

Optical Linewidth of a Low Density Fermi-Dirac Gas

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We study propagation of light in a Fermi-Dirac gas. We analytically obtain the leading density correction to the optical linewidth at zero temperature. This correction is a direct consequence of the quantum statistical correlations of atomic positions that modify the optical interactions between the atoms at small interatomic separations. The gas exhibits a dramatic line narrowing already at very low densities. [S0031-9007(99)09375-8]

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The observation of Bose-Einstein condensation in dilute atomic vapors [1] has spurred much interest in ultracold atomic gases. Another evident milestone of atomic physics would be the cooling of a Fermi-Dirac (FD) gas to the quantum degenerate regime. So far all probing of atomic Bose-Einstein condensates has been done optically, and obviously optical detection could also play an important role in the experimental studies of FD gases. Appropriately, theoretical studies of the FD gases are experiencing a renaissance [2–6].

In this paper we investigate propagation of low-intensity light in a FD gas in the limit of low atom density. At zero temperature we derive the leading quantum statistical

correction to the standard column-density refractive index analytically, by legitimately ignoring collective linewidths and line shifts generated in the processes in which a photon is repeatedly scattered between the same atoms. A fermion gas exhibits a striking line narrowing characteristic of the FD statistics, which behaves as $\rho^{2/3}$ at low atom densities.

In the dipole approximation it is advantageous to transform the Hamiltonian into the *length* gauge by the Power-Zienau-Woolley transformation [7]. Then the positive frequency component of the electric field \mathbf{E}^+ may be expressed [8,9] in terms of the positive frequency components of the driving electric displacement, \mathbf{D}_F^+ , and of the source field radiated by atomic polarization, \mathbf{P}^+ , as

$$\epsilon_0 \mathbf{E}^+(\mathbf{r}) = \mathbf{D}_F^+(\mathbf{r}) + \frac{1}{i\kappa} \int d^3r' \mathbf{G}(\mathbf{r} - \mathbf{r}') \mathbf{P}^+(\mathbf{r}'), \quad (1a)$$

$$\mathbf{G}_{ij}(\mathbf{r}) = i\kappa \left\{ \left[\frac{\partial}{\partial r_i} \frac{\partial}{\partial r_j} - \delta_{ij} \nabla^2 \right] \frac{e^{ikr}}{4\pi r} - \delta_{ij} \delta(\mathbf{r}) \right\}. \quad (1b)$$

Here $k = \Omega/c$, Ω is the frequency of the driving field, and the scalar constant $\kappa = \mathcal{D}^2/\hbar\epsilon_0$ is defined in terms of the reduced dipole moment matrix element \mathcal{D} . The three-dimensional monochromatic dipole radiation kernel $\mathbf{G}(\mathbf{r})$ coincides with the corresponding classical expression [10]. In second quantization the polarization reads

$$\mathbf{P}^+(\mathbf{r}) = \mathbf{d}_{ge} \psi_g^\dagger(\mathbf{r}) \psi_e(\mathbf{r}). \quad (2)$$

Here ψ_g and ψ_e are the ground state and the excited state atom-field operators in the Heisenberg picture, and \mathbf{d}_{ge} is the dipole matrix element for the transition $e \rightarrow g$. For

simplicity, we consider here two-level atoms with just a single ground state $|g\rangle$ and one excited state $|e\rangle$, using a constant real vector \mathbf{d} (such that $\mathcal{D} = |\mathbf{d}|$) as the dipole matrix element.

While Eqs. (1) describe the scattered light in a medium, in general, with a small atom-light detuning and for a dense atomic sample, there is no easy way to find the polarization $\mathbf{P}^+(\mathbf{r})$. By making a field theory version of the Born and Markov approximations, we have derived a hierarchy of equations of motion for correlation functions that contain one excited-atom field and one, three, five, etc., ground state atom fields, for the limit of low light intensity [8]. In the present case of two-level atoms the hierarchy reads

