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Quantum Limit of Decoherence: Environment Induced Superselection of Energy Eigenstates

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We investigate decoherence in the limit where the interaction with the environment is weak and the evolution is dominated by the self-Hamiltonian of the system. We show that in this case quantized eigenstates of energy emerge as pointer states selected through the predictability sieve. [S0031-9007(99)09512-5]

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The relevance of decoherence in the context of the quantum to classical transition has been recently recognized [1–3]. The basic idea is that classicality is an emergent property induced in open quantum systems by their environments. Because of the interaction with the environment, the vast majority of states in the Hilbert space of a quantum open system becomes highly unstable to entangling interaction with the environment, which in effect monitors selected observables of the system. After a decoherence time, which for macroscopic objects is typically many orders of magnitude shorter than any other dynamical time scale [4], a generic quantum state decays into a mixture of "pointer states." In this way the environment induces effective superselection rules ("einselection") thus precluding the stable existence of superpositions of pointer states. Experimental testing of decoherence has been recently initiated [5]. Here we investigate decoherence in the limit of weak interaction and show that it can enforce "quantum jumps" between discrete energy eigenstates which now become stable pointer states.

Pointer states are distinguished by their ability to persist in spite of the environmental monitoring and therefore are the ones in which quantum open systems are observed. Understanding the nature of these states and the process of their dynamical selection is of fundamental importance. This process has been studied first in a measurement situation: When the system is an apparatus whose intrinsic dynamics can be neglected, pointer states turn out to be eigenstates of the interaction Hamiltonian between the apparatus and its environment [1]. Even in this idealized limit, decoherence may be experimentally testable [6]. In more general situations, when the system's dynamics is relevant, einselection is more complicated. Pointer states result from the interplay between self-evolution and environmental monitoring.

To study einselection, an operational definition of pointer states has been introduced [7,8]. This is the "predictability sieve" criterion, based on an intuitive idea: Pointer states can be defined as the ones which become minimally entangled with the environment in the course of the evolution. The predictability sieve criterion is a way to quantify this idea by using the following algorithmic procedure: For every initial pure state $|\Psi\rangle$, one measures the entanglement generated dynamically between the system and the environment by computing the entropy $\mathcal{H}_\Psi(t) = -\mathrm{Tr}[\rho_\Psi(t)\log\rho_\Psi(t)]$ or some other measure of predictability [7–9] from the reduced density matrix of the system $\rho_\Psi(t)$ [which is initially $\rho_\Psi(0) = |\Psi\rangle\langle\Psi|$]. The entropy is a function of time and a functional of the initial state $|\Psi\rangle$. Pointer states are obtained by minimizing $\mathcal{H}_\Psi$ over $|\Psi\rangle$ and demanding that the answer be robust when varying the time t.

The nature of pointer states has been investigated using the predictability sieve criterion only for a limited number of examples [9]. Apart from the case of the measurement situation (where pointer states are eigenstates of the interaction Hamiltonian) the most notable example is that of a Brownian particle interacting through the position with a bath of independent harmonic oscillators. In such case pointer states are localized in phase space, even though the interaction Hamiltonian involves the position of the particle [8]. Pointer states are the result of the interplay between self-evolution and interaction with the environment and turn out to be coherent states.

One may think that the above is a generic situation. However in nature there are systems which are not found in localized states but in eigenstates of energy. Therefore, it is natural to ask how can this be possible. A conceivable answer could be that such systems (the electron in an atom, for example) are coupled to their environments through their self-Hamiltonian [10]. But this suggestion is easily dismissed because the form of the interaction is rather universal and generically local [1].

In this Letter we will show that, for a large class of circumstances, even when the interaction between the system and the environment depends on position (or other, essentially arbitrary observable), the pointer states selected through decoherence turn out to be energy eigenstates. We will illustrate this emergence of quantized energy as a pointer observable by using the simple but physically relevant example of a particle interacting locally with a quantum scalar field. The nature of pointer states—we shall see—strongly depends upon the spectral density of the environment and, when the dominant frequencies present in the environment are slow with respect to the system's own time scale, pointer states turn out to be eigenstates of energy. On the other hand, only when the environment modes include frequencies comparable or higher than the ones associated with the system are the pointer states localized in phase space.

