

## Differential Flow in Heavy-Ion Collisions at Balance Energies

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A strong differential transverse collective flow is predicted for the first time to occur in heavy-ion collisions at balance energies. We also give a novel explanation for the disappearance of the total transverse collective flow at the balance energies. It is further shown that the differential flow especially at high transverse momenta is a useful microscope capable of resolving the balance energy's dual sensitivity to both the nuclear equation of state and in-medium nucleon-nucleon cross sections in the reaction dynamics. [S0031-9007(99)09446-6]

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One of the main goals of nuclear physics is to study properties of the dense and hot nuclear matter created in heavy-ion collisions. Information about the equation of state (EOS) of the hot and dense matter is important for understanding the evolution of the early universe and the scenario of supernova explosions. Nuclear collective flow in heavy-ion collisions has been found to be a useful tool for extracting the nuclear EOS. During the last decade, intensive theoretical and experimental studies have revealed considerable interesting information about the reaction dynamics of heavy-ion collisions and the nuclear EOS; for a recent review see, e.g., [1–4]. In particular, the excitation function of transverse collective flow from low to ultrarelativistic energies has been found especially interesting. On the high energy side, a minimum of the collective flow is expected in reactions crossing the phase transition region from hadronic matter to quark-gluon plasma [2,5–7]. At intermediate energies, the transverse collective flow disappears at an incident energy, termed the balance energy  $E_{\text{bal}}$ . This phenomenon has been well established by many experiments during the last decade [1]. It has been found experimentally that the balance energy depends sensitively on the mass and isospin of the colliding nuclei as well as the impact parameter of the reaction [8]. Simultaneously, much theoretical work has been devoted to understand the mechanism responsible for the disappearance of transverse flow at balance energies. It has long been suggested that at the balance energies the attractive scattering dominant at energies around 10 MeV/nucleon balances the repulsive interactions dominant at energies around 400 MeV/nucleon. Moreover, to extract information about the nuclear EOS and in-medium nucleon-nucleon cross sections, extensive comparisons between experimental data and theoretical calculations on the balance energies have been carried out; for a review see, e.g., Refs. [9,10]. These efforts, however, have been severely hindered by the dual sensitivities of the balance energies to both the nuclear EOS and the in-medium nucleon-nucleon cross sections [11–19]. In this Letter, it is shown for the first time that there exists clearly a strong

*differential* transverse collective flow at the balance energies. Moreover, we give a novel explanation for the disappearance of the transverse flow at the balance energies, and further show that the differential flow is a useful microscope capable of resolving the dual sensitivity of the balance energy.

The anisotropic collective flow (also called directed flow) has been studied most commonly by analyzing the average transverse momentum per nucleon in the reaction plane as a function of rapidity  $y$  [20]

$$\begin{aligned} \langle p_x/A \rangle(y) &= \frac{1}{A(y)} \sum_{i=1}^{A(y)} p_{ix} \\ &= \frac{1}{dN/dy} \int p_t \frac{d^2N}{dp_t dy} \langle \cos(\phi) \rangle(y, p_t) dp_t, \end{aligned} \quad (1)$$

where  $A(y)$  is the number of nucleons at rapidity  $y$ ,  $\phi$  is the azimuthal angle of nucleons with respect to the reaction plane, and

$$\langle \cos(\phi) \rangle(y, p_t) \equiv \left( \frac{dN}{dp_t} \right)^{-1} \int \cos(\phi) \frac{d^2N}{dp_t d\phi} d\phi. \quad (2)$$

