Nonequilibrium Phase Transition in Rapidly Expanding Matter

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Nonequilibrium features of a first order phase transition from the quark-gluon plasma to a hadronic gas in relativistic heavy-ion collisions are discussed. It is demonstrated that strong collective expansion may lead to the fragmentation of the plasma phase into droplets surrounded by undersaturated hadronic gas. Subsequent hadronization of droplets will generate strong nonstatistical fluctuations in the hadron rapidity distribution in individual events. The strongest fluctuations are expected in the vicinity of the phase transition threshold. [S0031-9007(99)09365-5]

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The main goal of present and future experiments with relativistic heavy ions is to produce and study in the laboratory a new form of strongly interacting matter, the quark-gluon plasma (QGP). Because of the confinement of color charges, only colorless hadronic final states can be observed experimentally. Therefore, QGP properties can be studied only indirectly through the final hadron distributions or by penetrating electromagnetic probes.

The phase structure of QCD is not yet fully understood. Reliable lattice calculations exist only for baryon-free matter where they predict a second order phase transition or crossover at $T_c \approx 160$ MeV. Recent calculations using different models [1–4] reveal the possibility of a first order phase transition at large baryon chemical potentials and moderate temperatures. The predicted phase diagram in the (T, μ) plane contains a first order transition line (below called the critical line) with a (tri)critical point at $T \approx 120$ MeV [2,3]. Possible signatures of this point in heavy-ion collisions are discussed in Ref. [5]. Under certain nonequilibrium conditions, a first order transition is also predicted for baryon-free matter [6].

A striking feature of relativistic heavy-ion collisions, confirmed in many experiments (see, e.g., [7]), is a very strong collective expansion of matter. The applicability of equilibrium concepts for describing phase transitions under such conditions becomes questionable. The goal of this paper is to demonstrate that nonequilibrium phase transitions in rapidly expanding matter can lead to interesting phenomena which, in a certain sense, can be even easier to observe.

To make the discussion below more concrete, I adopt a picture of the chiral phase transition for which the mean chiral field $\Phi = (\sigma, \boldsymbol{\pi})$ serves as an order parameter. It is assumed that the theory respects chiral symmetry, which is spontaneously broken in the vacuum where $\sigma = f_\pi$, $\boldsymbol{\pi} = 0$. The effective thermodynamic potential $\Omega(T, \mu; \Phi)$ depends, besides Φ, on temperature T and baryon chemical potential μ. Since explicit symmetry breaking terms are supposed to be small, to a good approximation Ω is a function of $\Phi^2 = \sigma^2 + \boldsymbol{\pi}^2$.

The schematic behavior of $\Omega(T, \mu; \Phi)$ as a function of the order parameter field σ at $\boldsymbol{\pi} = 0$ is shown in Fig. 1. Each curve represents a certain point on the (T, μ) trajectory of expanding matter. The minima of Ω correspond to the stable or metastable states of matter under the condition of thermodynamical equilibrium, where the pressure is $P = -\Omega_{\min}$. The figure is based on the calculations within the linear σ model with constituent quarks [1], which predicts a rather weak first order phase transition. A similar structure of $\Omega(T, \mu; \Phi)$ but, possibly, with a stronger phase transition is predicted by the Nambu-Jona-Lasinio model [2] and by the random matrix model [3]. The discussion below is quite general.