We will consider the following simple model: The system is a particle with position $\vec{x}$ (moving in a N-dimensional space) and the environment is a quantum scalar field ϕ. The interaction is local and is described by the Hamiltonian $H_{\rm int} = e\phi(\vec{x})$. Expanding the scalar field in normal modes, the Hamiltonian can be written as $H_{\rm int} = \int d^N\vec{k}\,[h_{\vec{k}} \exp(i\vec{k}\vec{x}) + \text{H.c.}]$ where the Fourier components $h_{\vec{k}}$ are proportional to anihillation operators of the quantum field [i.e., $h_{\vec{k}} = ea_{\vec{k}}/(2\pi)^{N/2}(2\omega_k)^{1/2}$]. More generally, we consider models in which the particle-field interaction is slightly nonlocal taking into account the finite extent of the particle. In this case, the interaction Hamiltonian $\tilde{H}_{\rm int} = e\int d^N\vec{y}\, W(\vec{x}-\vec{y})\phi(\vec{y})$ depends upon the window function $W(\vec{r})$ (whose support is a sphere of radius R, the Compton radius of the particle, centered around the origin). This nonlocal interaction corresponds to a Hamiltonian whose Fourier components $h_{\vec{k}}$ are multiplied by $\hat{W}(\vec{k})$ [the transform of $W(\vec{r})$].

For this class of models we can derive a master equation for the reduced density matrix of the particle. This equation is obtained under two assumptions: (i) an expansion up to second order in perturbation theory, and (ii) initial states with no correlations between the system and the environment (the initial state of the environment being thermal equilibrium). The master equation reads [11]

$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho] - \frac{e^2}{\hbar^2}\int d^N\vec{k}\int_0^t dt_1 \times \big(G_H(\vec{k},t_1)\,[e^{i\vec{k}\vec{x}},[e^{-i\vec{k}\vec{x}(-t_1)},\rho]] - -iG_R(\vec{k},t_1)\,[e^{i\vec{k}\vec{x}},\{e^{-i\vec{k}\vec{x}(-t_1)},\rho\}]\big)\,. \tag{1}$$

Here, $\vec{x}(t)$ is the Heisenberg position operator for the particle (evolved with the free Hamiltonian H) and $G_{R,H}(\vec{k},t)$ are the Fourier transform of the retarded and symmetric two point functions of the scalar field (multiplied by the appropriate window function if the interaction is nonlocal). When the environment is a free field, $G_R(\vec{k},t) = W(\vec{k})\sin(\omega_{\vec{k}}t)/2\omega_{\vec{k}}$, $G_H(\vec{k},t) = W(\vec{k})\cos(\omega_{\vec{k}}t)\,(1+2N_k)/2\omega_{\vec{k}}$, where N_k is the number density of particles in the initial state of the quantum field (the above result is valid if the field is not free in which case the propagators are appropriately dressed). The master Eq. (1) is extremely rich. One of its most interesting features is that it is local in time [note that the density matrix appearing in the right-hand side of (1) is evaluated at time t]. We will use this equation to derive the main results of this Letter. But before, it is useful to show how some known results follow from Eq. (1).

The most widely used approximation for the particle-field model is the dipole approximation (dominant wavelengths in the environment are much larger than the length scale over which the position of the particle varies). Expanding up to second order ($\vec{k}\vec{x} \ll 1$) we obtain

$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho] - \frac{e^2}{\hbar^2}\int_0^t dt_1 \times \big(F_H(t_1)\,[\vec{x},[\vec{x}(-t_1),\rho]] - iF_R(t_1)\,[\vec{x},\{\vec{x}(-t_1),\rho\}]\big)\,, \tag{2}$$

where $F_{R,H}(t_1) = \int d^N\vec{k}\,\vec{k}^2 G_{R,H}(\vec{k},t_1)/N(2\pi)^{N/2}$.

One can recognize (2) as the master equation of a Brownian particle interacting with an environment of oscillators. It can be further simplified for a harmonic oscillator with frequency Ω since in this case, the Heisenberg operator is $x(t) = x\cos(\Omega t) + \frac{1}{m\Omega}p\sin(\Omega t)$. Using this, we rewrite the master equation as

$$\dot{\rho} = -\frac{i}{\hbar}\left[H + \frac{1}{2}m\tilde{\Omega}^2(t)x^2,\rho\right] + 2i\gamma(t)\,[\vec{x},\{\vec{p},\rho\}] - D(t)\,[\vec{x},[\vec{x},\rho]] - f(t)[\vec{x},[\vec{p},\rho]]\,. \tag{3}$$

