Effective Field Theory, Black Holes, and the Cosmological Constant

Andrew G. Cohen,[1,*] David B. Kaplan,[2,†] and Ann E. Nelson[3,‡]
[1]Department of Physics, Boston University, Boston, Massachusetts 02215
[2]Institute for Nuclear Theory, 1550, University of Washington, Seattle, Washington 98195-1550
[3]Department of Physics 1560, University of Washington, Seattle, Washington 98195-1560
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Bekenstein has proposed the bound $S \le \pi M_P^2 L^2$ on the total entropy S in a volume L^3. This nonextensive scaling suggests that quantum field theory breaks down in large volume. To reconcile this breakdown with the success of local quantum field theory in describing observed particle phenomenology, we propose a relationship between UV and IR cutoffs such that an effective field theory should be a good description of nature. We discuss implications for the cosmological constant problem. We find a limitation on the accuracy which can be achieved by conventional effective field theory. [S0031-9007(99)09399-0]

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It is generally assumed that particle physics can be accurately described by an effective field theory with an ultraviolet (UV) cutoff less than the Planck mass M_P, provided that all momenta and field strengths are small compared with this cutoff to the appropriate power. Computations performed with such effective field theories, for example, the standard model, have been extraordinarily successful at describing properties of elementary particles. Nevertheless, considerations involving black holes suggest that the underlying theory of nature is not a local quantum field theory. In this Letter we attempt to reconcile this conclusion with the success of effective quantum field theory by determining the range of validity for a local effective field theory to be an accurate description of the world. We accomplish this by imposing a relationship between UV and infrared (IR) cutoffs. We will argue that this proposed IR bound does not conflict with any current experimental success of quantum field theory, but explains why conventional effective field theory estimates of the cosmological constant fail so miserably.

For an effective quantum field theory in a box of size L with UV cutoff Λ the entropy S scales extensively, $S \sim L^3 \Lambda^3$ [1]. However, the peculiar thermodynamics of black holes [2,3] has led Bekenstein [2] to postulate that the maximum entropy in a box of volume L^3 behaves nonextensively, growing only as the area of the box. For any Λ, there is a sufficiently large volume for which the entropy of an effective field theory will exceed the Bekenstein limit. 't Hooft [4] and Susskind [5] have stressed that this result implies conventional $3 + 1$ dimensional field theories vastly overcount degrees of freedom: as these field theories are described in terms of a Lagrange density, they have extensivity of the entropy built in. The Bekenstein entropy bound may be satisfied in an effective field theory if we limit the volume of the system according to

$$L^3 \Lambda^3 \lesssim S_{\mathrm{BH}} \equiv \pi L^2 M_P^2 , \qquad (1)$$

where S_{BH} is the entropy of a black hole of radius L [2,3]. Consequently the length L, which acts as an IR cutoff, cannot be chosen independently of the UV cutoff, and scales as Λ^{-3}.

As startling as the Bekenstein-motivated constraint Eq. (1) seems, there is evidence that conventional quantum field theory fails at an entropy well below this bound. 't Hooft has stressed that ordinary field theories should fail on large scales if near the horizon of a black hole [4]. In the presence of even a very large black hole, a low energy description of particle physics is expected to be inadequate, since infalling particles experience Planck scale interactions with outgoing Hawking radiation near the horizon. Furthermore, it has been shown in string theory that local observables do not necessarily commute at a spacelike separation in the presence of a black hole [6]. These problems arise even in the absence of any large field strengths or momenta. Local quantum field theory appears unlikely to be a good effective low energy description of any system containing a black hole, and should probably not attempt to describe particle states whose volume is smaller than their corresponding Schwarzschild radius.

An effective field theory that can saturate Eq. (1) necessarily includes many states with the Schwarzschild radius much larger than the box size. To see this, note that a conventional effective quantum field theory is expected to be capable of describing a system at a temperature T, provided that $T \le \Lambda$; so long as $T \gg 1/L$, such a system has thermal energy $M \sim L^3 T^4$ and entropy $S \sim L^3 T^3$. When Eq. (1) is saturated, at $T \sim (M_P^2/L)^{1/3}$, the corresponding Schwarzschild radius L_S for this system is $L_S \sim L(LM_P)^{2/3} \gg L$.