Assume that at some early stage of the reaction thermal (but not necessarily chemical) equilibrium is established and partonic matter is in a "high energy density" phase Q. This state corresponds to the absolute minimum of Ω with the order parameter close to zero, $\sigma \approx 0$, $\boldsymbol{\pi} \approx 0$,

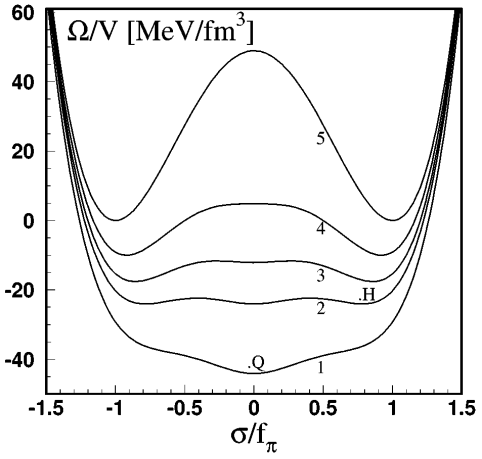

FIG. 1. Schematic view of the effective thermodynamic potential per volume Ω/V as a function of the order parameter field σ at $\boldsymbol{\pi} = 0$, as predicted by the linear σ model in the chiral limit $m_\pi = 0$ [1]. The curves from bottom to top correspond to the different stages of the isentropic expansion of homogeneous matter starting from $T = 100$ MeV and $\mu = 750$ MeV (curve 1). The upper curve 5 is the vacuum potential. The other curves are discussed in the text.

and chiral symmetry restored (curve 1). Because of a very high internal pressure, Q matter will expand and cool down. At some stage a metastable minimum appears in Ω at a finite value of σ corresponding to a "low energy density" phase H, in which chiral symmetry is spontaneously broken. At some later time, the critical line in the (T, μ) plane is crossed where the Q and H minima have equal depths, i.e., $P_H = P_Q$ (curve 2). At later times the H phase becomes more favorable (curve 3), but the two phases are still separated by the potential barrier. If the expansion of the Q phase continues until the barrier vanishes (curve 4), the system will freely roll down into the lower energy state corresponding to the H phase (spinodal instability).

According to the standard theory of homogeneous nucleation [8], supercritical bubbles of the H phase appear only below the critical line. Under condition of thermal equilibrium between the two phases, the supercritical bubbles can grow only through the conversion of new portions of the Q matter into the H phase on the bubble boundary. The bubble growth is then limited by a small viscosity of the Q phase resulting in a slow dissipation of the latent heat [8]. Therefore, a certain degree of supercooling is needed in order to convert a significant fraction of the Q matter into the H phase in the form of nucleation bubbles [8,9].

In rapidly expanding matter the nucleation picture might be very different. Let us consider first an isotropically expanding system with the collective velocity field which follows the Hubble law locally, $v(r) = \mathcal{H} r$. The Hubble "constant" $\mathcal{H}$ may in general be a function of time, e.g., $\mathcal{H} \sim 1/t$. Suppose that a bubble of the H phase has formed in the expanding Q matter because of a statistical fluctuation. In the thin-wall approximation the change in thermodynamic potential of the system can be decomposed into three parts,

$$\Delta\Omega = \Delta\Omega_{\rm bulk} + \Delta\Omega_{\rm surf} + \Delta\Omega_{\rm kin}\,. \tag{1}$$

The bulk and surface terms are expressed through the bubble radius R in a standard way,

$$\Delta\Omega_{\rm bulk} = -\frac{4\pi}{3} R^3 (P_H - P_Q)\,,$$

$$\Delta\Omega_{\rm surf} = 4\pi R^2 \gamma\,, \tag{2}$$

where P_H and P_Q are the pressures of the bulk H and Q phases, γ is the effective surface tension. The last term in Eq. (1) accounts for the change in the local kinetic energy of expanding matter,

$$\Delta\Omega_{\rm kin} = \frac{1}{2}\int_0^R 4\pi r^2\, dr \mathcal{E}(r) v^2(r)$$

$$\approx -\frac{2\pi}{5} R^5 \Delta\mathcal{E}\mathcal{H}^2, \tag{3}$$

where $\Delta\mathcal{E} \equiv \mathcal{E}_Q - \mathcal{E}_H$ is the difference of energy (more exactly, enthalpy) densities of the two bulk phases. Since this term is negative (typically, $\mathcal{E}_Q \gg \mathcal{E}_H$), the bubble formation is favored by the collective expansion. Moreover, the nucleation can start now even above the critical line, when $P_H < P_Q$, and the standard theory would predict no growing bubbles. In principle, the phase separation can start as early as the metastable H state appears in the thermodynamic potential, and a stable interface between the phases may exist.

