Severe Limits on Variations of the Speed of Light with Frequency

Bradley E. Schaefer*
Physics Department, Yale University, P.O. Box 208121, New Haven, Connecticut 06520-8121
(Received 29 October 1998)

Explosive astrophysical events at high redshift can be used to place severe limits on the fractional variation in the speed of light with frequency ($\Delta c/c$), the photon mass (m_γ), and the energy scale of quantum gravity (E_{QG}). I find $\Delta c/c < 6.3 \times 10^{-21}$ based on the simultaneous arrival of a flare in GRB 930229 with a rise time of 220 ± 30 μs for photons of 30 and 200 keV. The limit on m_γ is 4.2×10^{-44} g for GRB 980703 from radio to gamma ray observations. The limit on E_{QG} is 8.3×10^{16} GeV for GRB 930131 from 30 keV to 80 MeV photons. [S0031-9007(99)09404-1]

PACS numbers: 98.70.Rz, 03.30.+p, 04.60.–m, 14.70.Bh

The question of whether the speed of light varies with frequency is of fundamental and current interest: (1) Einstein's postulate of the invariance of light is the cornerstone of much of modern physics, so tests of the correctness of this postulate should be pushed as far as possible. (2) With the recent evidence from Super-Kamiokande that the neutrino likely has a mass [1], the question of the mass of the photon should be reexamined. (3) Quantum gravity models suggest [2,3] that the speed of light has an effective energy dependence.

Classical textbooks [4] and review articles [5] report the status as of the middle 1970s: Laboratory and accelerator experiments were not able to test for fractional variations in the speed of light with frequency ($\Delta c/c = [c_{\nu_1} - c_{\nu_2}]/c$) to better than roughly 10^{-8}. The first limit that took advantage of astronomical distances compared the arrival time of radio and optical emission from flare stars to constrain $\Delta c/c < c\,\Delta t/D = 10^{-6}$ [6], where Δt is the difference in arrival times and D is the source distance. Then Warner and Nather measured the phase difference for pulses of the Crab pulsar between optical wavelengths of 0.35 and 0.55 μm to be less than 10 μs [7]. At a distance of 2 kpc, this limits $\Delta c/c$ to be less than 5×10^{-17}.

Surprisingly, no further improvements on $\Delta c/c$ have been made on the Crab pulsar limit, and the topic has received little subsequent discussion in the literature. Indeed, the speed of light has been defined to be a constant for purposes of metrology. In the meantime, a key assumption for the Crab pulsar limit has been severely undermined since five out of six pulsars detected at high energy have pulse structures that vary strongly both in shape and phase as a function of frequency [8]. Within the last year, several groups have independently realized that gamma ray bursts (GRBs) provide a means to look for delays in light traversing extremely large distances. Amelino-Camelia *et al.* [2] set approximate limits on the dispersion scale for quantum gravity, while Biller *et al.* [9] set stricter limits by analysis of a short TeV flare seen in a nearby active galaxy (Mkn 421).

The old GRB galactic/cosmological distance scale debate has been definitely resolved with bursters residing in very distant galaxies. The high accuracy isotropy of burst positions is now adequate [10] and the time dilation of light curves is definitive [11], but it is the x-ray, optical, and radio counterparts which brought universal agreement. These include five radio transients with rapid scintillation implying high redshift, four optical transients with measured redshifts ($z = 0.835, 0.966, 5.3,$ and 1.60), two optical transients with host galaxies of measured redshifts ($z = 3.42, 1.096$), twelve of thirteen optical transients coinciding with faint host galaxies (magnitude ~25), and two x-ray transients with redshifted iron lines ($z = 0.83, 0.33$) [12]. The debate is now concerned with whether the distance scale is that associated with no evolution (and an average peak luminosity of 1×10^{57} photons $\cdot$ s^{-1}), evolution appropriate for star formation rates (2×10^{58} photons $\cdot$ s^{-1}), or even higher peak luminosity (~10^{59} photons $\cdot$ s^{-1}), with the higher luminosities in general favor [13,14].

In this paper, I present new limits based on a variety of explosive events at high redshifts. My original motivation was the discovery last year of a flare in a gamma ray burst (GRB 930229) with a 220 ± 30 μs rise time that occurs simultaneously from 30 to 200 keV [15], and the realization that this constrains the dispersion of light to be less than a millisecond out of a Hubble time. However, the relevant limit depends on the assumed functional form for the frequency dependence of "c," so different events are the most restrictive in the various cases.

