Indirect Collider Signals for Extra Dimensions

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(Received 30 November 1998)

A recent suggestion that quantum gravity becomes strong near the weak scale can be probed by the exchange of Kaluza Klein towers of massive gravitons in fermion pair production in e^+e^- annihilation and in Drell-Yan production, including contributions from gluon-gluon fusion, at hadron colliders. These processes are found to provide strong bounds which are essentially independent of the number of extra dimensions. We also demonstrate that angular distributions provide a smoking gun signal for low-scale quantum gravity which cannot be mimicked by other new physics scenarios. [S0031-9007(99)09317-5]

PACS numbers: 13.10.+q, 04.50.+h, 11.25.Mj, 13.85.Rm

It has recently been suggested [1] that the hierarchy problem, i.e., the smallness of the ratio of the weak scale to the Planck scale ($M_{\rm Pl}$), may be avoided by simply removing the hierarchy. In this case gravitational interactions become strong near the weak scale and take place mainly in n new large spatial dimensions, known as the bulk. Because of experimental constraints standard model (SM) fields cannot propagate into the bulk and are forced to lie on a wall, or 3-dimensional brane, in the higher-dimensional space. The relation between the scales where gravity becomes strong in the $4 + n$ and 4-dimensional theories can be derived from Gauss' Law and is given by $M_{\rm Pl}^2 \sim r^n M_{\rm eff}^{2+n}$, where r is the size of the additional dimensions and $M_{\rm eff}$ is the effective Planck scale in the bulk. The hierarchy dilemma is resolved by taking $M_{\rm eff}$ to be near a TeV, which yields $r \sim 10^{30/n-19}$ meters. In this scenario $n = 1$ theories are thus automatically excluded, while the case of $n = 2$ with r at a submillimeter will be probed by future gravitational experiments [2]. This framework can be embedded [3] into string models, where the effective Planck scale can be identified with the string scale M_s. We concentrate on this particular scenario, but note that there are other interesting suggestions [4] for a low effective Planck or string scale.

While this is a fascinating concept, what makes this theory exciting is that it has testable consequences. One manifestation of these theories is the existence of a Kaluza Klein (KK) tower of massive gravitons which can interact with the SM fields on the wall. Here we examine the indirect effects of these massive gravitons being exchanged in fermion pair production in e^+e^- annihilation and Drell-Yan production at hadron colliders. In the latter case we examine a novel feature of this theory, which is the contribution of gluon-gluon initiated processes to lepton pair production. As we will see, these processes provide strong bounds on the effective Planck scale which are essentially independent of the number of extra dimensions. We also quantify the extent to which the spin-2 nature of the graviton exchange is distinguishable from other new physics contributions.

The effective theory below $M_{\rm eff}$ of concern here consists of the interactions between the SM fields on the wall and gravity. The bulk metric can be written as $G_{\hat\mu\hat\nu} = \eta_{\hat\mu\hat\nu} + h_{\hat\mu\hat\nu}(x^\mu, x^a)/M_{\rm eff}^{n/2+1}$, where the indices $\hat\mu$ extend over the full $4 + n$ dimensions, μ over the $3 + 1$ dimensions on the wall, and a over the n bulk dimensions. The graviton field-strength tensor $h_{\hat\mu\hat\nu}$ can be decomposed into spin-2, -1, and -0 fields with the interactions

$$\int d^{4+n}x T^{\hat\mu\hat\nu}\, \frac{h_{\hat\mu\hat\nu}(x^\mu, x^a)}{M_{\rm eff}^{n/2+1}}\,, \qquad (1)$$

where $T^{\hat\mu\hat\nu}$ is the symmetric, conserved stress-energy tensor in the bulk. The bulk fields appear as KK towers in the 4-dimensional space arising from a Fourier analysis over the cyclic boundary conditions of the compactified dimensions. The induced metric on the wall is $G_{\mu\nu}(x^\mu, x^a = 0)$ and the interactions with the SM fields are obtained by decomposing (1) into the 4-dimensional states. Performing this, we see that $T_{\mu a} = 0$ and hence the spin-1 KK states do not interact with the wall fields. The scalar, or dilaton, states couple proportionally to the trace of the stress-energy tensor. For interactions with fermions, this trace is linear in the fermion mass, while for gauge bosons it is quadratic in the boson mass. Hence the dilaton does not contribute to the processes under consideration here.