Here the time dependent coefficients [the frequency renormalization $\tilde{\Omega}(t)$, the damping coefficient $\gamma(t)$, and the two diffusion coefficients $D(t)$ and $f(t)$] are

$$\begin{aligned}
\tilde{\Omega}^2(t) &= -\frac{2\hbar}{m}\int_0^t dt' \cos(\Omega t') F_R(t'), \\
\gamma(t) &= -\frac{\hbar}{2m\Omega}\int_0^t dt' \sin(\Omega t') F_R(t'), \\
D(t) &= \int_0^t dt' \cos(\Omega t') F_H(t'), \\
f(t) &= \frac{1}{m\Omega}\int_0^t dt' \sin(\Omega t') F_H(t').
\end{aligned} \tag{4}$$

The existence of a local master equation for linear Brownian motion was recognized some time ago [12]. In fact, for this system an *exact* local master equation can always be obtained under the sole assumption of uncorrelated initial conditions [13]. Equation (3) has the same form as the exact master equation and its coefficients (4) coincide with the perturbative expansion of the exact ones. The particularly simple form of the perturbative coefficients is worth noting. This equation has been used to study the nature of pointer states in Brownian motion models. In fact, if the particle moves in one dimension ($N = 1$) and the high frequency cutoff [present in $W(k)$] is much larger than Ω we have $F_R(t) \propto \delta'(t)$. Moreover, in the limit of high temperatures we also see that $F_H(t) \propto \delta(t)$. Using this in Eq. (4) we find that the coefficients appearing in the master equation are approximately constant and using this we can prove that pointer states tend to be localized in phase space [8].

To investigate the quantum limit of decoherence we analyze Eq. (3) in the opposite regime considering the case in which the frequencies in the environment are much lower than Ω (the frequency of the system). In this case $F_{R,H}(t)$ vary slowly in time and could be taken outside the integrals in (4). Therefore, the coefficients of the master equation turn out to be oscillatory functions [actually, as $F_R(0) = 0$ we have $\tilde{\Omega}(t) \approx \gamma(t) \approx 0$, while the diffusion coefficients oscillate with an amplitude proportional to $F_H(0)$]. Thus, one may be tempted to conclude that an adiabatic environment does not dynamically select any preferred basis of the system. However, we will now show that this type environment can lead to decoherence, although it can no longer impose its own preferences for the pointer states which turn out to be the eigenstates of the system Hamiltonian. To obtain this result we take a step back and analyze the master Eq. (1) without the dipole approximation. The master (1) is

$$\begin{aligned}
\dot{\rho} = -\frac{i}{\hbar}[H,\rho] - \sum_k \int_0^t dt_1 \\
\times \big(c_k(t_1)[S_k,[S_k^\dagger(-t_1),\rho]] \\
- -ic_k'(t_1)[S_k,\{S_k^\dagger(-t_1),\rho\}]\big),
\end{aligned} \tag{5}$$

where we have written the integral over $\vec{k}$ as a sum, and introduced the operators $S_k = \exp(i\vec{k}\vec{x})$, which act on the system's Hilbert space and the functions $c_k(t)$ and $c_k'(t)$ (proportional to the Fourier transform of the symmetric and retarded propagators of the field). When the environment behaves adiabatically we can rewrite this equation. Thus, if $c_k(t)$ and $c_k'(t)$ are slowly varying functions one can take them outside the temporal integrals which can then be performed after writing the operator $S_k(t)$ in the energy eigenbasis. Assuming that the system has a nondegenerate spectrum the resulting master equation [obtained from (5) by averaging over the largest Bohr period] can be written in the energy eigenbasis as

$$\dot{\rho}_{nm} = -i\omega_{nm}\rho_{nm} - \gamma_{nm}^2 t\rho_{nm} - t\sum_{k,j} A_{kjnm}\rho_{kj}, \tag{6}$$

where $\omega_{nm} = (E_n - E_m)/\hbar$ and γ_{nm}^2, A_{lnm} and B_{lnm} depend on c_k and c_k' and on the matrix elements of S_k in the energy eigenstates $|\phi_n\rangle$. For example, $\gamma_{nm}^2 = \sum_k c_k(S_k^{(nn)} - S_k^{(mm)})^2$ where $S_k^{(nm)} = \langle\phi_n|S_k|\phi_m\rangle$.