In Table I, I have gathered the data for the most restrictive events of various classes. These include short duration GRBs, GRBs with GeV photons, GRBs with associated x-ray/optical/radio transients, high redshift Type Ia supernovae, the active galaxy Mkn 421, and the Crab pulsar. The first seven columns give the source name, the observed bands, the observed maximum delay Δt between the rise of the light curve at two different frequencies, the reference for the observation, the low frequency, the high frequency, and the source distance D. Cosmological distances were calculated from the look-back time as a function of redshift for the most conservative reasonable case of $H_0 = 80$ km $\cdot$ s^{-1} $\cdot$ Mpc^{-1} and $\Omega = 1$. The GRBs

 0031-9007/99/82(25)/4964(3)$15.00

TABLE I. Limits on $\Delta c/c$, m_γ, $E_{\rm QG}$, and p from explosive events at high redshift.

Event	Bands	Δt	Ref.	$\nu_1(Hz)$	$\nu_2(Hz)$	D(Mpc)	$\Delta c/c$	m_γ(g)	$E_{\rm QG}$(GeV)	p
GRB 930229	$\gamma-\gamma$	0.5 ms	[15]	7.2×10^{18}	4.8×10^{19}	791	6.3×10^{-21}	6.1×10^{-39}	2.7×10^{16}	1.1×10^{21}
GRB 910711	$\gamma-\gamma$	2 ms	[21]	7.2×10^{18}	1.2×10^{20}	1413	1.4×10^{-20}	9.0×10^{-39}	3.3×10^{16}	1.2×10^{21}
GRB 910625	$\gamma-\gamma$	4 ms	⋯	7.2×10^{18}	1.2×10^{20}	1344	3.0×10^{-20}	1.3×10^{-38}	1.6×10^{16}	5.6×10^{20}
GRB 910607	$\gamma-\gamma$	8 ms	⋯	7.2×10^{18}	1.2×10^{20}	1677	4.8×10^{-20}	1.7×10^{-38}	9.9×10^{15}	3.5×10^{20}
GRB 930131	$\gamma-$GeV	25 ms	[22]	7.2×10^{18}	1.9×10^{22}	260	9.6×10^{-19}	7.4×10^{-38}	8.3×10^{16}	2.8×10^{21}
GRB 930131	$\gamma-$GeV	0.5 s	[22]	7.2×10^{18}	1.1×10^{23}	260	1.9×10^{-17}	3.3×10^{-37}	2.4×10^{16}	8.0×10^{20}
GRB 940217	$\gamma-$GeV	4800 s	[23]	7.2×10^{18}	4.3×10^{24}	385	1.2×10^{-13}	2.7×10^{-35}	1.4×10^{14}	4.8×10^{18}
GRB 970508	X$-\gamma$	5.6 h	[24]	2.4×10^{17}	1.2×10^{20}	1493	1.4×10^{-13}	9.2×10^{-37}	3.7×10^{9}	2.2×10^{18}
GRB 970508	U$-\gamma$	4.4 h	[25]	8.2×10^{14}	1.2×10^{20}	1493	1.1×10^{-13}	2.8×10^{-39}	4.7×10^{9}	1.4×10^{18}
GRB 970508	Radio$-\gamma$	121 h	[26]	8.6×10^{9}	1.2×10^{20}	1493	2.9×10^{-12}	1.5×10^{-43}	1.7×10^{8}	4.8×10^{21}
GRB 980703	X$-\gamma$	22 h	[27]	2.4×10^{17}	1.2×10^{20}	1592	5.0×10^{-13}	1.8×10^{-36}	1.0×10^{9}	1.0×10^{15}
GRB 980703	I$-\gamma$	21 h	[28]	3.3×10^{14}	1.2×10^{20}	1592	4.8×10^{-13}	2.4×10^{-39}	1.0×10^{9}	7.6×10^{17}
GRB 980703	Radio$-\gamma$	29 h	[29]	5.0×10^{9}	1.2×10^{20}	1592	6.6×10^{-13}	4.2×10^{-44}	7.6×10^{8}	3.6×10^{22}
SN 1997ap	I$-$R	240 h	[30]	3.3×10^{14}	4.3×10^{14}	1489	5.8×10^{-12}	1.3×10^{-38}	6.8×10^{1}	2.2×10^{11}
SN 1994G	I$-$R	72 h	[31]	3.3×10^{14}	4.3×10^{14}	1029	2.5×10^{-12}	8.8×10^{-28}	1.6×10^{2}	5.1×10^{11}
Mkn 421	TeV$-$TeV	280 s	[9]	1.2×10^{26}	4.8×10^{26}	112	2.5×10^{-14}	2.1×10^{-28}	6.0×10^{16}	1.6×10^{14}
Crab	V$-$U	0.01 ms	[7]	5.5×10^{14}	8.2×10^{14}	0.002	5.0×10^{-17}	5.4×10^{-41}	2.3×10^{7}	3.0×10^{16}