To avoid these difficulties we propose an even stronger constraint on the IR cutoff $1/L$ which excludes all states that lie within their Schwarzschild radius. Since the maximum energy density in the effective theory is Λ^4, the constraint on L is

$$L^3 \Lambda^4 \lesssim L M_P^2 . \qquad (2)$$

Here the IR cutoff scales like Λ^{-2}. This bound is far more restrictive than Eq. (1): when Eq. (2) is near saturation, the entropy is

$$S_{\max} \simeq S_{\mathrm{BH}}^{3/4}. \tag{3}$$

We propose that an effective local quantum field theory will be a good approximate description of physics when Eq. (2) is satisfied. This bound is more restrictive than Eq. (1) because we are explicitly considering only those states that can be described by conventional quantum field theory [7].

Can such a dramatic depletion of quantum states be relevant to the cosmological constant problem [11]?

If the standard model is valid in an arbitrarily large volume up to at least LEP energies, then the quantum contribution to the vacuum energy density computed in perturbation theory is $\sim(100\ \mathrm{GeV})^4$. The empirical bound on the cosmological constant corresponds to a vacuum energy density $\lesssim(10^{-2.5}\ \mathrm{eV})^4$. Conventionally this discrepancy is explained by either unknown physics at high energies which conspires to cancel this vacuum contribution to enormous precision, or else new physics at $\sim 10^{-2.5}$ eV which adjusts to cancel the vacuum energy while being devious enough to escape detection [14,15].

There is however a third possibility—that the usual perturbative computation of the quantum correction to the vacuum energy density, which assumes no infrared limitation to the quantum field theory, is incorrect. There is, in fact, no evidence that fields at present experimental energies can fluctuate independently over a region as large as our horizon. In fact, if we choose an IR cutoff comparable to the current horizon size, the corresponding UV cutoff from Eq. (2) is $\Lambda \sim 10^{-2.5}$ eV and the resulting quantum energy density of Λ^4 requires no cancellation to be consistent with current bounds. This observation does not predict the cosmological constant's value, as one can always add a constant to the quantum contribution. However it does eliminate the need for fine-tuning.

The peculiar relationship between IR and UV cutoffs in Eq. (2) is, in principle, testable as it limits the successful application of quantum field theory to experiment. For instance, if we wish to search for new physics (coming from new interactions or particles at high energies which do not violate low energy symmetries) using high precision experiments at low energies p, there is a maximal energy scale that can be probed without incorporating effects beyond conventional quantum field theory. Surprisingly, this scale depends on p, and can be much lower than M_P.

In order to perform an effective field theory calculation we simultaneously impose a UV and an IR cutoff consistent with Eq. (2). There will be small discrepancies between such a calculation and a conventional one performed in an infinite box. Such a discrepancy can be of interest when trying to discover new physics through radiative corrections. For example, consider $(g-2)$ for the electron. The UV and IR cutoffs that we must impose each lead to corrections to the usual calculation, whose total size is

$$\delta(g-2) \sim \frac{\alpha}{\pi}\left[\left(\frac{m_e}{\Lambda}\right)^2 + \left(\frac{1}{m_e L}\right)^2\right]. \tag{4}$$

If we were able to choose L independently of Λ we would simply ignore the IR corrections. However we must now comply with Eq. (2). Substituting this constraint on L gives

$$\delta(g-2) \gtrsim \frac{\alpha}{\pi}\left[\left(\frac{m_e}{\Lambda}\right)^2 + \left(\frac{\Lambda^2}{m_e M_P}\right)^2\right]. \tag{5}$$

This uncertainty in our calculation is minimized by choosing the UV cutoff to be $\Lambda \sim (m_e^2 M_P)^{1/3} \sim 14$ TeV, so that

$$\delta_{\min}(g-2) \sim \frac{\alpha}{\pi}\left(\frac{m_e}{M_P}\right)^{2/3} \sim \frac{\alpha}{\pi} \times 10^{-15}. \tag{6}$$