Using Eqs. (2) and (3) one can determine the critical bubble radius R_c, corresponding to the top of the potential barrier in $\Delta\Omega(R)$. The condition $\partial_R \Delta\Omega = 0$ leads to a cubic equation for R_c. When $\mathcal{H} \to 0$ the kinetic term vanishes and this equation gives a standard Laplace formula for the critical bubble [8]. However, for realistic parameters (see below) the kinetic term dominates. In particular, in the vicinity of the critical line, when $P_H \approx P_Q$, one can consider the bulk term perturbatively. Then one obtains

$$R_c = \left(\frac{4\gamma}{\Delta\mathcal{E}\mathcal{H}^2}\right)^{1/3}\left[1 - \frac{P_H - P_Q}{3(2\gamma^2\Delta\mathcal{E}\mathcal{H}^2)^{1/3}}\right]. \tag{4}$$

The bubbles with $R > R_c$ will expand further while those with $R < R_c$ will eventually shrink. One may even expect the formation of vacuum bubbles with $P \le 0$ inside [10].

Below the critical line the bubbles will grow faster due to increasing pressure difference, $P_H - P_Q > 0$, between the two phases. It is most likely that the conversion of Q matter on the bubble boundary is not fast enough to saturate the H phase. Therefore, a fast expansion may lead to a deeper cooling of the H phase inside the bubbles compared to the surrounding Q matter. At some stage the H bubbles percolate, and the topology changes to isolated regions of the Q phase (Q droplets) surrounded by the undersaturated vapor of the H phase.

Standard thermodynamical concepts cannot be used in this nonequilibrium situation. However, the characteristic droplet size can be estimated by applying the energy balance consideration first proposed by Grady [11,12] in the study of dynamical fragmentation. The idea is that the fragmentation of expanding matter is a local process minimizing the sum of surface and kinetic (dilational) energies per fragment volume. The predictions of this simple model are in reasonable agreement with molecular dynamics simulations [12,13] and experimental data on dynamical fragmentation of fluids and solids (see, e.g., [11,12,14]). As shown in Ref. [15], this prescription works fairly well also for multifragmentation of expanding nuclei, where the standard statistical approach fails.

Let us imagine an expanding spherical Q droplet embedded in the background of the dilute H phase. In the droplet rest frame the change of thermodynamic potential compared to the uniform H phase is given by the same expression (1) but with indexes H and Q interchanged. The kinetic term is positive now. According to the Grady's prescription, the characteristic droplet radius R^* can be

determined by minimizing

$$\left(\frac{\Delta\Omega}{V}\right)_{\text{droplet}} = -(P_Q - P_H) + \frac{3\gamma}{R} + \frac{3}{10}\Delta\mathcal{E}\mathcal{H}^2R^2. \quad (5)$$

It is worth noting that the collective kinetic energy term acts here as an effective long-range potential, similar to the Coulomb potential in nuclei. Since the bulk term does not depend on R the minimization condition constitutes the balance between the collective kinetic energy and interface energy. This leads to an optimum droplet radius

$$R^* = \left(\frac{5\gamma}{\Delta\mathcal{E}\ \mathcal{H}^2}\right)^{1/3}. \quad (6)$$

It should be noticed that this radius is expressed through the same combination of model parameters as the critical bubble radius at $P_H \approx P_Q$, Eq. (4), but with a slightly bigger numerical coefficient. This suggests that in the vicinity of the critical line the H and Q phases occupy roughly equal fractions of the total volume. This mixed state of matter is far from thermodynamical equilibrium because of the excessive interfacial energy and undersaturation of the H phase. One can say that the metastable Q matter is torn apart by a mechanical strain associated with the collective expansion. This has a direct analogy with the fragmentation of pressurized fluids leaving nozzles [13,14]. In a similar way, splashed water forms droplets.