in the first seven rows of Table I do not have measured redshifts, so I have constrained their distance with the conservative assumption that their peak luminosity is $>10^{57}$ photons $\cdot$ s^{-1} and have used Fenimore's peak flux versus redshift relation [16]. These assumptions used to derive the burst distances are all conservative, such that the constraints on the variations of the speed of light are made more severe by typically 1 order of magnitude when more plausible parameters ($H_0 = 65$ km $\cdot$ s^{-1} $\cdot$ Mpc^{-1}, $\Omega = 0.3$ for a flat Universe, and a peak luminosity of 10^{58} photons $\cdot$ s^{-1}) are used.

All the events at cosmological distances are explosive, so presumably the light curves of different wavelengths will start to rise simultaneously in the absence of dispersion. The lack of flux or the lack of observations might produce an apparent delay not associated with dispersion. Also, the explosive system might have an intrinsic delay in emission which could counter any effects of dispersion. Finally, the normal dispersion by electrons along the line-of-sight can cause significant delays in the arrival time of radio waves. Nevertheless, I will assume that no conspiracies of delays hides the effect of dispersion in the speed of light. The number of strict limits from widely disparate classes of events argues that any such conspiracy is unlikely.

The frequency dependence of c is not known, so no single number can represent the limit on variations in the speed of light. The most model independent parameter is one with no reference to the observed frequencies, $\Delta c/c$. This limit on $\Delta c/c$ will be $c \cdot \Delta t/D$. For a general dispersion relation like $V = c(1 + A\nu^{-2})^{0.5}$ with V as the velocity of light with frequency ν and both c and A as constants [4,17], the limit on A is $(2c\Delta t/D)(\nu_2^{-2} - \nu_1^{-2})^{-1}$ for observations at frequencies ν_1 and ν_2. This dispersion can be related to the photon mass as $m_\gamma = A^{0.5}h/c^2$. For quantum gravity models [2], the characteristic energy $E_{\rm QG}$ is greater than $h(\nu_2 - \nu_1)D/(c\Delta t)$. Previous papers (e.g., [7]) have defined a quantity $p = (c/\Delta c)(\nu_2/\nu_1)$ which is useful for comparison. The limits on $\Delta c/c$, m_γ, $E_{\rm QG}$, and p for each event are presented in columns 8–11 of Table I.

The limits in Table I are many orders of magnitude past those from the Crab pulsar. The strictest limit on $\Delta c/c$ is 6.3×10^{-21} for GRB 930229, with second place at 1.4×10^{-20} for GRB 910711. The two lowest limits on the photon mass are 4.2×10^{-44} and 1.5×10^{-43} g for GRB 980703 and GRB 970508, both from radio to gamma ray constraints for bursts with measured redshifts. The tightest limit on $E_{\rm QG}$ is 8.3×10^{16} GeV for GRB 930131 followed closely by 6.0×10^{16} GeV for Mkn 421.

The new limit on $\Delta c/c$ is close to 10^4 times better than the Crab pulsar limit, and it is comforting to know that Einstein's postulate is vindicated to this level. The limit on the photon mass is orders of magnitude worse than that obtained by considering interplanetary or interstellar magnetic fields [18]; however, this model dependent conclusion uses virtual photons with $\nu = 0$ so an independent method is still of interest [17]. The limits on $E_{\rm QG}$ are not yet close to the expected $\sim 10^{19}$ GeV [2,9].

