# Phase Transitions in Finite Nuclei and the Integer Nucleon Number Problem

R. F. Casten,[1] Dimitri Kusnezov,[2] and N. V. Zamfir[1,3]
[1]*WNSL, Yale University, New Haven, Connecticut 06520*
[2]*Center for Theoretical Physics, Yale University, New Haven, Connecticut 06520-8120*
[3]*Clark University, Worcester, Massachusetts 01610*
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The study of spherical-deformed ground-state phase transitions in finite nuclei as a function of $N$ and $Z$ is hindered by the discrete values of the nucleon number. A resolution of the integer nucleon number problem and evidence relating to phase transitions in finite nuclei are discussed from the experimental point of view and interpreted within the framework of the interacting boson model. [S0031-9007(99)09401-6]

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It has been known for decades that nuclear properties change, often dramatically, as a function of $N$ and $Z$. However, the possibility of true nuclear *phase* transitions and *phase* coexistence, in the sense of conventional condensed matter systems, has generally been discounted. The reason has to do with the finite nature of atomic nuclei and the fact that they contain integer numbers of nucleons. In order to discuss the concept of phase transitions [1] one needs to identify a *control* parameter as well as an *order* parameter. If the fluctuations in the order parameter are small, that is, if the data follow a compact trajectory as a function of the control parameter, a phase transition would be signaled by a critical point where the order parameter is discontinuous.

Here we are interested in structural changes in nuclei as a function of $N$ and $Z$. The finite nature of nuclei means that any nuclear phase transition cannot be abrupt. The fact that nuclei contain *integer* numbers of nucleons means that their properties change *discretely* with $N$ and $Z$. This is shown in Fig. 1a where a typical collective observable is plotted against a neutron number for the even-even Sm ($Z = 62$) isotopes. Clearly, an interesting (and well known) change in structure is occurring at $N \sim 90$, but the abscissa values are, by definition, discrete. Therefore, regardless of what the data do, the integer nucleon number requires that, at best, one can connect only adjacent points by straight line segments and one can define only *differences* in properties, not derivatives.

It is the purpose of this Letter to discuss phase transitions in finite nuclei as a function of $N$ and $Z$ and to suggest a resolution of the integer nucleon number problem. When one considers low energy nuclear structure, nuclear models offer the flexibility of having one or more continuous parameters that can serve as control parameters, allowing the study of critical phenomena. The situation in actual nuclei is different. In contrast to phase transitions in a specific nucleus, where excitation energy (or temperature) can be introduced as a control parameter, in the evolution of nuclear structure at low energy we have seen that $N$ and $Z$ are not useful as control parameters. However, we will suggest an empirical quantity which is continuous and does correlate extremely well the structural changes across large regions of nuclei such as the $A = 150$ region [3–5]. Finally, a theoretical analysis with the interacting boson model (IBM) [6] shows why sharp transition regions are indeed an expected feature of structural evolution.

We noted in Fig. 1 that the data from a single element do not allow a discussion of phase transitional behavior. The data for collective nuclei in an entire region ($50 < Z \leq 66$), shown in Fig. 1b, only exacerbate the problem. The abscissa values remain discrete, but now another prerequisite for a phase transition disappears, namely, the absence of fluctuations. The scattering of the data obliterates any evidence of sharply discontinuous behavior.

How can we get around this situation in finite nuclei? A possible answer is to choose a qualitatively different quantity to play the role of a control parameter, one that is at least potentially continuous. Of course, even if the abscissa points become continuously distributed, we need to produce an approach in which the fluctuations in the data are small. To do so, consider Figs. 2a–2c. In this figure, the same $E(4_1^+)$ data as in Fig. 1 are plotted, not against $N$, $Z$, or $A$, but against another collective observable, the energy, $E(2_1^+)$, of the first $2^+$ state which, in principle, can have any value. The top left panel, with clearly separated data points, is no better for discussing phase transitional behavior than is Fig. 1a.