$$\begin{aligned} \dot{\mathbf{P}}_l(\mathbf{r}_1, \dots, \mathbf{r}_{l-1}; \mathbf{r}_l) &= (i\delta - \gamma) \mathbf{P}_l(\mathbf{r}_1, \dots, \mathbf{r}_{l-1}; \mathbf{r}_l) + \sum_{k=1}^{l-1} \mathbf{P} \cdot \mathbf{G}'(\mathbf{r}_l - \mathbf{r}_k) \mathbf{P}_l(\mathbf{r}_1, \dots, \mathbf{r}_{k-1}, \mathbf{r}_{k+1}, \dots, \mathbf{r}_l; \mathbf{r}_k) \\ &+ i\kappa \rho_l(\mathbf{r}_1, \dots, \mathbf{r}_l) \mathbf{P} \cdot \mathbf{D}_F^+(\mathbf{r}_l) + \int d^3r_{l+1} \mathbf{P} \cdot \mathbf{G}'(\mathbf{r}_l - \mathbf{r}_{l+1}) \mathbf{P}_{l+1}(\mathbf{r}_1, \dots, \mathbf{r}_l; \mathbf{r}_{l+1}), \end{aligned} \quad (3)$$

where $\gamma = \mathcal{D}^2 k^3 / 6\pi \hbar \epsilon_0$ denotes the spontaneous linewidth, δ is the atom-light detuning. We have defined a projection operator $\mathbf{P} \equiv \mathbf{d}\mathbf{d}/|\mathbf{d}|^2$ whose purpose is to eliminate all but the two atomic states from consideration, and the correlation

functions

$$\mathbf{P}_l(\mathbf{r}_1, \dots, \mathbf{r}_{l-1}; \mathbf{r}_l) \equiv \langle \psi_g^\dagger(\mathbf{r}_1) \cdots \psi_g^\dagger(\mathbf{r}_{l-1}) \mathbf{P}^+(\mathbf{r}_l) \psi_g(\mathbf{r}_{l-1}) \cdots \psi_g(\mathbf{r}_1) \rangle, \quad (4a)$$

$$\rho_l(\mathbf{r}_1, \dots, \mathbf{r}_l) \equiv \langle \psi_g^\dagger(\mathbf{r}_1) \cdots \psi_g^\dagger(\mathbf{r}_l) \psi_g(\mathbf{r}_l) \cdots \psi_g(\mathbf{r}_1) \rangle. \quad (4b)$$

The quantity \mathbf{P}_l reflects correlations between the dipole moment of one atom and the positions of $l - 1$ other atoms, and ρ_l is simply the density correlation function for l ground state atoms.

The terms in the sum on the right-hand side of Eq. (3) represent processes in which the l atoms at $\mathbf{r}_1, \dots, \mathbf{r}_l$ repeatedly exchange photons. Such processes are the microscopic mechanism for collective linewidths and line shifts. The integral stands for a process in which yet another atom shines its light on the atom at \mathbf{r}_l .

Because of the resulting divergent dipole-dipole interactions, all correlation functions \mathbf{P}_l vanish whenever two position arguments are the same [9]. The Lorentz-Lorenz (LL) local-field correction follows mathematically from this observation. Moreover, without changing the outcome of the hierarchy, we may, and will, remove all contact interactions between different atoms in Eq. (3) by introducing the field propagator \mathbf{G}' defined by

$$\mathbf{G}'_{ij}(\mathbf{r}) = \mathbf{G}_{ij}(\mathbf{r}) + i\kappa\delta_{ij}\delta(\mathbf{r})/3. \quad (5)$$

This definition indicates that the integral of \mathbf{G}' over an infinitesimal volume enclosing the origin vanishes.

The coupled theory for light and matter fields [Eqs. (1) and (3)] may be solved, in principle exactly, by means of stochastic simulations [6]. This is because the correlation hierarchy (3) is the same as the hierarchy describing the classical electrodynamics of charged harmonic oscillators with the position correlations ρ_l . By synthesizing a stochastic ensemble of samples of dipoles that have the position correlation functions ρ_l and calculating the ensemble-averaged response to classical light, we have a solution to Eq. (3). Unfortunately, such simulations are demanding on computer time. The computations of Ref. [6] were therefore performed within a one-dimensional (1D) model electrodynamics. While the predictive power of 1D electrodynamics may be questioned, the simulation results for a FD gas at $T = 0$ show clear signatures of the quantum statistics: Even in the limit of zero density, the optical linewidth of the FD gas is only half of the resonance linewidth of an isolated atom [6].

