Comment on "General Method to Determine Replica Symmetry Breaking Transitions"

In a recent Letter Marinari *et al.* [1] introduced a new method to study spin glass transitions and argued that by probing replica symmetry (RS), as opposed to time reversal symmetry (TRS), their method unambiguously shows that replica symmetry breaking (RSB) occurs in short-range spin glasses. In this Comment we show that, while the method introduced in [1] is indeed useful for studying transitions in systems where TRS is absent (such as the p-spin model studied by them), the conclusion that it shows the existence of RSB in short-range Ising spin glasses is wrong.

The analysis of Marinari *et al.* is based on a new quantity which for systems with TRS is given by

$$G(T,L) = \frac{[\langle q^2\rangle^2] - [\langle q^2\rangle]^2}{[\langle q^4\rangle] - [\langle q\rangle^2]^2}, \tag{1}$$

where $\langle\cdots\rangle$ and $[\cdots]$ denote thermal and disorder averages, respectively, T is the temperature, L is the system size, and q is the standard overlap between two replicas. For the Ising spin glass without a field in four dimensions, Marinari *et al.* found that $G(T,L)$ exhibits two distinct kinds of behavior quite clearly: a high temperature phase where $G(T,L)$ decreases with system size as $1/L^d$ in d dimensions and a low temperature phase where $G(T,L)$ increases with system size and saturates at a constant value close to $1/3$. They interpreted this as indicating the existence of RSB in these systems. (They also studied the Ising spin glass in a field, which we will discuss later in this Comment.)

However, within a dropletlike picture it is not clear how $G(T,L)$ behaves as $L \to \infty$ because $P(q) \to \frac{1}{2}[\delta(q - q_{\mathrm{EA}}) + \delta(q + q_{\mathrm{EA}})]$ and both the numerator and the denominator are zero in this limit. Motivated by this, we studied $G(T,L)$ for the three-dimensional Ising spin glass within the Migdal-Kadanoff approximation used recently by Moore, Bokil, and Drossel [2]. These authors showed that, even though the asymptotic behavior is described by the droplet theory, finite systems can exhibit many of the features associated with RSB. In Fig. 1 we show $G(T,L)$ as a function of T for a range of system sizes $L = 4$, 8, and 16, the averages being performed over 20 000–50 000 samples. Just as in the simulation reported in [1], we find two distinct regimes, a high temperature phase with $G(T,L)$ decreasing with system size and a low temperature phase where $G(T,L)$ increases with the system size saturating at a value about $1/3$. The lowest temperature is in the asymptotic regime, with the numerator and denominator separately decreasing with increasing L. Clearly, then, this behavior of $G(T,L)$ cannot be interpreted as evidence of RSB. What is happening here is that finite systems show non-self-averaging of the overlap distribution function [and a nonzero limit for $G(T,\infty)$ in the low temperature phase], but there is self-averaging in the infinite system limit where one has a dropletlike $P(q)$. The mistake in Ref. [1] was to note that the numerator in Eq. (1) would vanish in the thermodynamic limit with RS, while overlooking the fact that the denominator would vanish as well. Thus, $G(T,L)$ can have a nonzero limit. As an aside, we note that even for the rather trivial case of a single random bond (connecting two spins) it is possible to prove that $G = 1/3$ at $T = 0$ provided that the bond distribution function has a nonzero weight at the origin.

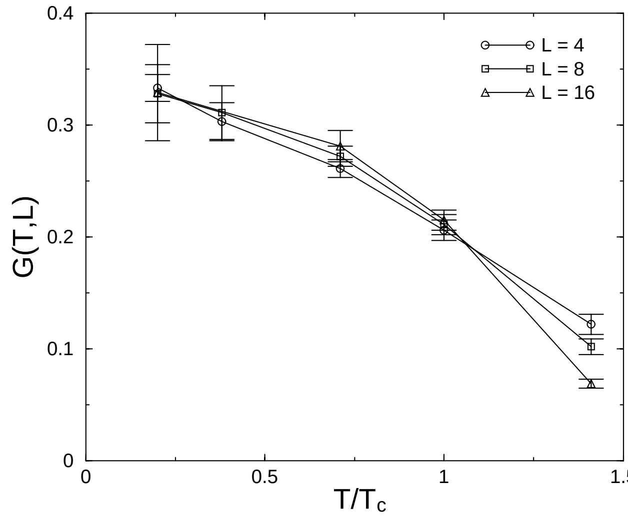

FIG. 1. $G(T,L)$ in the Migdal-Kadanoff approximation for $L = 4$, 8, and 16.

Thus we have demonstrated that the data reported in [1] for the Ising spin glass without a field do not give any evidence for RSB in this system. If the authors of [1] had found convincing evidence for a transition in the case with a field, that would indeed have been evidence for RSB. But we believe that their data for that case do not allow any conclusive statement to be made (there is no apparent crossing of the curves nor is it clear that there are two distinct kinds of behavior corresponding to high and low temperatures). In summary, while the method developed in [1] is useful for some problems, it does not give evidence for RSB in Ising spin glasses. The Reply [3] does not address this central point.

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[3] E. Marinari, C. Naitza, G. Parisi, M. Picco, F. Ritort, and F. Zuliani, following Reply, Phys. Rev. Lett. **82**, 5175 (1999).