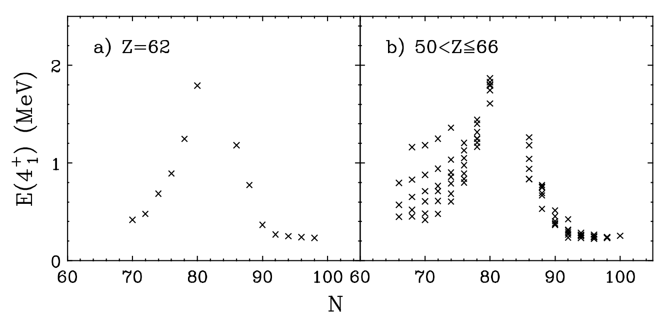


FIG. 1. $E(4_1^+)$ against neutron number for collective nuclei [nuclei for which $R_{4/2} \equiv E(4_1^+)/E(2_1^+) > 2.05$]. (a) Sm; (b) the $50 < Z \leq 66$ region. Data from Ref. [2].

 

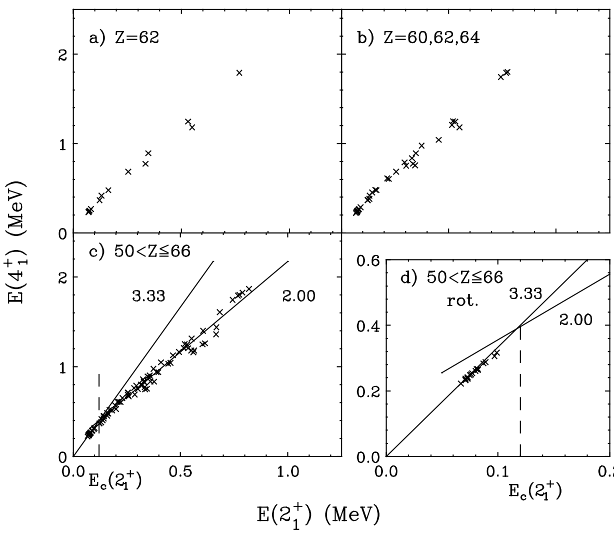


FIG. 2. $E(4_1^+)$ against $E(2_1^+)$. Panels (a)–(c) show the same data as Fig. 1 for sequentially more elements. Panel (d) is an expanded view of the rotor region, showing the data points with $E(2_1^+) < E_c(2_1^+)$.

However, when the data for additional elements are added (Figs. 2b and 2c) we see a behavior that is qualitatively different from Fig. 1. The distribution of points as a function of $E(2_1^+)$ successively fills in, yielding in Fig. 2c a nearly continuous distribution. Next, we note that $E(2_1^+)$ clearly correlates nuclear equilibrium properties extraordinarily well. The data for different elements lie along essentially *identical* paths and thus the ensemble of data also lies along a *single compact* curve, with very small fluctuations. This thereby enables a discussion of the trajectory and a potential interpretation in terms of phase transitions. Of course, other observables that reflect the equilibrium configuration could have been chosen. $E(2_1^+)$ is preferred, however, since it is well known in many nuclei and is easy to measure in new nuclei.

Observables such as $E(2_1^+)$, $E(4_1^+)$, separation energies, and other measures of structure cannot, rigorously speaking, be considered as control parameters since they are not independently variable as is the temperature in a condensed matter system. Nevertheless, $E(2_1^+)$, de facto, plays a similar role to a control parameter, and Fig. 2c suggests evidence for phase transitional behavior as seen by the nearly discontinuous change in slope (order parameter) from 2.00 to 3.33 at a specific value of $E(2_1^+)$ denoted $E_c(2_1^+)$. The expanded view of the rotor region in Fig. 2d clearly shows the different slope for these nuclei. In this mass region the change in slope occurs at $E_c(2_1^+) \sim 120$ keV.

Figure 2c itself is not new. We discussed the $E(4_1^+)$-$E(2_1^+)$ and related correlations in Ref. [7] and even broached the subject of phase transitional behavior. The correlation in Fig. 2c has been discussed theoretically [8] in the context of the $1/N$ expansion for the IBM and extended [9,10] empirically to observables for intrinsic excitations and to odd $A$ and odd-odd nuclei. What is new here is the explicit discussion of the *process* of reaching Fig. 2c as a way of resolving the finite nucleon number problem and identifying phase transitional behavior. Of course, as we stressed earlier, since nuclei are finite systems, the phase transition is naturally smoothed out over a narrow region of $2_1^+$ energies.