At ultrarelativistic collision energies the expansion will be very anisotropic, with its strongest component along the beam direction. Applying the same consideration for the anisotropic flow, one can see that resulting fractures will have smaller size in the direction of stronger flow. Therefore, in the case of strong one-dimensional expansion the inhomogeneities associated with the phase separation will rearrange themselves into pancakelike slabs of Q matter layered by the dilute H phase. The characteristic width of the slab is given by Eq. (6) with a slightly different geometrical factor. At a later stage the slabs will further fragment into smaller droplets.

The driving force for expansion is the pressure gradient $\nabla P = c_s^2\nabla\mathcal{E}$, which depends on the sound velocity in the matter c_s. In the vicinity of the critical line one may expect a "soft point" [16,17] where c_s is smallest and the ability of matter to generate the collective expansion is minimal (small $\mathcal{H}$). If the initial state of the Q phase is close to this point, the primordial droplets will be biggest. Increasing initial pressure will result in a faster expansion and smaller droplets. For numerical estimates I choose two values of the Hubble constant: $\mathcal{H}^{-1} = 20$ fm/c to represent the slow expansion from the soft point [16] and $\mathcal{H}^{-1} = 6$ fm/c for the fast expansion [9].

One should also specify two other parameters, γ and $\Delta\mathcal{E}$. The surface tension γ is a subject of debate at present. Lattice simulations indicate that at the critical point it could be as small as a few MeV/fm^2. However, for the nonequilibrium situation discussed here the values of 10–20 MeV/fm^2, which follow from effective chiral models, should be more appropriate. As a compromise, the value $\gamma = 10$ MeV/fm^2 is used below. It is clear that $\Delta\mathcal{E}$ should be close to the latent heat of the transition, i.e., about 0.5–1 GeV/fm^3. One can also estimate $\Delta\mathcal{E}$ by realizing that nucleons and heavy mesons are the smallest droplets of the Q phase. For estimates I take $\Delta\mathcal{E} = 0.5$ GeV/fm^3, i.e., the energy density inside the nucleon. Substituting these values in Eq. (6) one gets $R^* = 3.4$ fm for $\mathcal{H}^{-1} = 20$ fm/c and $R^* = 1.5$ fm for $\mathcal{H}^{-1} = 6$ fm/c.

In the lowest-order approximation the characteristic droplet mass is $M^* \approx \Delta\mathcal{E}V$. For spherical and slablike droplets one gets, respectively

$$M^*_{sp} \approx \frac{20\pi}{3}\frac{\gamma}{\mathcal{H}^2}, \quad M^*_{sl} \approx 2S\left(\frac{3\gamma(\Delta\mathcal{E})^2}{\mathcal{H}^2}\right)^{1/3}, \quad (7)$$

where S is the slab transverse area. It is interesting to note that in this approximation M^*_{sp} is independent of $\Delta\mathcal{E}$. For the two values of R^* given above M^*_{sp} is ~100 and ~10 GeV, respectively. The slablike fractures could have even larger mass, since S could be of order of the transverse system size. Using the minimum information principle one can show [12,15] that the distribution of droplets should follow an exponential law, $\exp(-\frac{M}{M^*})$. Therefore, with 1% probability one can find droplets as heavy as $5M^*$.

After separation, the droplets recede from each other according to the global Hubble expansion, predominantly along the beam direction. Therefore, their center-of-mass rapidities are in one-to-one correspondence with their spatial positions. Presumably they will be distributed more or less evenly between the target and projectile rapidities. At this late stage it is unlikely that the thermodynamical equilibrium will be reestablished between the Q and H phases or within the H phase alone.