This equation enables us to derive our main result. On the one hand, we can see that an environment behaving adiabatically does not produce any change in the population of energy eigenstates. On the other hand, the evolution of nondiagonal elements is dominated by the second term on the right-hand side of (6) which implies that they decay at a rate γ_{nm} which is determined by the sum of the squared differences between the corresponding expectation values of the operators S_k. Thus, neglecting the contribution of the last terms in (6) (they will generally have alternating signs and therefore a small net effect), we find that $\rho_{nm}(t) \approx \rho_{nm}(0)\exp(-i\omega_{nm}t)\exp(-t^2\gamma_{nm}^2)$. It is now straightforward to see that energy eigenstates are perfect pointer states and (at the approximation level adopted above) *produce no entropy*. Thus, we have shown that energy eigenstates are the ones which are selected by the environment as the pointer states when the environment behaves adiabatically. As the decay rate for nondiagonal elements depends on the difference between the corresponding diagonal elements of S_k in the energy eigenbasis, for decoherence to take place these operators must have nonvanishing diagonal elements in the energy eigenbasis. This is the reason why this effect could not be found using the master equation for quantum Brownian motion, which is obtained by applying the dipole approximation to Eq. (1): In fact, in that case $S_k \approx 1 - i\vec{k}\vec{x}$, whose $\vec{x}$-dependent part has vanishing diagonal elements in energy eigenbasis. The range of times over which our conclusions are valid is not limited by the use of a perturbative expansion in the equations above but by the adiabatic approximation. The following simple argument shows that this is indeed the case: Consider an initial pure state of the system-environment ensemble $|\Psi(0)\rangle = \sum_n \alpha_n|\phi_n\rangle|E_0\rangle$ where $|E_0\rangle$ is the initial state of the environment. Solving the Schrödinger equation in the adiabatic approximation one finds $|\Psi(t)\rangle = \sum_n \exp(-i\omega_n t)|\phi_n\rangle|E_n(t)\rangle$, where the states of the environment correlating with different energy eigenstates are $|E_n(t)\rangle = \exp(-it\langle\phi_n|H_{\rm int}|\phi_n\rangle/\hbar)\,|E_0\rangle$. The overlap $\langle E_n(t)\,|\,E_m(t)\rangle$ can be computed for some simple examples. Thus, when $\langle\phi_n|H_{\rm int}|\phi_n\rangle$ acts as a displacement operator for the environment (as in the above particle-field model) it can be estimated to be $\langle E_n(t)\,|\,E_m(t)\rangle \approx \exp(-\gamma_{nm}^2 t^2)$ where γ_{nm} is the coefficient appearing in (6). Thus, decoherence occurs at the predicted rate as long as the adiabatic approximation is accurate (times must be shorter than the environmental time scale, which we assumed to be the longest in the problem).

There are three basically distinct regimes in which one can analyze the properties of the pointer states selected through decoherence. They differ through the relative strength of the self-Hamiltonian and of the Hamiltonian of interaction. The first one is the measurement situation [1] where the self-Hamiltonian of the system is negligible and the evolution is completely dominated by the interaction with the environment. In such case, pointer states are eigenstates of the interaction Hamiltonian. The second, most common and complex situation occurs when neither the self-Hamiltonian nor the interaction with the environment is clearly dominant. Then the pointer states arise from a compromise between self-evolution and interaction. The most widely studied example of this situation is the Brownian motion model for which pointer states become localized in phase space. The third situation completing this picture is the one we analyzed in this Letter. It corresponds to the case where the dynamics is dominated by the system's self-Hamiltonian. In this case einselection produces pointer states which coincide with the energy eigenstates of the self-Hamiltonian.

Our conclusion conforms with the heuristic picture of decoherence and einselection: The environment "monitors" certain states and, by doing so, elevates them to the pointer state status. In absence of the self-Hamiltonian this leads to the selection of eigenstates of the interaction Hamiltonian. However when energy eigenstates are separated by more than the highest energies present in the environment a "protective monitoring" of energy eigenstates will ensue [14]. Thus, an environment coupling to the system through nearly any observable will become correlated with an energy eigenstate, because time average of any observable over a time necessary to establish correlations (achieving orthogonality of records imprinted in the environment) can depend on the only nonoscillating quantity: energy [15].

