

Fluctuations and Evidence for Charging in the Quantum Hall Effect

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We find that mesoscopic conductance fluctuations in the quantum Hall regime in silicon MOSFETs display simple and striking patterns. The fluctuations fall into distinct groups which move along lines parallel to loci of integer filling factor in the gate voltage–magnetic field plane. Also, a relationship appears between the fluctuations on quantum Hall transitions and those found at low densities in zero magnetic field. These phenomena are most naturally attributed to charging effects. We argue that they are the first unambiguous manifestation of interactions in dc transport in the integer quantum Hall effect. [S0031-9007(99)09312-6]

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The microscopic situation in a two-dimensional (2D) quantum Hall (QH) conductor is strongly influenced by interactions, as evinced by the observation of fractional resistance plateaus when the disorder is weak. In the integer QH regime, for a sufficiently smooth disorder potential, interactions cause the electron liquid to separate into metallic (compressible) and insulating (incompressible) regions [1], as has recently been confirmed by direct imaging techniques [2–6]. Nevertheless, it may still be argued that all linear-response dc transport properties in the integer regime are demonstrated by noninteracting models in which electrons penetrate through a disordered potential landscape [7]. Modifications by interactions appear to be subtle or negligible—for example, the localization length exponent is unaffected. This has helped to justify the continuing study of noninteracting models.

Here we report that the mesoscopic conductance fluctuations in small Si metal-oxide-semiconductor field-effect transistors (MOSFETs) provide strong evidence that interactions, in the form of charging effects, can have a profound effect on conduction in the integer QH regime. Previous experiments [8,9] on these quantum Hall fluctuations (QHF) have focused on their shape and periodicity, for comparison with predictions of the peak shapes [10] and the conductance distribution [11] and correlation [12] functions. Unfortunately, owing to the critical dependence on magnetic field and density it is hard to make detailed statistical measurements near a QH transition. Here we concentrate instead for the first time on the evolution of the fluctuations in the magnetic field-density plane. Our principal result is as follows: the extrema (peaks and dips) in the QHFs fall into groups moving along linear trajectories parallel to lines of constant integer filling factor $\nu = p$, where $p = 0, 1, 2, \dots$. We conclude that a realistic picture of the QH transition must go well beyond noninteracting models, to incorporate not only the existence of insulating and metallic regions, but also the charging conditions for electrons and holes in the metallic regions.

The MOSFETs used have oxide thickness $d_{\text{ox}} = 25$ nm and a range of effective channel dimensions L and W from 0.4 to 4 μm [9]. Each device is approximately a rectangle of disordered two-dimensional electron gas with a metallic contact (n+ diffusion) at each end, as sketched in the left inset in Fig. 1. The electron density ρ is linear in the voltage V_g on the metallic polysilicon gate: $d\rho/dV_g = C/e = 8.6 \times 10^{11} \text{ cm}^{-2} \text{ V}^{-1}$, where $C = \epsilon_{\text{ox}}\epsilon_0/d_{\text{ox}}$ and $\epsilon_{\text{ox}} = 3.9$ [13]. The conductance G was measured using

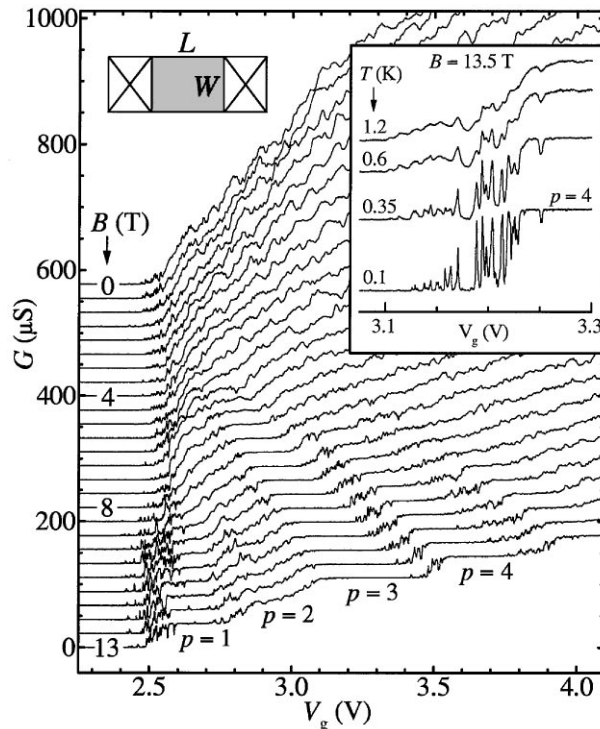


FIG. 1. $G - V_g$ characteristics of device M1 ($L = W = 0.6 \mu\text{m}$) in a perpendicular magnetic field B . Left inset: schematic device. Right inset: T dependence of the $p = 3 \rightarrow 4$ transition.

an ac bias of $10 \mu\text{V}$. The mobility at low temperature is of order $0.2 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$, corresponding to a mean free path $l \sim 20 \text{ nm}$, and the phase coherence length was $L_\phi \sim 0.4 \mu\text{m}$ [9] at base temperature ($T \approx 100 \text{ mK}$) in the dilution refrigerator.