Our next point is to relate this phase transitional behavior to the recent evidence [4,5] for phase *coexistence* in $^{152}$Sm. Theoretical analysis [4] of $^{152}$Sm points to two coexisting phases, a deformed ground band and a spherical anharmonic vibrator built on the $0_2^+$ level. Phase coexistence must occur at the critical point and so it is reassuring that $E(2_1^+)$ in $^{152}$Sm (122 keV) coincides with the change in slope from 2.00 to 3.33 in Fig. 2c.

Thus, two different perspectives—phase coexistence in $^{152}$Sm and the relation of yrast energies across the *region* of nuclei—give evidence for a phase transition from spherical to deformed structures, near $A = 150$. Note that, while phase *coexistence* occurs in a specific nucleus ($^{152}$Sm), the phase *transition* does not characterize a single nucleus, or the isotopes of an element, but is a property of, and only definable in terms of, an entire region. We stress here that this type of phase transition and phase coexistence is different from the shape coexistence picture known in other regions [11,12]. Here the structural changes develop within the context of a single shell and do not involve an intruder state mechanism.

Other observables, such as two nucleon separation energies, $S_{2n}$, reflect these rapid structural changes. In Fig. 3a we show the empirical values of $S_{2n}$ for the $50 < Z \leq 66$ nuclei (for all collective nuclei—those with $R_{4/2} > 2.05$). The $S_{2n}$ values have a well known, essentially parallel, shift in values for each successive $Z$. To compare values for different elements, we therefore shift the separation energies for each $Z$ by a constant amount chosen to give equal $S_{2n}$ values at $N = 88$ for the $N > 82$ shell and at $N = 76$ for the $N < 82$ shell.

For $N > 82$, the results for $S_{2n}$ in Fig. 3a are as striking as for the $4_1^+$ energies. The behavior is compact, with small fluctuations and a sharp break in trajectory. This break occurs at a slightly lower $2_1^+$ energy than the slope change in Fig. 2c. Apparently, $S_{2n}$ displays a different dependence on the shape of the potential than low spin yrast levels (which are most sensitive to the details of the potential near its minimum). For the lighter shell, $N < 82$, there are greater fluctuations and a gradual structural change but no evidence for a sharp phase transition.

In Fig. 3b we show $B(E2 : 2_1^+ \rightarrow 0_1^+)$ values against $S_{2n}$. The kink shows that $E(4_1^+)$ is not a unique measure of the structural transition: $B(E2)$ values provide additional evidence for it.

The question arises *why* nuclei should behave in this way. As we have noted, it is easier to look at phase transitional behavior in a model since the parameters are inherently continuous. We consider the IBM here but similar results characterize the geometric collective model (GCM) [15]. We use the IBM Hamiltonian [16,17] $H = \epsilon n_d - \kappa Q \cdot Q$, where $Q = (s^\dagger \tilde{d} + d^\dagger s) + \chi(d^\dagger \tilde{d})^2$.

