75 ± 40) Hz/V (expt.); $\partial\varphi/\partial V = (2\pi t_r)\partial\nu/\partial V = 2.9 \times 10^{-6}$ rad/V (sim.) = $(4 \pm 2) \times 10^{-6}$ rad/V (expt.).

Based on the good agreement between the simulation code and the experiment, we found the code reliable and useful for calculating other technical noise and fundamental linewidth parameters of the FEL radiation. Figure 4 displays FEL oscillator response parameters to perturbations of the steady state saturation condition, computed with FEL3D for the experimental parameters.

Figure 4(a) displays the *intracavity* saturated power P_c in the net gain frequency range of the FEL 96–104 GHz, for which steady state oscillation is possible. This saturation power curve was computed with the FEL3D "oscillator code" for the nominal (fixed) experimental parameters $V = 1.4$ MeV, $I = 1.5$ A, $R = 90\%$. Note that the free-running oscillator is likely to operate only at the highest gain (dominant mode) frequency $\nu_{\mathrm{mo}} \cong 99.98$ GHz, however other frequencies (modes) in the net gain range may evolve to single mode operation in the laser by statistical chance or by mode injection [12]. Figures 4(b) and 4(c) display the phase sensitivity parameters $\partial\varphi/\partial V$, $\partial\varphi/\partial I$ computed for each frequency at its saturation power point [Fig. 4(a)]. The experimental measurement of $\partial\varphi/\partial V$, deduced from the frequency chirp data (Fig. 3), is also marked in Fig. 4(b) for the dominant mode frequency $\nu_{\mathrm{mo}} \cong 99.98$ GHz, giving benchmark credibility to the computation model.

At the steady state single mode operation, the phase accumulation of the circulating mode in the resonator in each round-trip is exactly $2\pi m$. The phase Φ and amplitude $|\tilde{E}|$

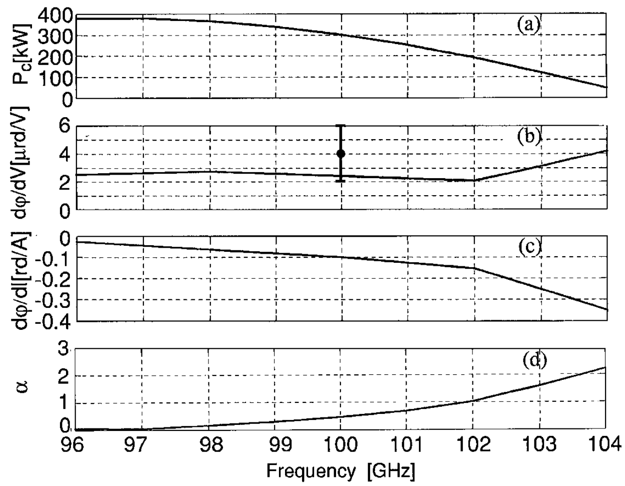

FIG. 4. Computed FEL parameters: (a) oscillator saturation circulated power at different frequencies. (b) Single traversal phase shift sensitivity to voltage variation at saturation (experimental data point marked). (c) Sensitivity of phase shift to current variation. (d) Sensitivity of phase shift to stored power variation (α).

of the circulating wave $E(t) = \mathrm{Re}(|\tilde{E}|\exp(i\Phi + i\omega_m t))$ at any point in the resonator are *fixed* (here ω_m is the frequency of the mode in the *loaded* cavity). Laser line spectral broadening takes place due to instantaneous random fluctuations of the phase Φ. These fluctuations around the saturation steady state condition can be caused by continuous admixture of incoherent radiation (spontaneous emission or black body radiation) with the resonator coherent circulating power, or by phase shifts in the FEL transfer function phase (φ) due to random changes in operating parameters, such as beam voltage and current. The line broadening of the first kind is the Schawlow-Towns and Gordon-Zeiger-Towns fundamental noise limits (extended in [6] to FEL). The second kind (along with other causes such as vibrations and thermal effects) is technical noise linewidth and can be reduced by technical improvements.

Statistical analysis of the spectral power of the laser field [10], under the assumption that the phase fluctuations result from additive white Gaussian noise, results in a Lorenzian line-shape function expression of width,

$$\Delta\nu_{\mathrm{laser}} = \langle(\Delta\Phi)^2\rangle/2\pi t_c\,, \quad (2)$$

where $\Delta\Phi = \Phi - \langle\Phi\rangle$, and t_c is the cavity decay time: $(2\pi t_c)^{-1} = \nu/Q = \Delta\nu_{1/2} = \Delta\nu_{\mathrm{FSR}}/F$, where $F = \pi\sqrt{R}/(1 - R)$ is the finesse of the resonator. Another parameter, useful for frequency stability and "clock" applications is the rms value of the frequency deviation, $\Delta\nu(t) = (d\phi/dt)/2\pi$,

$$\Delta\nu_{\mathrm{rms}} = \langle(\Delta\Phi)^2\rangle^{1/2}/4\pi t_c\,. \quad (3)$$

We can now improve the Schawlow-Towns expression for the fundamental FEL linewidth derived in [6] by including the effect of phase noise due to amplitude noise modulation [10]. Adding an input noise field of random phase Ψ to the circulating power field $E(t) = \mathrm{Re}[(|\tilde{E}_c| +$