We have only to consider the interactions of the KK spin-2 gravitons with the SM fields. All gravitons in the KK tower, including the massless state, couple identically. We may thus use the couplings as obtained in linearized general relativity [5]. In this theory, the matrix element for $e^+e^- \to f\bar f$ generalized for the case of n massive graviton exchanges is

$$\mathcal{M} = \frac{1}{M_{\rm Pl}^2} \sum_n \frac{T^e_{\mu\nu} P^{\mu\nu\lambda\sigma} T^f_{\lambda\sigma}}{s - m_{\rm gr}^2[n]}\,, \qquad (2)$$

where the sum extends over the KK modes. $P_{\mu\nu\lambda\sigma}$ represents the polarization sum of the product of two graviton fields and is given in [5]. Since the spacing of the KK states is given by $\sim 1/r$, the sum over states in (2) can be approximated by an integral which is log divergent for $n = 2$ and power divergent for $n > 2$. A cutoff must be applied to regulate these ultraviolet divergences, and is generally taken to be the scale of the new physics. For $n > 2$ it can be shown [1] that the dominant contribution to this integral is of order $\sim M_{\rm Pl}^2/M_s^4$, where we have taken the cutoff to be the string scale, while for $n = 2$ this result is multiplied by a factor of order $\ln(M_s^2/s)$, where $\sqrt{s}$ is the center-of-mass energy of the process. Combining these results yields

$$\mathcal{M} = \frac{\lambda}{M_s^4}\{\overline{e}(p_1)\gamma_\mu e(p_2)\overline{f}(p_3)\gamma^\mu f(p_4)(p_2 - p_1)(p_4 - p_3)\overline{e}(p_1)\gamma_\mu e(p_2)\overline{f}(p_3)\gamma_\nu f(p_4)(p_2 - p_1)^\nu(p_4 - p_3)^\mu\}. \tag{3}$$

Note that graviton exchange is C and P conserving, and is independent of the flavor of the final state. The coefficient λ is of $\mathcal{O}(1)$ and cannot be explicitly calculated without knowledge of the full quantum gravity theory. It is dependent on the number of extra dimensions, how they are compactified, and is in principle a power series in s/M_s^2. We neglect this possible energy dependence in λ and note that the limits obtained here, which go as $|\lambda|^{1/4}$, are only very weakly dependent on its precise value and hence on the specific model realization. In principle the sign of λ is undetermined and we examine the constraints that can be placed on M_s with either choice of signs.

The angular distribution for $e^+e^- \to f\overline{f}$ with massive fermions is then given by

$$\begin{aligned}\frac{d\sigma}{dz} = N_c\,\frac{\pi\alpha^2}{2s}\,\beta\Big\{&P_{ij}[A_{ij}^eA_{ij}^f(1 + \beta^2z^2) + 2\beta B_{ij}^eB_{ij}^f z + A_{ij}^eC_{ij}^f(1 - \beta^2)]\\ &- \frac{\lambda s^2}{2\pi\alpha M_s^4}P_i[2\beta^3z^3v_i^ev_i^f - \beta^2(1 - 3z^2)a_i^ea_i^f]\\ &+ \frac{\lambda^2s^4}{16\pi^2\alpha^2M_s^8}[1 - 3\beta^2z^2 + 4\beta^4z^4 - (1 - \beta^2)(1 - 4\beta^2z^2)]\Big\},\end{aligned} \tag{4}$$

where the indices i, j are summed over γ and Z exchange, $z = \cos\theta$, P_{ij} and P_i are the usual propagator factors (defined in, e.g., [6]), $\beta = (1 - 4m_f^2/s)^{1/2}$, $A_{ij}^f = (v_i^fv_j^f + a_i^fa_j^f)$, $B_{ij}^f = (v_i^fa_j^f + v_j^fa_i^f)$, $C_{ij}^f = (v_i^fv_j^f - a_i^fa_j^f)$, and N_c represents the number of colors of the final state. In the case of Bhabha scattering, t- and u-channel graviton exchanges will also be present. If polarized beams are available a z-dependent left-right asymmetry can also be formed:

$$\begin{aligned}A_{\rm LR}(z) = &P_{ij}[B_{ij}^eA_{ij}^f(1 + \beta^2z^2) + 2\beta A_{ij}^eB_{ij}^f z + B_{ij}^eC_{ij}^f(1 - \beta^2)]/D\\ &- \frac{\lambda s^2}{2\pi\alpha M_s^4}P_i[2\beta^3z^2a_i^ev_i^f - \beta^2(1 - 3z^2)v_i^ea_i^f]/D,\end{aligned} \tag{5}$$

where D is given by the curly bracket in (4) above. Note that the total cross section and integrated left-right asymmetry are *unaltered* by graviton exchanges, independently of fermion flavor, up to terms of order s^4/M_s^8, and hence only the angular distributions for these quantities will be sensitive to these new exchanges.

