Bokil *et al.* Reply: Using Monte Carlo simulations (MCS) and the Migdal-Kadanoff approximation (MKA), Marinari *et al.* study in their Comment [1] on our paper [2] the link overlap between two replicas of a three-dimensional Ising spin glass in the presence of a coupling between the replicas. They claim that the results of the MCS indicate replica symmetry breaking (RSB), while those of the MKA are trivial, and that moderate size lattices display the true low temperature behavior. Here we show that these claims are incorrect, and that the results of MCS and MKA both can be explained within the droplet picture.

The link overlap is defined as $q^{(L)}(\epsilon) = (1/3V) \times \sum\langle\sigma_i\sigma_j\tau_i\tau_j\rangle$ where the sum is over all nearest-neighbor pairs $\{ij\}$, and the brackets denote the thermal and disorder average. σ and τ denote the spins in the two replicas. The Hamiltonian used for the evaluation of the thermodynamic average is $H[\sigma,\tau] = H_0[\sigma] + H_0[\tau] - \epsilon\sum\sigma_i\sigma_j\tau_i\tau_j$, where H_0 is the ordinary spin glass Hamiltonian. For the subsequent discussion, it is useful to write $q^{(\infty)}(\epsilon)$ in the form $q^{(\infty)}(\epsilon) = q_+ + A_+|\epsilon|^{\lambda_+}$ for $\epsilon > 0$ and $q^{(\infty)}(\epsilon) = q_- + A_-|\epsilon|^{\lambda_-}$ for $\epsilon < 0$.

In the mean-field RSB picture, $q_+ > q_-$, and $\lambda_+ = \lambda_- = 1/2$, and Marinari *et al.* claim to see a trend towards this discontinuous behavior in their MCS data (Fig. 1 of [1]). Alternatively, if they assume continuous behavior, they find a value $\lambda_\pm \simeq 0.25$. These conclusions are based on the assumptions that there are no corrections to the pure power-law behavior, and that $\lambda_+ = \lambda_-$. However, neither assumption is justified, and the most natural interpretation of Fig. 1 of [1] is $q_+ = q_-$, and $\lambda_\pm \simeq 1/2$.

This result, as well as the results of the MKA, is, in fact, fully compatible with the droplet picture. Using scaling arguments similar to those in [3], the value of λ_- and λ_+ at low temperatures can be derived in the following way: The energy cost of the formation of a spin-flipped "droplet" of radius l in one of the replicas is of the order $l^\theta + \epsilon l^{d_s}$, where d_s denotes the fractal dimension of the droplet surface, and θ is the scaling dimension for the domain wall energy. For negative ϵ, droplets of a characteristic size $l^* \sim (1/|\epsilon|)^{1/(d_s-\theta)}$ are formed, since they lower the energy of the system. Since flipping a cluster affects only links on the surface of the cluster, this gives $q^{(\infty)}(\epsilon) \simeq q^{(\infty)}(0) - C|\epsilon|^{(d-d_s)/(d_s-\theta)}$. Within the MKA $d_s = d - 1$ and $\theta \simeq 0.24$, leading to $\lambda_- \simeq 0.57$. For a cubic lattice, one has $\theta \simeq 0.2$, and $d_s \simeq 2.2$, leading to $\lambda_- \simeq 0.4$. For positive ϵ, the leading correction to the link overlap comes from the suppression of the thermal excitation of large droplets and has for low temperatures the form $q^{(\infty)}(\epsilon) - q^{(\infty)}(0) \simeq k_BT(\epsilon/k_BT)^{(d+\theta-d_s)/d_s}$, leading to $\lambda_+ = (d + \theta - d_s)/d_s$. Its value in MKA is $\lambda_+ \simeq 0.62$, very close to λ_-.

For finite temperatures and small systems, there are corrections to this asymptotic behavior due to finite size effects which replace the nonanalyticity at $\epsilon = 0$ with a linear behavior for small $|\epsilon|$, and due to the influence of the critical fixed point, where the leading behavior is linear in ϵ. As we have argued in [2], the influence of the critical fixed point changes the apparent value of the low temperature exponents for the system sizes studied in the MCS and the MKA. The MCS data shown in [1] with an apparent value of 0.5 for $\lambda_\pm$ are fully compatible with these predictions of the droplet picture. For the MKA, the apparent exponent at $0.7T_c$ is close to 1 for $L \simeq 16$, leading to the "trivial" behavior found in [1]. However, at lower temperatures, for the same small system sizes the above-mentioned nontrivial features predicted by the droplet picture become clearly visible, as shown in Fig. 1.

The apparent system size dependence of the exponents $\lambda_\pm$ allows us even to estimate numerically the system sizes needed to see the true low temperature scaling behavior. By iterating the recursion relations for the coupling constants within MKA, we find that these system sizes are of the order $L \simeq 100$ at $T \simeq 0.7T_c$. We expect that similar system sizes would be needed for MCS to see the scaling behavior predicted by the droplet picture.

This work was supported by EPSRC Grants No. GR/K79307 and No. GR/L38578.

H. Bokil, A. J. Bray, B. Drossel, and M. A. Moore
Department of Physics and Astronomy
University of Manchester
Manchester M13 9PL, United Kingdom

Received 22 February 1999 [S0031-9007(99)09275-3]
PACS numbers: 75.10.Nr, 75.50.Lk

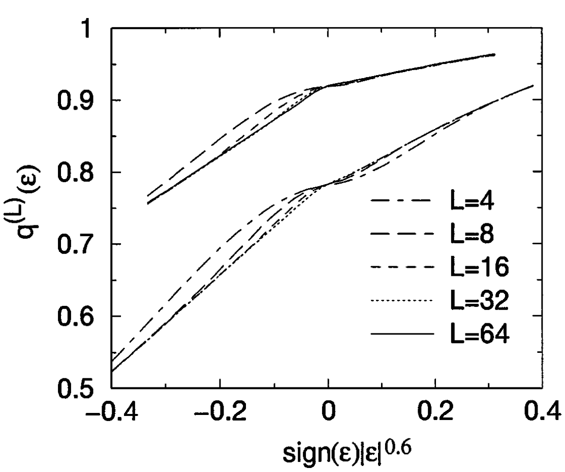

FIG. 1. $q^{(L)}(\epsilon)$ in MKA at $T \simeq 0.38T_c$ (bottom) and $0.14T_c$ (top) as function of $\text{sign}(\epsilon)|\epsilon|^{0.6}$ for various system sizes.

[1] E. Marinari, G. Parisi, J. J. Ruiz-Lorenzo, and F. Zuliani, preceding Comment, Phys. Rev. Lett. **82**, 5176 (1999).
[2] M. A. Moore, H. Bokil, and B. Drossel, Phys. Rev. Lett. **81**, 4252 (1998).
[3] A. J. Bray and M. A. Moore, Phys. Rev. Lett. **58**, 57 (1987).