Significant improvements in limits on $\Delta c/c$ are unlikely since we are already dealing with delays of under a millisecond out of a Hubble time. Limits on the photon mass can be improved by several orders of magnitude with radio detection of prompt emission from GRBs (as with the FLIRT telescope [19]) or with radio studies of millisecond pulsars. The detection of $\sim$100 GeV photons near the start of a GRB of moderate brightness is possible with the GLAST satellite mission [20], and this would test the expected quantum gravity threshold.

*Electronic address: schaefer@grb2.physics.yale.edu

[1] Y. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1562 (1998).

[2] G. Amelino-Camelia *et al.*, Nature (London) **393**, 763 (1998).
[3] R. Gambini and J. Pullin, gr-qc/9809038, 1998.
[4] A.P. French, *Special Relativity* (W. W. Norton, New York, 1968); J.D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975).
[5] J.L. Hall, in *Atomic Masses and Fundamental Constants*, edited by J.H. Sanders and A.H. Wapstra (Plenum, New York, 1976), p. 322; R.H. Garstang, Astrophys. J. **79**, 1260 (1974).
[6] B. Lovell, F. Whipple, and L. Solomon, Nature (London) **202**, 377 (1964).
[7] B. Warner and R. Nather, Nature (London) **222**, 157 (1969).
[8] D.J. Thompson, in *Pulsars, Problems and Progress*, IAU Colloquium 160, edited by S. Johnston, M. Walker, and M. Bailes (Astronomic Society of the Pacific, San Francisco, 1996), p. 307.
[9] S.D. Biller *et al.*, gr-qc/9810044.
[10] C.A. Meegan *et al.*, in *Gamma-Ray Bursts*, edited by C.A. Meegan, R.D. Preece, and T.M. Koshut, AIP Conf. Proc. No. 428 (AIP, New York, 1998), p. 3; J.M. Kommers *et al.*, *ibid.*, p. 45.
[11] M. Deng and B.E. Schaefer, Astrophys. J. **502**, L109 (1998).
[12] http://gcn.gsfc.nasa.gov/gcn/gcn_main.html; C.A. Meegan, R.D. Preece, and T.M. Koshut, *Gamma-Ray Bursts* (Ref. [10]).
[13] B.E. Schaefer, Astrophys. J. **511**, L79 (1999).
[14] S.R. Kulkarni *et al.*, Nature (London) **393**, 35 (1998).
[15] B.E. Schaefer and K.C. Walker, Astrophys. J. **511**, L89 (1999).
[16] E.E. Fenimore *et al.*, Nature (London) **357**, 140 (1992).
[17] G. Feinberg, Science **166**, 879 (1969); Z. Bay and J.A. White, Phys. Rev. D **5**, 796 (1972).
[18] A.S. Goldhaber and M.M. Nieto, Rev. Mod. Phys. **43**, 277 (1971); J.D. Barrow and R.R. Burman, Nature (London) **307**, 14 (1984).
[19] R.J. Balsano *et al.*, in *Gamma Ray Bursts* (Ref. [10]), p. 585.
[20] J.T. Bonnell *et al.*, in *Gamma Ray Bursts* (Ref. [10]), p. 884.
[21] P.N. Bhat *et al.*, Nature (London) **359**, 217 (1992).
[22] M. Sommer *et al.*, Astrophys. J. **422**, L63 (1994).
[23] K. Hurley *et al.*, Nature (London) **372**, 652 (1994).
[24] L. Piro *et al.*, International Astronomical Union Circular No. 6656 (1997).
[25] A.J. Castro-Tirado *et al.*, International Astronomical Union Circular No. 6657 (1997).
[26] D.A. Frail *et al.*, Nature (London) **389**, 261 (1997).
[27] T.J. Galama *et al.*, GRB Coordinates Network No. 127 (1998).
[28] P.M. Vreeswijk *et al.*, GRB Coordinates Network No. 132 (1998).
[29] D.A. Frail *et al.*, GRB Coordinates Network No. 128 (1998).
[30] S. Perlmutter *et al.*, Nature (London) **391**, 51 (1998).
[31] S. Perlmutter *et al.*, Astrophys. J. **483**, 565 (1997).