The 1D simulations have also allowed us to test predictions of the density expansion introduced by Morice *et al.* [11] in their studies of the optical response of a quantum degenerate Bose-Einstein gas. At least in one dimension this expansion is in qualitative agreement with numerical simulations [6]. The first two orders of the analytical density expansion of the optical line shape agree with direct numerical computations to better than on the 10% level by slightly underestimating the effects of the quantum statistics. In the low density limit the agreement is excellent. With this in mind, we venture to use the approximation of Morice *et al.* [11] to truncate

the correlation hierarchy (3) also in the present three-dimensional case.

We consider the steady-state solution of (3). The atoms are assumed to fill the half-infinite space $z > 0$ with a constant density ρ . The incoming free field is written $\mathbf{D}_F(\mathbf{r}) = D_F \hat{\mathbf{e}} \exp(ikz)$, and we assume that $\hat{\mathbf{e}} \parallel \mathbf{d}$. The hierarchy of Eq. (3) is truncated by writing [11]

$$\mathbf{P}_3(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}_3) \approx \mathbf{P}_2(\mathbf{r}_2; \mathbf{r}_3) \rho_2(\mathbf{r}_1, \mathbf{r}_2) / \rho. \quad (6)$$

By ignoring the interatomic short-range interactions, which in the case of a low-temperature single-species FD gas may be described in terms of the weak p -wave scattering [5], the pair correlation function for a homogeneous gas has the following general form:

$$\rho_2(\mathbf{r}_1, \mathbf{r}_2) = \rho^2 [1 + \varphi(\mathbf{r}_1 - \mathbf{r}_2)]. \quad (7)$$

Here the function $\varphi(\mathbf{r})$ characterizes two-atom position correlations analogous to bunching or antibunching for photons. We introduce the dimensionless quantities $\bar{\delta} \equiv \delta/\gamma$, $\bar{\rho} \equiv \rho/k^3$,

$$\bar{\alpha} \equiv -\frac{6\pi}{\bar{\delta} + i}, \quad \bar{\mathbf{G}}'(\mathbf{r}) \equiv \frac{\mathbf{P} \cdot \mathbf{G}'(\mathbf{r})}{i\kappa k^3}, \quad (8)$$

where $\bar{\alpha}$ denotes the dimensionless atomic polarizability. In the present three-dimensional case, with the ansatz $\mathbf{P}_1(\mathbf{r}) = P \hat{\mathbf{e}} \exp(ik'z)$ for $\text{Im}(k') > 0$, we obtain the susceptibility of the sample as

$$\chi = \frac{k'^2}{k^2} - 1 = \frac{\bar{\alpha} \bar{\rho}}{1 - \bar{\alpha} \bar{\rho}/3 + C}, \quad (9)$$

with

$$C = -\bar{\rho} \int d^3\bar{\mathbf{r}} \hat{\mathbf{e}}^* \cdot \left[\frac{\bar{\alpha}^3 \bar{\mathbf{G}}'^3 e^{-i\bar{z}} + \bar{\alpha}^2 \bar{\mathbf{G}}'^2}{1 - \bar{\alpha}^2 \bar{\mathbf{G}}'^2} \right] \cdot \hat{\mathbf{e}} \\ - \bar{\rho} \int d^3\bar{\mathbf{r}} \varphi \hat{\mathbf{e}}^* \cdot \left[\frac{\bar{\alpha} \bar{\mathbf{G}}' e^{-i\bar{z}} + \bar{\alpha}^2 \bar{\mathbf{G}}'^2}{1 - \bar{\alpha}^2 \bar{\mathbf{G}}'^2} \right] \cdot \hat{\mathbf{e}}. \quad (10)$$

Here we use the dimensionless integration variable $\bar{\mathbf{r}} = k\mathbf{r}$. The quantity C , and hence also k' , have been forced to be independent of position by essentially ignoring the effects of the surface of the atomic sample [6].