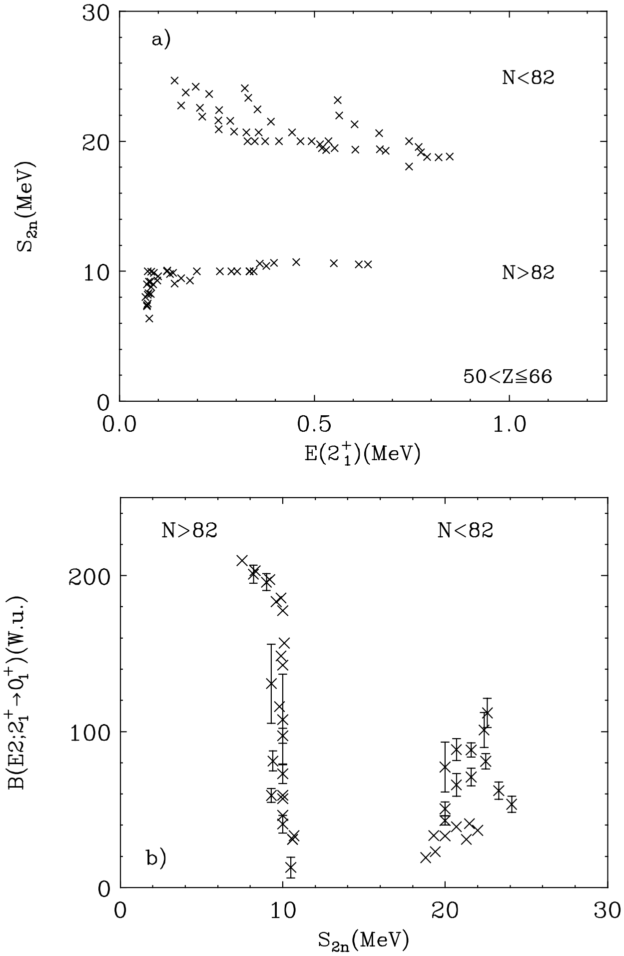


FIG. 3. (a) Separation energies $S_{2n}$ as a function of $E(2_1^+)$. $S_{2n}$ data from Ref. [13]. (b) $B(E2: 2_1^+ \rightarrow 0_1^+)$ values as a function of $S_{2n}$. $B(E2)$ data from Ref. [14].

The Hamiltonian has parameters $\epsilon$, $\kappa$, $\chi$ and the boson number $N_B$. IBM calculations of $E(4_1^+)$ versus $E(2_1^+)$ reproduce Fig. 2c: they follow a slope of 2.00 above $E_c(2_1^+)$ for virtually any choice of $\epsilon$, $\chi$, and $N_B$ that gives $R_{4/2} = 2.05-3.15$ as long as $\kappa$ is constant [18]. Indeed, the value of $\kappa$ determines the intercept $\epsilon_4$ [and $E_c(2_1^+)$ as well]. Were the data different from Fig. 2c (e.g., scattered, or following a slope other than 2.00) the only way it could be reproduced would be to adjust $\kappa$ for each nucleus, a clearly unlikely scenario that is inconsistent with microscopic analyses of IBM parameters [19].

We now study the IBM results through the intrinsic state formalism [20,21], computing the energy surfaces