While still small, this deviation is far larger than the usual effects one would ascribe to gravity. In fact, the minimal discrepancy in the calculation of $(g-2)$ that arises in this way is equivalent to the contribution from a lepton of mass $M \sim 100$ GeV, and is roughly twice the contribution to $(g-2)$ from the top quark [16]. These effects are enormously larger than conventional estimates of Planck scale corrections which are of order $(m_e/M_P)^2 \sim 10^{-44}$.

More generally, we may consider processes of characteristic energy p which receive contributions from dimension D operators with $D > 4$, characterizing new physics. The correction due to a finite UV cutoff is of the order of $(\alpha/\pi)(p/\Lambda)^{(D-4)}$. The required IR cutoff $L \lesssim M_P/\Lambda^2$ leads to additional corrections $\sim(\alpha/\pi)(1/L^2p^2)$, which are at least as big as $(\alpha/\pi)(\Lambda^2/pM_P)^2$, according to our constraint Eq. (2). Minimization of this theoretical uncertainty occurs for a UV cutoff $\Lambda \sim p(M_P/p)^{2/D}$. Thus in a given experiment there is a maximum energy scale that can be probed and a maximum accuracy that can be achieved using conventional quantum field theory, with the energy scale depending on M_P to a remarkably small fractional power. For operators of dimension five this scale is $M_P^{2/5}$, while for operators of dimension six it is $M_P^{1/3}$.

Note that the relative size of these effects grows with p. When p is the weak scale and the effective theory is the standard model, new physics at short distances appears in the effective theory through dimension six operators, and the maximum energy scale that can be conventionally probed is 10^8 GeV, with a corresponding uncertainty of 10^{-13}. If the new high energy physics appears through dimension five operators the maximum energy scale would be 10^9 GeV with an uncertainty of 10^{-9}.

We might worry that the low scale which can be probed by electroweak physics eliminates the possibility of computing coupling constant unification, which involves an energy scale $M_{\rm GUT} \sim 10^{16}$ GeV. However it is still possible to consider running of dimension four operators up to energies as high as M_P. In order to compute coupling constant running in the presence of our IR and UV cutoffs, we may use a renormalization group treatment, matching the S matrices of two theories with parameters $\{L, \Lambda\}$ and $\{L', \Lambda'\}$ [each of which obey Eq. (2)] in their combined domain of validity. There is an inherent uncertainty in the beta function at a given energy scale due to the effects of these cutoffs. Choosing $\{L, \Lambda\}$ at each energy scale to minimize the uncertainty [18] leads to corrections of the relation between the unified coupling at $M_{\rm GUT}$ and the standard model gauge couplings at M_Z,

$$\frac{4\pi}{\alpha_i(M_Z)} = \frac{4\pi}{\alpha_{\rm GUT}(M_{\rm GUT})} + b_i \ln\frac{m_z}{M_{\rm GUT}} + \mathcal{O}\big((M_{\rm GUT}/M_P)^{2/3}\big)\,. \tag{7}$$

These corrections are small, but comparable to the usual 2-loop corrections, and are not obviously out of experimental reach. Thus if one had a compelling reason to believe in a particular grand unified theory with a unification scale well below M_P, one might be able to use visible deviations from its low energy coupling constant predictions as evidence for the limitations of quantum field theory proposed here.

As our renormalization group (RG) analysis of gauge coupling flow differs from the conventional analysis by only small corrections, one might expect to obtain conventional results for the RG flow of the vacuum energy as well, recovering the usual fine-tuning problem associated with the cosmological constant, arising from quartic divergences. However, in order to match two theories with cutoffs $\{L, \Lambda\}$ and $\{L', \Lambda'\}$ by requiring that they reproduce the same physical vacuum energy density λ—by comparing graviton propagators about a flat metric, for example—the lengths L and L' both must be larger than the length scale $M_P/\sqrt{\lambda}$, in order to avoid spurious finite volume effects. This implies that one cannot perform the RG scaling to UV cutoffs larger than $\lambda^{(1/4)}$, and that consequently one never sees a fine-tuning problem for the vacuum energy.