The final fate of individual droplets depends on their sizes, expansion rate, and details of the equation of state. Because of the counteracting pressure of the H phase and additional Laplace pressure, their residual expansion will slow down. In smaller droplets the expansion and cooling may even reverse to the contraction and reheating. The conversion of Q droplets into the H phase may proceed through formation of a deflagration front [17] or evaporation of hadrons from the surface [18]. Bigger droplets may expand further until they enter the region of spinodal instability. As shown in Ref. [19], the characteristic time of the "rolling down" process is rather short, ~1 fm/c, so that the Q droplets will be converted rapidly into the H phase. The energy released in this process can be transferred partly into the collective oscillations of the $(\sigma, \boldsymbol{\pi})$ fields. Numerical simulations [10,20] show that these oscillations persist for a long time and give rise to soft pion radiation. One should also expect the formation of disoriented chiral condensates (DCC) in the voids between droplets.

Since rescatterings in the dilute H phase are rare, most hadrons produced from individual droplets will go directly into detectors. One may guess that the number of produced hadrons is proportional to the droplet mass. Each droplet will give a bump in the hadron rapidity distribution around its center-of-mass rapidity [19]. If emitted particles have a Boltzmann spectrum, the width of the bump will be $\delta y \sim 2\sqrt{T/m}$, where T is the droplet temperature and m is the particle mass. At $T \sim 100$ MeV this gives $\delta y \approx 2$ for pions and $\delta y \approx 1$ for nucleons. These spectra might be slightly modified by the residual expansion of droplets and their transverse motion. The resulting rapidity distribution in a single event will be a superposition of contributions from different droplets, and therefore it will exhibit strong nonstatistical fluctuations. The fluctuations will be more pronounced if primordial droplets are big. If droplets as heavy as 100 GeV are formed, each of them will produce up to $\sim$300 pions within a narrow rapidity interval, $\delta y \sim 1$. Such bumps can be easily resolved and analyzed. Critical fluctuations of a similar nature were discussed recently in Ref. [21].

Some unusual events produced by high-energy cosmic nuclei have been already seen by the JACEE collaboration [22]. Unfortunately, they are very few. We should be prepared to see plenty of such events in the future collider experiments. It is clear that the nontrivial structure of the hadronic spectra will be washed out to a great extent when averaging over many events. Therefore, more sophisticated methods of the event sample analysis should be used. The simplest one is to search for nonstatistical fluctuations in the hadron multiplicity distributions measured in a fixed rapidity bin [23]. One can also study the correlation of multiplicities in neighboring rapidity bins, bump-bump correlations, etc. Such standard methods as intermittency and commulant moments [21], wavelet transforms [24], and HBT interferometry [25] can also be useful. All these studies should be done at different collision energies to identify the phase transition threshold. The predicted dependence on the Hubble constant and the geometry of reaction, Eq. (7), can be checked in collisions with different ion masses and impact parameters.

One should bear in mind two important points. First, if the expansion trajectory goes close to the (tri)critical point, both γ and $\delta\mathcal{E}$ will tend to zero and the critical fluctuations will be less pronounced. Second, if a first order phase transition is possible only in the baryon-rich matter, then the Q droplets should have much higher baryon density than the hadronic phase [3]. In this case one should expect strong nonstatistical fluctuations in the distribution of the net baryon charge.

In conclusion, it is demonstrated that a first order phase transition in rapidly expanding matter should proceed through a nonequilibrium stage when the metastable phase fragments into droplets. If QCD matter undergoes such a phase transition, it will manifest itself in relativistic heavy-ion collisions by the formation of droplets of quark-gluon plasma. The primordial droplets should be biggest in the vicinity of the soft point where the expansion is slowest. The fragmentation of plasma might be accompanied by the formation of multiple DCC domains and enhanced soft-pion radiation. Subsequent hadronization of QGP droplets will lead to large nonstatistical fluctuations in the hadron rapidity density in individual events. These novel phenomena can only be detected through dedicated event-by-event analysis of experimental data.

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