$E_n e^{i\Psi})e^{i\omega_m t}]$ not only changes the resultant field phase instantaneously, but also changes its amplitude, and, consequently, causes an additional phase shift due to the dependence of the FEL transfer function phase φ on the input field in the nonlinear gain regime: $\Delta\Phi = (E_n/|\tilde{E}_c|) \times [\sin\Psi + |\tilde{E}_c|(\partial\varphi/\partial|\tilde{E}_c|)\cos\Psi]$. This results in

$$\langle(\Delta\Phi)^2\rangle = \pi S_n(\omega)\Delta\nu_{\mathrm{FSR}}(1 + \alpha^2)/P_{\mathrm{gen}}\,, \qquad (4)$$

where $S_n(\omega) \equiv dP_n(\omega)/d\omega = R_q \mathrm{sinc}^2(\overline{\theta}/2)$ [where $\mathrm{sinc}x = \sin(x)/x$] is the FEL spectral power due to spontaneous emission [6], $P_{\mathrm{gen}} = (1 - R)P_c$ is the coherent power generation in the resonator at steady state (equal to the out-coupled power if there are no losses), and $\alpha = 4(\partial\varphi/\partial P_c)/(3R\partial G/\partial P_c)$. The parameter α is the amplitude-noise phase-modulation parameter of the standard laser linewidth theory [10], generalized to any kind of laser medium (not necessarily uniform but with unidirectional gain or in a ring resonator) of gain G and round-trip reflectivity R. For the parameters of the tandem FEL, α was computed with the aid of the FEL3D code and drawn in Fig. 4(d) for different frequencies at their corresponding saturation power point. Equation (4), substituted in (2) results in the Schawlow-Towns fundamental linewidth expression for FEL [6] with the additional α term.

We can take advantage of the FEL3D code simulations, corroborated by our spectral measurements, in order to estimate the technical noise contribution to the linewidth and frequency stability of the FEL due to voltage and current fluctuations. These fluctuations produce a variance in the resonator field according to

$$\langle(\Delta\Phi)^2\rangle = (\partial\varphi/\partial V)^2\langle(\Delta V)^2\rangle + (\partial\varphi/\partial I)^2\langle(\Delta I)^2\rangle\,. \quad (5)$$

If the voltage and current fluctuations could be described in terms of a white Gaussian noise statistical model [7], one could use (5) in (2) and (3) and find a technical noise line broadening correction to the fundamental noise expressions. Our experimental observation was that the significant high voltage fluctuations (besides the constant voltage droop) were of long term (msec) scale and approximate amplitude $\Delta V_{\mathrm{rms}} \approx 1$ kV. Their effect should rather be described as in the chirp effect by substituting (5) into (1). For these parameters and $\partial\varphi/\partial V = 53$ Hz/V [Fig. 4(b)], we obtain $(\Delta\nu)_{V,\mathrm{rms}} = \Delta V_{\mathrm{rms}}(\partial\varphi/\partial V)/2\pi t_r = 53$ kHz. Similarly, for estimated current instability $\Delta I_{\mathrm{rms}} = 10$ mA and $\partial\varphi/\partial I = 1.8$ MHz/A [Fig. 4(c)] we obtain $(\Delta\nu)_{I,\mathrm{rms}} = \Delta I_{\mathrm{rms}}(\partial\varphi/\partial I)/(2\pi t_r) = 18$ kHz. The estimated technical noise frequency instability is consequently $(\Delta\nu/\nu)_{\mathrm{tech}} = 5.5 \times 10^{-7}$.

Based on the experimental data analysis and on the computed parameters, we can now assess the potential of the EA-FEL as a prospective high quality spectroscopic source. We evidenced attainability of the single mode operation even with pulses as short as 20 μsec and despite a voltage droop of 20–40 keV. We showed that wide range tunability of 60% around 100 GHz is possible. Continuous frequency tuning across the free spectral range $\Delta\nu_{\mathrm{FSR}}$ can be easily attained with mirror adjustment. Furthermore, we found out that electronic fine tuning is possible. Tuning range of $\Delta\nu = (\partial\nu/\partial V)\Delta V \cong 1$ MHz may be attained by voltage control of up to $\Delta V = 30$ kV. We suggest that this handle can be used also for phase-locked-loop stabilization of the laser.

The fundamental *spectral linewidth* limit [(2) and (4)] is minuscule for the tandem FEL: $(\Delta\nu/\nu)_{\mathrm{laser}} = 4.3 \times 10^{-16}$. The α^2 parameter, which in some lasers has a dominant contribution, was found [Fig. 4(d)] to be less than unity (at least for the parameters of the present tandem FEL example) despite the fact that the FEL gain curve is not symmetrical [10]. The fundamental *rms frequency instability* [(3) and (4)] is $(\Delta\nu/\nu)_{\mathrm{rms}} = 6 \times 10^{-11}$.

The tandem-FEL linewidth was measured to be below $\Delta\nu = 1$ MHz and is probably limited presently (after elimination of the chirp) only by the pulse duration ($T \approx 20$ μs) Fourier-transform limit: $(\Delta\nu/\nu)_T = 5 \times 10^{-7}$. This and the line center frequency instability $(\Delta\nu/\nu)_{\mathrm{tech}}$ are 4 orders of magnitude above the fundamental rms value $(\Delta\nu/\nu)_{\mathrm{rms}} = 6 \times 10^{-11}$. With voltage stabilization and with phase-locked-loop frequency locking to a stable subharmonic source, it may be possible to get close to this fundamental limit. Of course, in order to observe it, the stabilized FEL must operate quasicontinuously ($T > 1$ sec), to facilitate a narrower Fourier transform linewidth broadening $[(\Delta\nu/\nu)_T < 10^{-11}]$.

We conclude that if efforts to develop cw or quasi-cw EA-FEL are successful, it should be possible to attain a source that is not only extremely powerful but also exhibits narrow linewidth, fine tuning, and frequency control over a wide spectral range in the mm-FIR regime.

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