Figure 1(a) shows the $G - V_g$ characteristics of the small square device M1. As B is increased, quantum Hall plateaus develop, which we label by the integers $p = 0, 1, 2, \dots$, where $p = (h/e^2)G$. All the fluctuations visible in these characteristics are reproducible. Similar fluctuations are seen in every device, though their amplitude decreases as the device area increases. The fluctuations can be separated into three categories, as follows. The first, seen at low B and $V_g \lesssim 2.6 \text{ V}$ (where $G \gtrsim e^2/h$), are small and have a short period in V_g . We refer to them as rapid threshold fluctuations (RTFs). Similar features have earlier been attributed to hopping or resonant tunneling [14]. The second, seen at low B and higher V_g , exhibit the properties of universal conductance fluctuations, characteristic of the diffusive regime [15]. The third, seen in the transition regions between the QH plateaus, are the QHFs. As T is decreased, the QH transitions narrow while the QHFs sharpen and grow, as illustrated in the right inset in Fig. 1. At low T the peak-to-dip amplitude near the center of the transition approaches e^2/h , yielding a roughly top-hat-shaped conductance distribution function as reported previously [9].

To study the QHFs as a joint function of B and V_g we make grey-scale plots of G such as those in Fig. 2. A smooth background is subtracted so that the plateaus appear grey, while peaks and dips produce, respectively, darker or lighter regions. Figure 2(a) shows data for device M2 at high B . It can be seen that the extrema follow long, straight trajectories. In an expanded view of the third ($p = 2 \rightarrow 3$) transition [Fig. 2(b)] it can further be seen that the extrema in this region fall into two groups. Those in one group move parallel to the dashed line drawn on the $p = 2$ plateau, while those in the other move parallel to the dotted line on the $p = 3$ plateau. Each group contains both peaks and dips. A similar pattern is repeated on every transition and in every device, irrespective of size and geometry. Figure 2(c) shows data for the third transition in a larger device, M3. This plot has been sheared parallel to the V_g axis to make the center of the transition vertical. In this case it can be seen that the two components of the fluctuations coexist right across the transition region.

Combining results for the fluctuations on the first few transitions in several devices, we find that, to within a few percent accuracy in all cases, the trajectories of the extrema are parametrized by

$$\frac{C}{e} \frac{\partial V_g}{\partial B} = p \frac{e}{h}, \quad (1)$$

where p is an integer. Assuming $d\rho/dV_g = C/e$, this can also be written as $\partial\rho/\partial B = pe/h$. Hence the

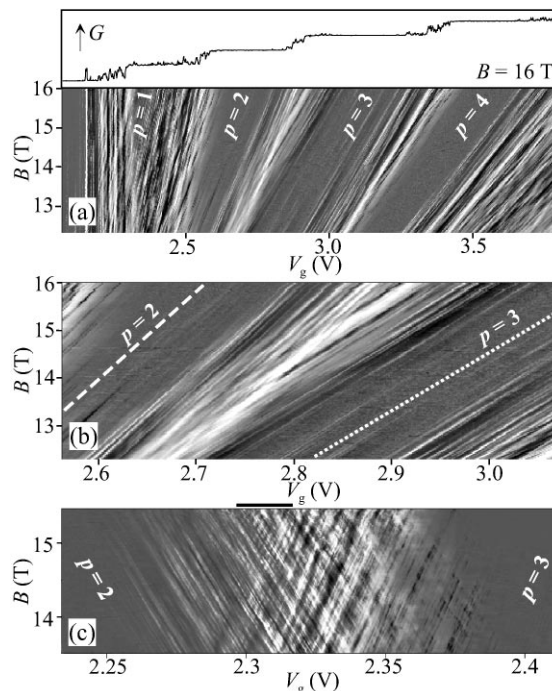


FIG. 2. Grey-scale plots of conductance with a smooth background subtracted (lighter = smaller G). (a) Device M2 ($L = W = 0.6 \mu\text{m}$), showing the first four QH plateaus, as indicated in the trace above. (b) Zoom on the third transition in (a). (c) Third transition in device M3 ($L = W = 3.4 \mu\text{m}$). This plot has been sheared to make the center of the transition vertical.

extrema move parallel to lines of integer filling factor, $\nu = (h/e)\rho/B = p$. Let us now discuss the implications of this, our key result.