The second term in the denominator in Eq. (9) gives LL shift. In the absence of the C term the electric susceptibility is the standard column-density result augmented with a local-field correction. Atom statistics and atom-field collective effects are encapsulated in the integral C . The expansion of Ref. [11] is such that the parameter C takes into account quantum statistical position correlations between any pair of atoms and the exchange of photons between them to arbitrary order, but ignores all repeated photon exchange involving more than two atoms [12]. Correspondingly, one may expand the functions inside the integrals in Eq. (10) as power series in $\bar{\alpha} \bar{\mathbf{G}}'$, and

interpret the n th order as a process in which a photon is radiated between a pair of atoms n times.

In this paper we consider only the leading order in the modifications of the optical response of an atom due to the presence of the other atoms, and write

$$C \simeq -\bar{\rho}\bar{\alpha} \int d^3\bar{r} e^{-i\bar{z}} \varphi(\bar{\mathbf{r}}) \hat{\mathbf{e}}^* \cdot \bar{\mathbf{G}}'(\bar{\mathbf{r}}) \cdot \hat{\mathbf{e}}. \quad (11)$$

The expression (11) arises from processes in which any “probe” dipole is subject to the external driving field, and in addition to the primary radiation from the other dipoles. Collective linewidths and line shifts, processes that involve the repeated scattering of a photon between the same atoms, are ignored. For uncorrelated locations of the dipoles with $\varphi = 0$, the effects of the primary radiation from the other dipoles on a probe dipole average to zero. However, for the FD statistics Eq. (11) gives a nontrivial result. This reflects the short-range ordering, within the correlation length, of the atoms in the gas.

In the thermodynamic limit $\varphi(\mathbf{r})$ in the pair correlation function [Eq. (7)] is given by

$$\varphi(\mathbf{r}) = -\frac{1}{\rho^2(2\pi)^6} \left| \int d^3k \bar{n}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} \right|^2, \quad (12)$$

where the occupation numbers $\bar{n}_{\mathbf{k}} = (e^{\beta\epsilon_{\mathbf{k}}}/z + 1)^{-1}$ obey the FD statistics with $\beta = 1/k_B T$ and $\epsilon_{\mathbf{k}} = \hbar^2 k^2/2m$. Fermions at $T = 0$ fill the Fermi sphere $\bar{n}_{\mathbf{k}} = \Theta(k_F - |\mathbf{k}|)$, with the Fermi wave number $k_F = (6\pi^2\rho)^{1/3}$. In this case we may evaluate Eq. (12) in closed form. The result is

$$\varphi(\mathbf{r}) = -\frac{9}{k_F^4 r^4} \left[\frac{\sin k_F r}{k_F r} - \cos k_F r \right]. \quad (13)$$

After inserting Eq. (13) into Eq. (11) we obtain a (complicated) analytical expression, whose density expansion reads

$$C = \frac{3i}{10} \left(\frac{\pi}{6} \right)^{1/3} \bar{\alpha} \bar{\rho}^{2/3} + \mathcal{O}(\bar{\rho}). \quad (14)$$

In our 1D electrodynamics, the entire expression (10) may be integrated analytically for a FD gas at $T = 0$ [6]. It is then easy to see that the lowest-order density contribution is correctly introduced by the expansion (11). This is also true in the present three-dimensional case, although the demonstration is more indirect. First, the terms in Eq. (10) that do not depend on $\varphi(\bar{\mathbf{r}})$ are linearly proportional to $\bar{\rho}$. Second, expanding the contributions to Eq. (10) that do depend on φ into a series of $\bar{\alpha}\bar{\mathbf{G}}'$, for orders beyond the one included in (11) we find radial integrals of the form

$$\int d\bar{r} \bar{r}^2 \frac{e^{\xi i\bar{r}}}{\bar{r}^{n-1}} \varphi(\bar{\mathbf{r}}) \propto \mathcal{O}(\bar{\rho}),$$

$$\int d\bar{r} \bar{r}^2 \frac{1}{\bar{r}^n} \varphi(\bar{\mathbf{r}}) \propto \mathcal{O}(\bar{\rho})^{n/3}.$$