The bin integrated angular distributions are displayed in Fig. 1 for the sample case of $b\overline{b}$ final states with $\sqrt{s} = 500$ GeV. The solid histogram corresponds to the SM expectations and the "data" points represent the case with graviton exchanges with $M_s = 1.5$ TeV. The two sets of data points correspond to the two sign choices for λ. The errors on the data points represent the statistics in each bin for an integrated luminosity of 75 fb^{-1}. We have assumed a 60% heavy quark tagging efficiency corresponding to the expectations for linear colliders, an electron beam polarization of 90%, a 10° angular cut around the beam pipe and included of initial state radiation. We see that these distributions provide a statistically significant and outstanding signal for graviton exchanges. It is clear that the spectra with the graviton exchanges do not have the $(1 + z^2)$ shape that is typical of the SM or any spin-1 exchange. Summing over e, μ, τ, c, b, and t final states, including the τ polarization asymmetry, and performing a χ^2 analysis results in the 95% C.L. search reaches shown in Table I. Note that the effects of string scales up to $6\sqrt{s}$ are discernible. The results from performing this same procedure for LEP II, but excluding top final states and the $A_{\rm LR}(z)$ observable and using heavy quick tagging efficiencies applicable for LEP II, are also given in Table I. Note that these constraints are actually placed on the quantity $|\lambda|^{-1/4}M_s$. We find that the difference in the search reach due to the sign ambiguity in λ is only a few GeV.

Next, we quantify the extent to which these spin-2 exchanges are distinguishable from other new physics sources. As an example, we perform a fit to generated $e^+e^- \to \gamma, Z, G_n \to f\overline{f}$ data assuming that the unpolarized and polarized angular distributions take the forms expected for new vector boson exchange. For both these angular distributions, we include e, μ, τ, b, and c final states, the top quark is excluded as its mass effects would alter the constants. The value of χ^2 per degree of freedom is computed and the resulting confidence level of the fit is presented in Fig. 2 as a function of the string scale. We see that the quality of the fit is quite poor for string scales up to $\sim 5\sqrt{s}$, which is almost up to the discovery limit. This demonstrates that spin-2 graviton exchanges are easily separated from that of new vector bosons. Similar studies can also be performed for comparison with new scalar exchange [7].

We now examine lepton pair production in hadronic collisions. The subprocess contribution of the graviton exchanges to ordinary Drell-Yan production is given by Eq. (4) in the massless limit. However, gravitons can also mediate gluon-gluon contributions to lepton pair production via s-channel exchange. Following an analogous procedure as outlined above for the four-fermion case, the matrix element for $gg \to \ell^+\ell^-$ via graviton exchanges is found to be

$$\begin{aligned}\mathcal{M} = {} & \frac{-4\lambda}{M_s^4}\bar{f}(p')[(p'-p)_\mu\gamma_\nu + (p'-p)_\nu\gamma_\mu]f(p) \\ & \times \{k'_\alpha(k_\mu\eta_{\beta\nu} + k_\nu\eta_{\beta\mu}) + k_\beta(k'_\mu\eta_{\alpha\nu} + k'_\nu\eta_{\alpha\mu}) - \eta_{\alpha\beta}(k'_\mu k_\nu + k_\mu k'_\nu) + \eta_{\mu\nu}(k'\cdot k\eta_{\alpha\beta} - k_\beta k'_\alpha) \\ & \quad - k\cdot k'(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha})\}\epsilon_g^\beta(k')\epsilon_g^\alpha(k)\,. \end{aligned} \tag{6}$$

Because the graviton couplings and the summation over the KK tower of states for $2 \to 2$ processes are universal, λ is the same $\mathcal{O}(1)$ coefficient as in Eq. (3). This yields the $gg \to \ell^+\ell^-$ differential cross section for massless leptons

$$\frac{d\sigma}{dz} = \frac{\lambda^2\hat{s}^3}{64\pi M_s^8}(1-z^2)(1+z^2)\,. \tag{7}$$