A nonvanishing  $\langle \cos(\phi) \rangle(y, p_t)$  indicates the existence of an azimuthally anisotropic transverse flow at the rapidity  $y$  and transverse momentum  $p_t$ , and we name it the *differential* flow. In particular, adopting the Fourier expansion [21,22]

$$\frac{d^2N}{dp_t d\phi} = \frac{dN}{dp_t} \left[ 1 + \sum_{i=1}^{i=\infty} 2v_i(y, p_t) \cos(i\phi) \right], \quad (3)$$

one finds that  $\langle \cos(\phi) \rangle(y, p_t) = v_1(y, p_t)$  is the strength of the azimuthal angle distribution to the first order. Information about the nuclear EOS and in-medium cross sections can be preserved and revealed more completely by decoupling the differential flow from the integrand in Eq. (1). The analysis of  $v_1(y, p_t)$  in heavy-ion collisions at high energies has been found especially useful for studying properties of the transverse collective flow and

the nuclear EOS [21–24]. At the balance energies the total in-plane average transverse momentum of Eq. (1) vanishes in almost the whole range of rapidity. It is thus sufficient to study the  $p_t$  dependence of differential flow around the projectile and/or target rapidities where transverse flow peaks, should it exist at all.

Much of the current understanding about the nuclear EOS [25,26] and in-medium nucleon-nucleon cross sections [15–18] comes from comparing flow measurements with predictions based on transport theories such as the Boltzmann-Uehling-Uhlenbeck (BUU) model. Details of the BUU model used in the present study can be found in Refs. [27–29]. To identify the balance energy we first performed the standard transverse momentum analysis for the reaction of Au + Au at an impact parameter of 5 fm and several beam energies. Typical results of this analysis are shown in the upper window of Fig. 1. These calculations were carried out by using the stiff nuclear EOS of compressibility  $K = 380$  MeV and the free-space nucleon-nucleon cross sections given in Refs. [27–29]. It is seen that the transverse flow is attractive, almost zero and repulsive at the beam energy of 30, 50, and 100 MeV/nucleon, respectively. To investigate the

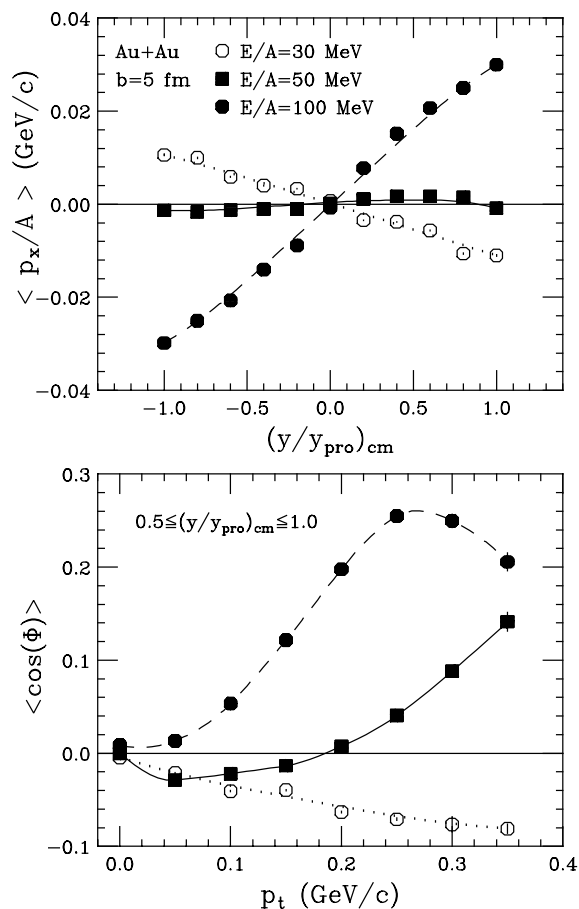


FIG. 1. Total (upper) and differential (lower) transverse flow analysis for the reaction of Au + Au at an impact parameter of 5 fm and beam energies of 30, 50, and 100 MeV/nucleon.