Here ξ are integers independent of $\bar{\rho}$, and $n \geq 3$. Most of these integrals formally diverge at the origin, but in

a manner that must eventually cancel to give a finite result (10). Besides, the divergences do not depend on ρ . Omitting the divergences, the integrals scale with $\bar{\rho}$ as indicated. All told, Eq. (11) not only represents the lowest-order correction to the optical properties in terms of the number of microscopic optical interaction processes between the atoms, but it also correctly gives the leading density correction $\propto \bar{\rho}^{2/3}$ to the optical response.

We have plotted the linewidth and the line shift, including both the effect of the FD statistics and LL shift,

$$\Gamma = \gamma[1 - 6\pi \text{Im}(C/\bar{\alpha})],$$

$$\Delta = \gamma[2\pi\bar{\rho} - 6\pi \text{Re}(C/\bar{\alpha})], \quad (15)$$

as a function of density in Fig. 1 using the full form of C from Eq. (11). A zero-temperature result is obtained analytically and the finite temperature result numerically from Eq. (12) for ^{40}K with the 767 nm optical transition [5]. A low-temperature FD gas exhibits a striking linewidth narrowing already at low densities. For $\bar{\rho} = 1.5 \times 10^{-3}$ the optical linewidth of the gas at $T = 0$ is $\Gamma \simeq 0.94\gamma$, and with $\bar{\rho} = 1.5 \times 10^{-2}$ we have $\Gamma \simeq 0.79\gamma$. For the 767 nm transition the corresponding densities would be $\rho \simeq 8.2 \times 10^{11} \text{ cm}^{-3}$ and $\rho \simeq 8.2 \times 10^{12} \text{ cm}^{-3}$. At $\bar{\rho} = 0.1$ ($\rho \simeq 5.5 \times 10^{13} \text{ cm}^{-3}$) and at $T = 0$ the optical linewidth of a FD gas would be approximately half of the linewidth of an isolated atom. However, at the latter density we may already have to consider collective linewidths and line shifts to obtain a reliable quantitative prediction, a task we do not undertake in the present paper. From Fig. 1(b), the line shift at $T = 0$ is negative at low densities and completely vanishes at $\bar{\rho} \simeq 0.03$. At higher densities the line shift turns positive.

The dramatic line narrowing may be attributed to the regular spacing between the atoms characteristic of the FD statistics. As discussed in Ref. [6], the mechanism is particularly transparent in one dimension. An alternative description of the line narrowing may be obtained in the momentum representation. At $T = 0$ the fermions fill the Fermi sphere. Because of the Pauli exclusion principle only either strict forward scattering or scattering events that take the recoiling atom out of the Fermi sea are allowed. The change of the wave vector of an atom upon scattering satisfies $|\Delta\boldsymbol{\kappa}| = 2k \sin(\theta/2)$, where θ is the scattering angle for photons. All atoms are scattered out of the Fermi sea if $|\Delta\boldsymbol{\kappa}| > 2k_F$. Thus, we see that for the photon scattering angles θ satisfying

$$\sin(\theta/2) > k_F/k = (6\pi^2\bar{\rho})^{1/3}, \quad (16)$$

scattering is not inhibited by the FD statistics. On the other hand, for $\sin(\theta/2) < k_F/k$ some recoil events would lead to an already occupied state in the Fermi sea, and are forbidden. The suppression of light scattering is strongest in the near-forward direction corresponding to small values of θ . When the density is increased, at $\bar{\rho} \geq 1/6\pi^2$ we have $k_F \geq k$, and scattering is at least partially suppressed in all nonforward directions.