It is conceivable that black holes and their interactions with particles can be described by some effective field theory, eliminating the motivation for the bound of Eq. (2). It remains difficult to understand the necessary nonextensive behavior of the entropy without *some* infrared limitation of effective field theory at least as strong as the Bekenstein-motivated bound of Eq. (1). However, even this latter bound leads to conclusions qualitatively similar to those above. For example, experiments at a scale p sensitive to new physics which arises through dimension D operators ($D > 4$) would be limited to probing energies below $\Lambda \sim (p^{D-2}M_P^4)^{1/(D+2)}$, and the maximum theoretical accuracy would be $\sim(\alpha/\pi)\,(p/M_P)^{4(D-4)/(D+2)}$.

Both bounds give relatively large corrections to effective field theory computations compared to conventional computable quantum gravitational effects. The latter are generally expected to be suppressed by integral powers of M_P; such expectations are born out by explicit constructions of effective field theories from string theory [19].

It is tempting to consider a less drastic solution: patching up conventional effective field theory (with a Planck scale UV cutoff and no IR cutoff) by eliminating "by hand" those states corresponding to black holes. We do not know how to prune a Hilbert space in this manner; the result would likely be a bizarre, nonlocal theory. Still, one could imagine that even though most of the degrees of freedom in an effective field theory in an arbitrarily large box have no sensible physical interpretation, for some reason the theory accurately describes the properties of few particle states. This would leave conventional calculations which contain no intermediate states approaching black hole formation unchanged to low orders in perturbation theory, while rejecting the numerous states predicted by the same theory which lie within their own Schwarzschild radius. However, there would be drastic effects on thermal distributions even at temperatures $T \ll \Lambda$. Instead, our main assumption is that a local effective field theory which correctly describes all single particle states with momenta up to $p \sim \Lambda$ should also describe multiparticle excitations, and would have a normal density matrix for thermal distributions with $T \ll \Lambda$. While conventional, this assumption may not be valid when the underlying theory is not local. The alternative that an effective field theory can be valid up to a scale Λ for certain calculations, but fails to correctly describe a thermal system at temperature $T \ll \Lambda$, seems at least as strange as our assumption.

In conclusion, many different results about the physics of black holes imply that, in the presence of quantum gravity, there are no fundamental extensive degrees of freedom. Furthermore, considerations of the maximum possible entropy of systems which do not contain black holes suggest that ordinary quantum field theory may not be valid for arbitrarily large volumes, but would apply provided the UV and IR cutoffs satisfy a bound given by Eq. (2). The experimental success of quantum field theory survives, as long as this effective theory is not applied to calculations which simultaneously require both a low infrared cutoff and an overly high UV cutoff. The simultaneous UV and IR sensitivity of computations relevant for current laboratory experiments never comes close to requiring cutoffs which violate Eq. (2). In contrast, the computation of the quantum contribution to the vacuum energy of the visible universe within quantum field theory requires a UV cutoff of less than $10^{-2.5}$ eV. With this cutoff, no fine-tuned cancellation of the cosmological constant is required. Recognition that quantum field theory vastly overcounts states can help resolve the enormous discrepancy between conventional estimates of the vacuum energy and the observed cosmological constant and eliminate a celebrated fine-tuning problem.

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*Email address: cohen@andy.bu.edu
†Email address: dbkaplan@phys.washington.edu
‡Email address: anelson@phys.washington.edu

[1] For example, a free Weyl fermion on a lattice of size L and spacing $1/\Lambda$ has $4^{(L\Lambda)^3}$ states and entropy $S = (L\Lambda)^3 \ln 4$ (ignoring lattice doublers); a lattice theory of bosons represented by a compact field likewise has entropy scaling as $(L\Lambda)^3$.
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