differential flow, the value of  $\langle \cos(\phi) \rangle$  is shown as a function of  $p_t$  in the lower window for nucleons with rapidities in the range of  $0.5 \le (y/y_{proj})_{c.m.s.} \le 1.0$ . Although quantitatively different, our results are qualitatively independent of the chosen rapidity bin and impact parameters [30]. Of course,  $\langle \cos(\phi) \rangle(p_t)$  reverses its sign on the negative rapidity side. The reactions at beam energies of 30 and 100 MeV/nucleon show clearly the characteristic attractive and repulsive differential flow, respectively, as one expects in the whole range of  $p_t$ . It is most interesting to see clearly also a strong differential flow at the balance energy of 50 MeV/nucleon in the whole range of  $p_t$ . It changes from being attractive to repulsive at a balance transverse momentum of about  $p_{bal} = 0.2$  GeV/c. Although altogether having a zero average transverse momentum in the reaction plane particles with higher and lower  $p_t$  move preferentially towards the positive and negative flow directions, respectively. This is because the lower  $p_t$  particles are more affected by the attractive mean field while the higher  $p_t$  particles are more affected by the repulsive nucleon-nucleon scatterings. It is thus clear that the disappearance of transverse flow is due to the canceling of positive and negative differential flow of particles with high and low transverse momenta. We also notice that the magnitude of the observed differential flow at the balance energy is compatible with the azimuthal asymmetry normally observed in heavy-ion collisions at intermediate energies [31]. A comparison of the two analyses above clearly indicates that the differential flow probes more microscopically the interesting features which are otherwise smeared out by the integration in the standard flow analysis.

It is well known that the transverse collective flow in general and the balance energy in particular are dually sensitive to both the nuclear EOS and the in-medium nucleon-nucleon cross sections. For the Au + Au reaction we found that a soft EOS of compressibility  $K = 210$  MeV with a 30% reduction in the nucleon-nucleon cross sections results in about the same balance energy as the stiff one with the free-space nucleon-nucleon cross sections. This is qualitatively in agreement with the previous findings [15–18]. More quantitatively, an average in-plane transverse momentum per nucleon of 0.86 and  $-0.02$  MeV/c in the range of  $0.5 \le (y/y_{proj})_{c.m.s.} \le 1.0$  is obtained by using the stiff and soft EOS, respectively. Both of these values are equivalent to zero transverse flow within the statistical errors of the calculations. Shown in Fig. 2 is an analysis of the differential flow for the above two cases which give the same balance energy of 50 MeV/nucleon. A clear, experimentally distinguishable separation in the differential flow is seen at high transverse momenta above about 0.2 GeV/c. The soft EOS with the reduced in-medium cross sections lowers the differential flow at high transverse momenta by about a factor of 2. A counterbalancing shift in the  $p_t$  spectrum  $\langle p_t dN/dp_t/A \rangle$  is found simultaneously and will be

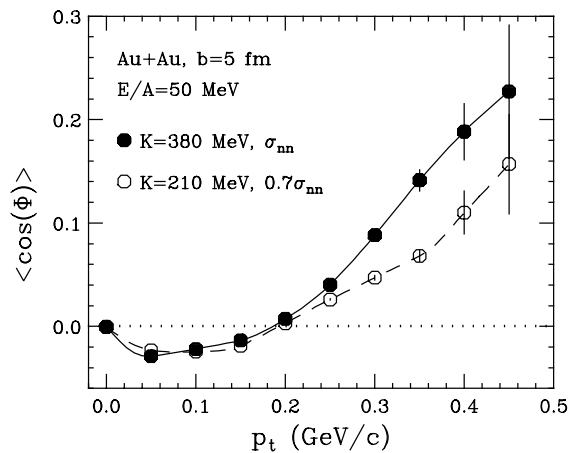


FIG. 2. Differential flow analysis for Au + Au at an impact parameter of 5 fm using two different parameter sets leading to the same balance energy of 50 MeV/nucleon.

published elsewhere [30]. The differential flow is thus a useful microscope capable of resolving the balance energy's dual sensitivity to both the nuclear EOS and the in-medium cross sections.

