## Comment on "Evidence for the Droplet Picture of Spin Glasses"

In a recent Letter Moore *et al.* [1] claim to exhibit evidence for a non-mean-field behavior of the 3D Ising spin glass. We show here that their claim is insubstantial, and by analyzing in detail the behavior of the Migdal-Kadanoff approximation (MKA) as compared to the behavior of the Edwards-Anderson (EA) spin glass we find further evidence of a mean-field-like behavior of the 3D spin glass.

The main point of [1] does not concern the validity of the MKA in describing spin glasses, since it is well known, after the work of [2], that already at the mean-field level the MKA describes a trivial droplet structure, completely missing the structure of the phase space of the model in any dimension.

Reference [1] shows instead that the probability distribution of the order parameter $P_{\mathrm{MK}}(q)$ computed in the MKA at $T = 0.7$, close to the temperature where most of the numerical simulations have been run, has a spurious small $q$ "plateau," very similar to the nontrivial $P(q)$ one finds numerically for the EA model. In these conditions, for values of the lattice size comparable to the ones used in numerical simulations, $L \leq 16$, the small $q$ region of $P_{\mathrm{MK}}(q)$ does not seem to depend on $L$, even if one knows that eventually, for very large values of $L$, it will have to become trivial. The authors of [1] explain this coincidence as a hint of the fact that asymptotically the EA model will also behave as a droplet model.

Here we show that this similarity in the behavior of the MKA and the true EA 3D spin glass does not concern observables that are crucial for determining replica symmetry breaking (RSB). We look at the *link overlap* (on a system of linear size $L$ and volume $V = L^3$) $q^{(L)} \equiv (1/3V)\sum{}'\langle\sigma_i\sigma_{i+\hat{\mu}}\tau_i\tau_{i+\hat{\mu}}\rangle$, where the sum runs over first-neighbor site pairs. $q^{(L)}$ is more sensitive than the usual overlap $q$ to the difference between a droplet and a mean-field-like behavior. The link overlap is of crucial importance, since a nontrivial $P(q)$ could be simply due to the presence of interfaces, while a nontrivial $P(q^{(L)})$ is a nonambiguous signature of RSB.

We show that one can see a clear difference, already at $T = 0.7$ on medium-sized lattices, among the MKA and the EA model. So, not only does our observation make the point of [1] obsolete, but it also shows that simulations on reasonable-sized lattices are useful, when studying either disordered systems or normal statistical mechanical models (from the point of view of the advocates of [1] in the case of disordered systems, only simulations on systems of a huge size could make the true nature of the system manifest).

We have analyzed the MKA of the 3D spin glass (averaging over 1000 disorder samples), and the 3D EA model by numerical simulations (using a tempering algorithm and an annealing scheme, checking convergence and averaging over 64 or more samples). In all cases we have considered binary couplings and a Hamiltonian $H_\epsilon[\sigma,\tau] = H_0[\sigma] + H_0[\tau] - \epsilon\sum{}'\sigma_i\sigma_{i+\hat{\mu}}\tau_i\tau_{i+\hat{\mu}}$, where $H_0$ is the usual EA 3D Hamiltonian.

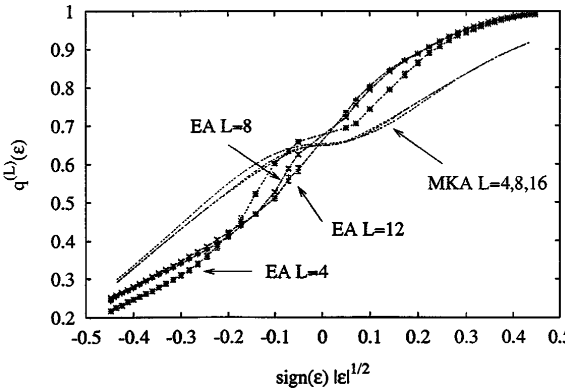


FIG. 1. $q^{(L)}$ in the MKA (lines without points) and from simulations of the 3D EA spin glass.

In Fig. 1 we show our results for $q^{(L)}(\epsilon)$ versus $\epsilon^{1/2}$. The MKA gives a smooth behavior: for small $\epsilon$, $q^{(L)}(\epsilon)$ behaves like $\epsilon^\lambda$, with $\lambda \simeq 1$. Finite-size effects look very small for these sizes (from 4 to 16). The EA model behaves in a completely different way. Here finite-size effects are large, and the behavior for small $\epsilon$ becomes more singular for larger sizes. The $L = 4$ lattice is reminiscent of the MKA behavior, but already at $L = 8$ the difference is clear. From our data we are not able to definitely establish the existence of a discontinuity, but the numerical evidence is strongly suggestive of that. The data are suggestive of the building up of a discontinuity as $L \to \infty$, i.e., $q = q_+ + A_+\epsilon^\lambda$ for $\epsilon > 0$ and $q = q_- + A_-|\epsilon|^\lambda$ for $\epsilon < 0$, with $q_+ \neq q_-$ and an exponent $\lambda$ close to $\frac{1}{2}$: a continuous behavior (i.e., $q_+ = q_-$) cannot be excluded from these data, but in this case we find an upper limit $\lambda < 0.25$, totally different from the behavior of MKA, $\lambda \simeq 1$. This is what is needed to show that when looking at observables that are very sensitive to RSB the difference among the trivial behavior of the MKA and true spin glasses is already clear at $T \simeq 0.6T_c$ on lattices of size $L \simeq 16$, as opposed to the claims of [1].

E. Marinari,[1] G. Parisi,[2] J. J. Ruiz-Lorenzo,[3] and F. Zuliani[1]
[1]Università di Cagliari, Cagliari, Italy
[2]Università di Roma, Roma, Italy
[3]Universidad Complutense, Madrid, Spain

Received 18 December 1998 [S0031-9007(99)09274-1]
PACS numbers: 75.10.Nr, 75.50.Lk

[1] M. A. Moore, H. Bokil, and B. Drossel, Phys. Rev. Lett. **81**, 4252 (1998); e-print cond-mat/9808140.
[2] E. Gardner, J. Phys. (Paris) **45**, 1755 (1984).

 