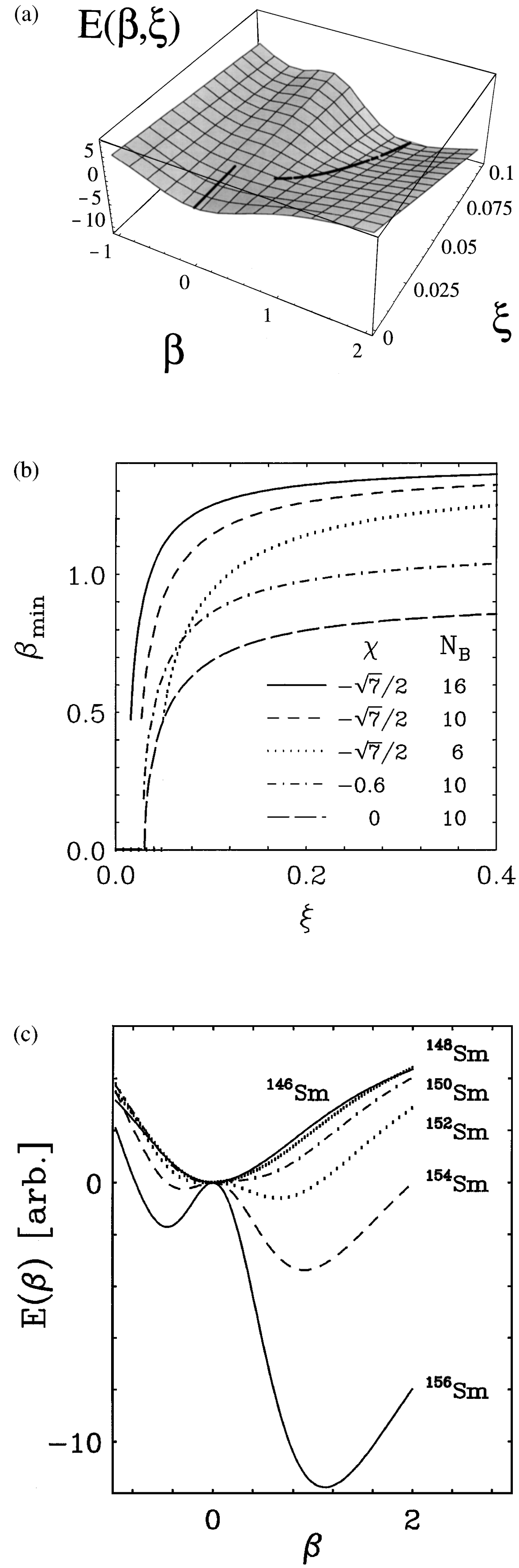


FIG. 4. Classical limit analysis of IBM calculations that reproduce Fig. 2c. (a) Energy surface as a function of $\xi$ [see Eq. (1)] near the critical point for $N_B = 10$ and $\chi = -\sqrt{7}/2$; (b) location of the minima, $\beta_{\rm min}$, as a function of $\xi$, for several values of $N_B$ and $\chi$; (c) the IBM energy $E(\beta)$ for the $\xi$ values and boson numbers corresponding to the Sm isotopes (see Ref. [22]). The minima occur only for $\beta = 0$ or large finite values. There is no gradual evolution of $\beta$ from 0 to saturation levels.

corresponding to different parameter values. It is convenient, for this purpose, to rewrite the Hamiltonian in terms of a control parameter $\xi = (1 + \epsilon/\kappa)^{-1}$. The resulting scaled Hamiltonian has the form

$$H' = (1 - \xi)n_d - \xi Q \cdot Q \,. \tag{1}$$

For $\chi = -\sqrt{7}/2$, $0 \leq \xi \leq 1$ maps the transition from U(5) to SU(3).

In Fig. 4a we show the IBM energy surface corresponding to the classical limit of Eq. (1), for $N_B = 10$ and $\chi = -\sqrt{7}/2$, against $\xi$. The figure shows the key point that the location of the minimum in the energy, $\beta_{\min}$, changes suddenly at a particular $\xi$ value, from $\beta_{\min} = 0$ to a large value, as indicated by the dark line cutting through the contour plot. There are virtually no $\xi$ values for which intermediate $\beta_{\min}$ values result. This is consistent with the calculations in Ref. [4]. The IBM indicates that nuclei change abruptly from near spherical to deformed at a critical value $\xi_{\mathrm{crit}}$. The evolution of the energy surface can be seen as the competition between two minima, spherical and deformed, rather than a gradual evolution from spherical to weakly deformed to large deformation. Such a level crossing scenario is, in fact, characteristic of a first order phase transition.

The qualitative behavior of these results is not sensitive to boson number $N_B$ or $\chi$. We show this in Fig. 4b, which gives the values of the location of the lowest minimum of the energy surface, as a function of $\xi$ for a set of $N_B$ and $\chi$ values. The curves all show the same behavior: $\beta_{\min}$ is zero for small $\xi$, and then rises rapidly to a saturation value within a very narrow range of $\xi$ values (which define a $\xi_{\mathrm{crit}}$ for each $N_B$ and $\chi$).

Figure 4c shows energy surfaces as a function of $\beta$ for IBM parameters applicable to the Sm isotopes. These surfaces range from near vibrator shapes for $^{146,148}$Sm to softer in $^{150}$Sm, to the coexistence nucleus $^{152}$Sm where two shallow minima occur, to the prolate deformed nuclei $^{154,156}$Sm. The actual minimum in the energy surface occurs *only* for $\beta_{\min} \sim 0$ or large positive $\beta_{\min}$.

In summary, we have shown that phase transitional character in finite nuclei can be assessed by proposing a way to resolve the integer nucleon number problem. We have discussed an empirical quantity, $E(2_1^+)$, that is nearly continuous and in terms of which other quantities, such as $E(4_1^+)$ or $S_{2n}$, follow simple, compact trajectories with small fluctuations for large regions of nuclei. These observables have distinct anomalies at $E(2_1^+)$ values similar to that for $^{152}$Sm where phase coexistence has been suggested. Other observables, such as $S_{2n}$, may also correlate structural changes. Through a model, such as the IBM (the GCM gives similar results), we have associated the order parameter with a physical quantity, the deformation $\beta_{\min}$, at which the potential has a minimum. In a spherical-deformed transition in the IBM, $\beta_{\min}$ has two characteristic values, zero and near-saturation deformation. It appears that nuclear structural evolution in this mass region entails two basic phases (spherical and deformed) rather than a gradual softening (with valence nucleon number) traditionally associated with the onset of deformation in nuclei. Although this view is unconventional, the analysis of the IBM suggests that it may be an important basic feature of structural evolution.

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