Quantum jumps are a proverbial characteristic of quantum theory, and an old subject of heated debates. Schrödinger had hoped that his equation would do away with the discreteness, replacing jumps with a comprehensible or at least continuous process. He became physically ill when Bohr convinced him otherwise, and was never reconciled with this conclusion [16]. The jumps were a phenomenological, but phenomenally successful rule of thumb introduced by Bohr in the old quantum theory. However, in absence of some sort of collapse postulate they are difficult to understand within the purely unitary evolution of a closed quantum system, as it is illustrated by recent exchanges of comments [17]. We have now seen in one—quite generic—set of circumstances how the telltale quantum discreteness emerges when the continuum of Schrödinger evolution is sieved out by einselection. Remarkably, even the discreteness of quantum physics appears to be in part traceable to decoherence.

We have shown how the Schrödinger equation alone suffices to bring about discreteness by enforcing einslection of energy eigenstates, providing that the environment is included in the considerations. Granted, the discrete spectrum of the self-Hamiltonian is a necessary, but—alone—an insufficient condition: Superposition principle would demand "equal rights" for arbitrary superpositions of energy eigenstates. Yet, in the presence of an adiabatic environment, eigenstates of the self-Hamiltonian are selected assuming the role of pointer states.

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[1] W. H. Zurek, Phys. Rev. D **24**, 1516 (1981); *ibid.* **26**, 1862 (1982).
[2] E. Joos and H. D. Zeh, Z. Phys. B **59**, 223 (1985); D. Giulini *et al., Decoherence and the Appearance of Classical World in Quantum Theory* (Springer-Verlag, Berlin, 1996).
[3] W. H. Zurek, Phys. Today **44**, No. 10, 36–44 (1991); see also Philos. Trans. R. Soc. London A **356**, 1793 (1998).
[4] W. H. Zurek, in *Frontiers in Nonequilibrium Statistical Physics,* edited by G. T. Moore and M. O. Scully (Plenum, New York, 1984); J. P. Paz, S. Habib, and W. H. Zurek, Phys. Rev. D **47**, 488–501 (1993).
[5] M. Brune *et al.,* Phys. Rev. Lett. **77**, 4887 (1996); see also Phys. Today **51**, No. 7, 36 (1998).
[6] J. F. Poyatos, J. Cirac, and P. Zoller, Phys. Rev. Lett. **77**, 4728 (1996) propose environment engineering to "dial up" pointer states in an ion trap setting where "Schrödinger kittens," ideal for studies of decoherence, were already seen [see C. Monroe, D. Meekhof, B. E. King, and D. J. Wineland, Science **272**, 1131 (1996)].
[7] W. H. Zurek, Prog. Theor. Phys. **89**, 281–312 (1993).
[8] W. H. Zurek, S. Habib, and J. P. Paz, Phys. Rev. Lett. **70**, 1187 (1993).
[9] M. Tegmark and H. S. Shapiro, Phys. Rev. E **50**, 2538–2547 (1994); M. R. Gallis, Phys. Rev. A **53**, 655–660 (1996); J. R. Anglin and W. H. Zurek, Phys. Rev. D **53**, 7327 (1996); S. M. Barnett, K. Burnett, and J. A. Vacarro, J. Res. Natl. Inst. Stand. Technol. **101**, 593–600 (1996); H. M. Wiseman and J. A. Vaccaro, quant-ph/9709014.

[10] D. F. Walls, M. J. Collet, and G. J. Milburn, Phys. Rev. D **32**, 3208–3215 (1985).

[11] This equation can be derived in the weak coupling non-Markovian limit following methods presented in S. Chaturvedy and F. Shibata, Z. Phys. B **35**, 297 (1979) [see also M. Despósito and S. H. Hernández, Physica (Amsterdam) **227A**, 248 (1996)]. A simpler derivation follows the argument presented in D. F. Walls and G. J. Milburn, *Quantum Optics* (Springer-Verlag, Berlin, 1994).

[12] B. L. Hu, J. P. Paz, and Y. Zhang, Phys. Rev. D **45**, 2843 (1992).

[13] L. Dávila Romero and J. P. Paz, Phys. Rev. A **53**, 4070 (1997).

[14] Y. Aharonov, J. Anandan, and L. Vaidman, Phys. Rev. A **47**, 4616 (1993); W. Unruh, e-print hep-th/9410168.

[15] The stability of the widely separated energy eigenstates can be anticipated to some extent from Fermi's "golden rule," which implies that transitions which almost conserve energy should dominate in presence of slow perturbations.

[16] E. Schrödinger, Brit. J. Phil. Sci. **3**, 109 (1952).

[17] H. D. Zeh, Phys. Lett. A **172**, 189 (1992); N. Gisin and I. C. Percival, Phys. Lett. A **167**, 315 (1992).