Our findings here can be further understood by studying the competitive roles of the nuclear EOS and in-medium nucleon-nucleon cross sections in forming the differential flow at the balance energies. Results of such a study is shown in Fig. 3 for the Au + Au reaction at  $E/A = 50$  MeV/nucleon. In the upper window, the differential flow is studied by varying the in-medium nucleon-nucleon cross section  $\sigma$  in the range of current theoretical predictions [32–34]. It is seen that the differential flow increases as the  $\sigma$  increases because it enhances the repulsive scatterings. In the lower window, the effect of the nuclear mean field is studied by varying the compressibility  $K$  in the range currently considered in the literature of astrophysics and nuclear physics [35–37]. In the energy range studied here, the mean field is attractive and its effect on the differential flow increases with the increasing compressibility. The differential flow at balance energies is a remnant of the competition between the in-medium cross section and the nuclear mean field. Since a 30% reduction in the  $\sigma$  reduces the differential flow more than the increase caused by a softening of the nuclear EOS from  $K = 380$  to 210 MeV, especially at high transverse momenta, our findings in Fig. 2 are easily understandable. The differential flow reveals more microscopically and directly the competition between the negative scattering due to the attractive mean field and the positive scattering due to the repulsive nucleon-nucleon collisions. This strong competition also makes the differential flow a much more sensitive probe than the balance energy itself. It is therefore interesting to note that the balance energy has been investigated extensively as a function of mass, isospin, and impact parameter of the reaction in many experiments

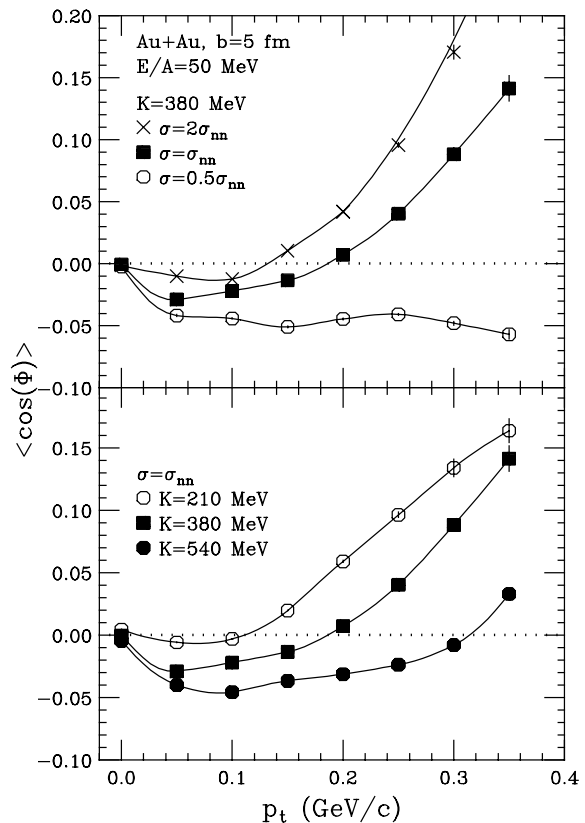


FIG. 3. Dependence of the differential flow on the in-medium cross section (upper) and compressibility (lower) for Au + Au at an impact parameter of 5 fm and a beam energy of 50 MeV/nucleon.

during the last decade. The differential flow analysis can thus be carried out using available data. This study will be both physically very interesting and highly economical since the data already currently exist. In addition, our present work has important physical implications to the study of “zero” flow of secondary particles, such as pions and kaons, produced in heavy-ion collisions. Many interesting predictions relating their collective flow and in-medium dispersion relations have been made [38]; however, an almost zero flow has been found experimentally [39,40]. Our differential flow analysis for these particles is in progress and will be published elsewhere.

In summary, it is predicted for the first time that a strong differential collective flow exists at balance energies where the normal transverse collective flow disappears. We also give a novel explanation for the disappearance of total transverse flow at the balance energies. It is further shown that the differential flow especially at high transverse momenta is a useful microscope capable of resolving the balance energy's dual sensitivity to both the nuclear EOS and the in-medium nucleon-nucleon cross sections in the reaction dynamics. Experimental analysis of differential flow at balance energies using available data will be very interesting and fruitful.

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