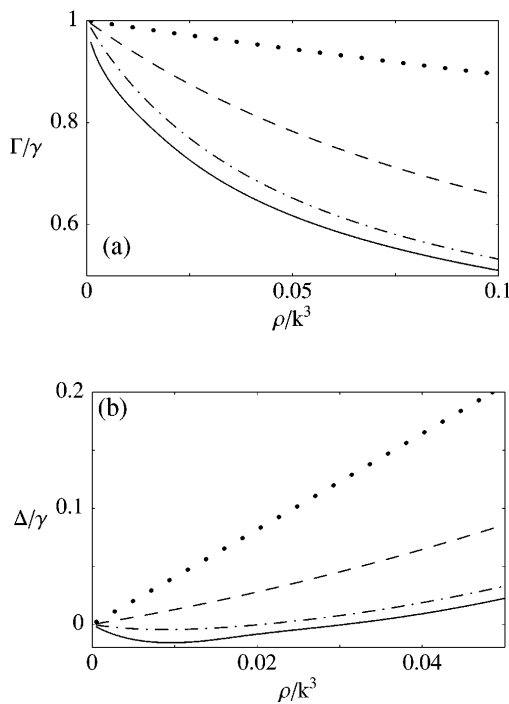


FIG. 1. The optical (a) linewidth and (b) the line shift of a Fermi-Dirac gas as a function of the atom density per cubic optical wave number of the driving light. The solid line represents $T = 0$, the dash-dotted line $T = 150$ nK for ^{40}K , the dashed line $T = 500$ nK, and the dotted line $T = 2$ μ K.

Correspondingly, Fig. 1 shows that $\bar{\rho} = 1/6\pi^2 \approx 0.017$ is a relevant scale for the density.

It is instructive to note the difference between different atom statistics. For the Bose-Einstein condensate the standard factorization of the correlation functions, $\rho_l = \rho^l$, corresponds to an uncorrelated atomic sample, and gives $\varphi = 0$. The leading correction to the standard column-density linewidth therefore results from the *co-operative* optical effects, the collective optical linewidths, and line shifts, and it is proportional to atom density, just as LL shift. FD statistics is different because the correlations have a length scale k_F^{-1} that enters the argument, and the length scale itself depends on density: $k_F^{-1} \propto \bar{\rho}^{-1/3}$. The result is that, at low densities, the effects of the FD statistics dominate over LL shift. A Maxwell-Boltzmann ideal gas has another nontrivial correlation function φ , but the length scale is determined by temperature and does not depend on $\bar{\rho}$. The leading density correction to the optical response is then once more proportional to atom density.

The dipole approximation for atoms neglects possible short-range atom-atom interactions. In the case of cold bosonic atoms the short-range interactions may be described to leading order in terms of the binary *s*-wave scattering. For cold FD atoms in the same internal atomic state the dominant contribution must be significantly weaker *p*-wave scattering due to the Pauli exclusion principle [3,5]. Between FD atoms in different internal levels also *s*-wave interactions are possible. As shown in Ref. [13] the two-body short-range interactions introduce a local

shift in the equation of motion for \mathbf{P}_2 [Eq. (3)]. This can generate an optical line shift that nevertheless for low density gas and for low-intensity light is suppressed in the absence of multiple spin states due to the FD statistics [13]. Apart from this line shift the dominant effect of the short-range interactions is to modify the ground state atom correlation functions. In the present study we ignored these effects which for a FD gas, in the absence of the BCS superfluid state [3,13], are expected to be significantly weaker than for bosonic atoms [5].

We assumed a homogeneous gas in our analysis. Now, a FD gas may be considered locally homogeneous [3] when the length scale over which the density varies is much larger than the spatial correlation length. Given the length scale of a harmonic trap $l = (\hbar/m\omega)^{1/2}$ and the correlation length $1/k_F = (6\pi^2\rho)^{-1/3}$ from Eq. (13), the criterion reads $l^3\rho \gg 1$. A simple dimensional argument shows that this is the same as requiring that the number of trapped atoms be much larger than one. Furthermore, if the size scale of the atomic sample is much larger than the wavelength of light, $l \gg \lambda$, it is reasonable to expect that the refractive index as appropriate for the local density applies in the bulk of the gas.

In conclusion, we studied propagation of light in a FD gas. We discussed quantum statistical corrections to the refractive index, and calculated the leading density correction to the standard column density susceptibility. Already at low densities, fermions exhibit a dramatic narrowing of the resonance line. This might serve as a signature of quantum degeneracy in a cold FD gas.

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