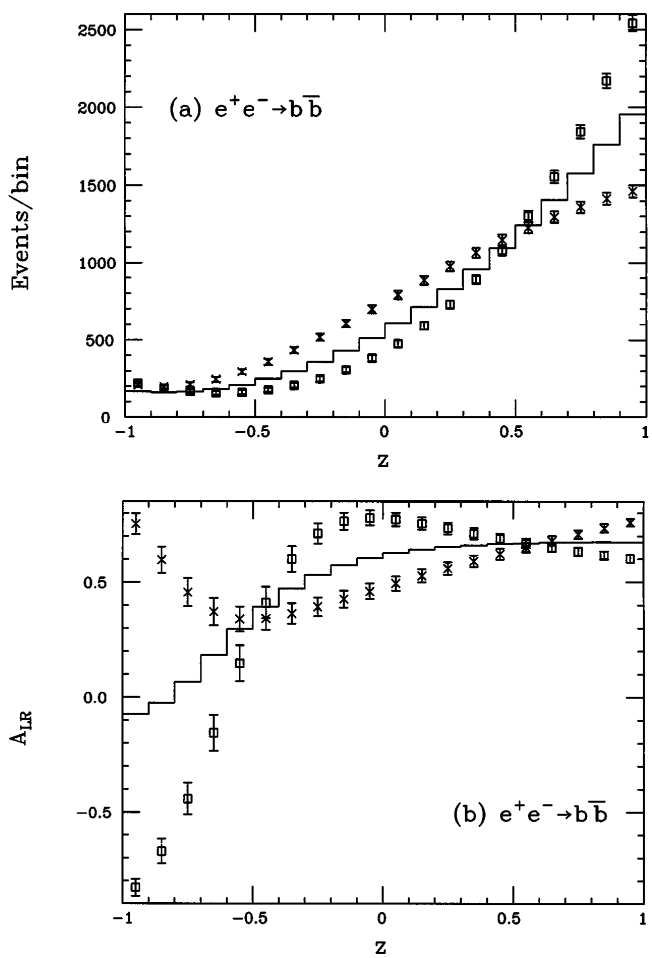

FIG. 1. Bin integrated angular distribution and z-dependent left-right asymmetry for $e^+e^- \to b\bar{b}$. In each case, the solid histogram represents the SM, while the "data" points are for $M_s = 1.5$ TeV with $\lambda = \pm 1$. The error bars correspond to the statistics in each bin.

The large parton luminosity for gluons at higher energy colliders may compensate for the M_s^{-8} dependence. Since this cross section is also even in $\cos\theta$, the gluon-gluon contributions will affect only the total cross section and not the forward-backward asymmetry.

The bin integrated lepton pair invariant mass distribution and forward-backward asymmetry $A_{\rm FB}$ is presented in Fig. 3 for the Large Hadron Collider (LHC) and the Tevatron main injector. The solid histogram represents the SM expectations, and the data points include the graviton exchanges with the error bars representing the statistics in each bin. We have summed over electron and muon final states. For the Tevatron we show the sample case of $M_s = 800$ GeV and the sign ambiguity in λ is visible in $A_{\rm FB}$. For the LHC we display the effects of a $M_s = 2.5$ and 4 TeV string scale on the M_{ll} spectrum (with the smaller string scale having the larger effect). Since graviton exchanges affect only the invariant mass distribution at order λ^2/M_s^8, we would expect only minor modifications to this spectrum. This holds true for the Tevatron, however, large string scales have a sizable effect on the M_{ll} spectrum at the LHC due to the large gluon luminosity at these center-of-mass energies. The deviations in $A_{\rm FB}$ are not as pronounced at the LHC, whereas the two cases $\lambda = \pm 1$ are statistically distinguishable at the Tevatron for this sample case. The resulting 95% C.L. search reaches are given in Table I for both machines. We also find that present

TABLE I. 95% C.L. search for the string scale in TeV for various colliders with center-of-mass energies and integrated luminosities as indicated.

	$\sqrt{s}$ (TeV)	$\mathcal{L}$ (fb^{-1})	$\lambda = \pm 1$
LEP II	0.195	2.5	1.1
Linear collider	0.5	75	3.4
	0.5	500	4.1
	1.0	200	6.6
Tevatron	1.8	0.11	0.99
	2.0	2	1.3
	2.0	30	1.7
LHC	14	10	5.2
	14	100	6.0

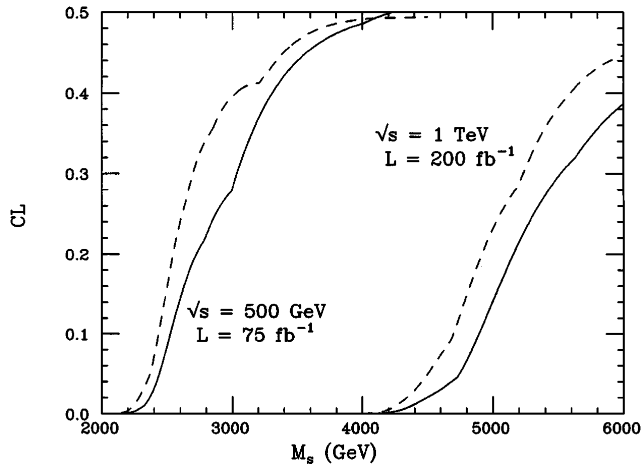

FIG. 2. The percentage of confidence level for the fit procedure described in the text. The assumed center-of-mass energy and luminosity is as labeled, and the dashed and solid curves in each case correspond to the choice $\pm\lambda$.

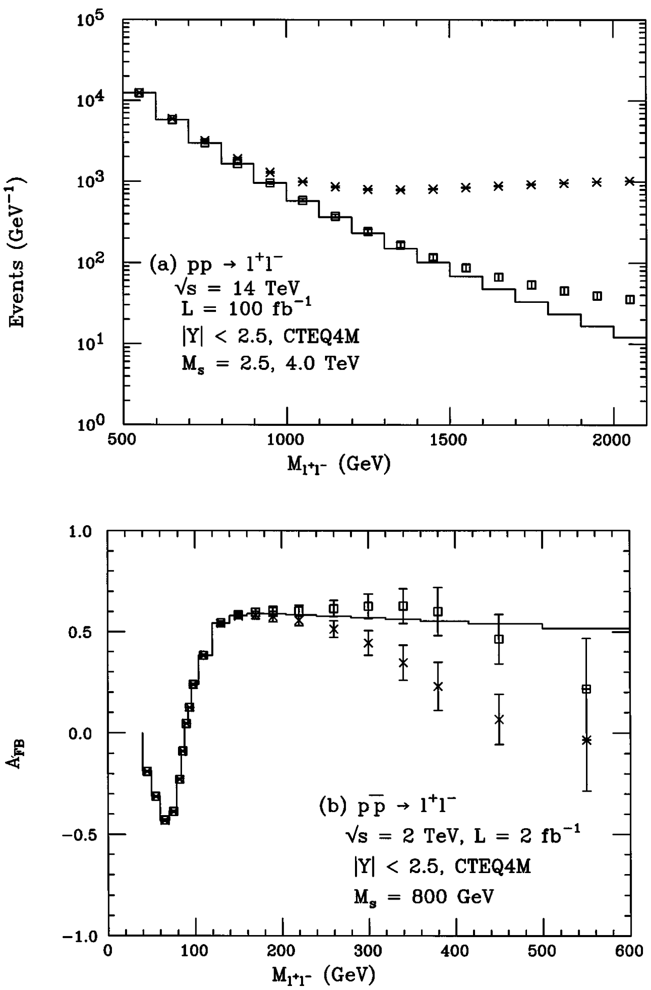

FIG. 3. Bin integrated lepton pair (a) invariant mass distribution at the LHC with $M_s = 2.5$ and 4.0 TeV and $\lambda = +1$ or -1, and (b) forward-backward asymmetry at the Tevatron main injector with $M_s = 800$ GeV and $\lambda = +1$ and -1.

Tevatron data from run I with 110 pb^{-1} of integrated luminosity excludes a string scale up to 980 (920) GeV at 95% C.L. for $\lambda = -1$ $(+1)$.

In conclusion, we have studied the indirect effects at high energy colliders of a TeV string scale resulting from new large extra dimensions. We derived the form of the interactions of the massive KK gravitons with the SM fields, examined their effect in $2 \rightarrow 2$ processes, and found that present colliders can exclude a string scale up to $\sim$1 TeV and that future colliders can extend this reach up to several TeV. The phenomenology of these models is just beginning to be explored and we look forward to the continued investigations of these theories.

This work was supported by the Department of Energy, Contract No. DE-AC03-76SF00515.

Note added.—After this work was completed related material discussing the phenomenological implications of large extra dimensions [8,9] appeared. Two classes of collider tests have emerged for these theories: (i) graviton emission and (ii) graviton exchange, which is the subject of this work. In the case of graviton emission, gravitons are produced in processes, such as $e^+e^- \rightarrow G + \gamma$ and $p\overline{p} \rightarrow G + g$, and radiate into the bulk appearing as missing energy in a detector. In this case the search reach for the string scale is quite sensitive to the number of extra dimensions. The bounds that can be placed on M_s from the two processes listed above are found [8] to be roughly equal to the results of this paper for $n = 2$ and degrade by a factor of (30–60)% for $n = 6$.

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