We begin by noting that Eq. (1) contradicts the predictions of noninteracting models, in which extrema are associated with alignment of the Fermi level with scattering resonances in particular Landau bands [10]. First, such resonances are expected either to follow *half-integer* filling factors or to show no clear patterns at all [16]. Second, owing to the nonuniform density of states at high B , the lines given by Eq. (1) in the V_g - B plane correspond to distorted trajectories in the E_F - B plane which do not resemble the expected paths of resonances [16]. Third, the distance between resonances in different Landau bands should depend on the energy gaps between bands. In MOSFETs the first four transitions are caused by spin and valley gaps *within the first orbital Landau level* [13]. In contrast, we find that Eq. (1) holds irrespective of spin, valley, and orbital indices.

However, if we assume instead that interactions are strong enough to make the electron density weakly dependent on B , we can make more headway with Eq. (1). Let $\rho(x, y, V_g, B)$ be the spatial density profile in the 2D (x - y) plane of the device, and consider an extremum of index p which passes through (V_{g1}, B_1) in the V_g - B plane. Equation (1), together with $d\rho/dV_g = C/e$, then implies

the following: along the path of the extremum, the contour in the x - y plane defined by $\rho(x, y, V_{g1}, 0) = p(e/h)B_1$ is unchanged. This is illustrated in Figs. 3(a)–3(d). We take a smooth random density profile and plot it at (V_{g1}, B_1) in 3(a) and at $(V_{g1} + \Delta V_g, B_1 + \Delta B)$ in 3(b), where $(C/e)\Delta V_g/\Delta B = e/h$. To help interpret these plots we show cross sections through them in 3(c) and 3(d). Because we have chosen $\Delta V_g/\Delta B$ corresponding to a $p = 1$ trajectory, the contour (black) at $\rho = eB/h$ does not change shape between Figs. 3(a) and 3(b). Meanwhile, the contour (white) at $\rho = 2eB/h$ shrinks.

In other words, each extremum can be linked to the occurrence of a density matching an integer ν at a particular spatial contour in the device. Now, it has been predicted [1] and observed [5,6] that for smooth disorder the electron liquid becomes incompressible and insulating wherever the local density matches an integer filling factor [17]. Accordingly, an extremum with index p may be associated with a particular shape of the $\nu = p$ incompressible strip. This leads naturally to an explanation for Eq. (1) [18,19]. Wherever the $\nu = p$ incompressible strip completely surrounds a metallic region, as occurs several times in Fig. 3, the charge on the metallic puddle should be quantized. The charging condition is determined mainly by the capacitance to the nearby gate and therefore by the puddle's area. Its charge state will thus not change as long as its shape is maintained, which is the case if V_g and B are varied according to Eq. (1).

Let us now consider other properties of the QHFs to see if they support this charging picture. The peak separation ΔV_g is always ≥ 1 mV at base T . The conductance at an extremum, followed along its trajectory, varies in amplitude on a scale of 1 to 2 T [see, e.g., Fig. 2(b)]. Also, the QHFs sometimes exhibit periodicity, such as may

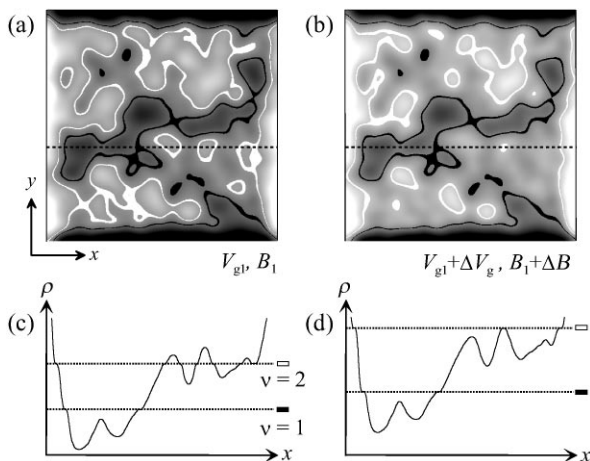


FIG. 3. Density $\rho(x, y, V_g, B)$ at two values of V_g and B . (c) and (d) are cross sections along the dotted lines through contour plots (a) and (b), respectively. Density contours corresponding to integer filling factor are indicated by black ($\nu = 1, \rho = eB/h$) or white ($\nu = 2, \rho = 2eB/h$), while other densities are shown in grey scale.

be discerned in the region indicated by a horizontal bar above Fig. 2(c). In all these properties, as well as in their qualitative T dependence (Fig. 1 inset), they resemble the RTFs. There is however a more direct link between the QHFs and RTFs, which is illustrated in Fig. 4. Figure 4(a) shows the first two transitions in device M4. As on higher transitions, the extrema on the first transition ($p = 0 \rightarrow 1$) fall into two groups. One group is associated with the $p = 1$ plateau. The other is independent of B , i.e., it corresponds to $p = 0$ in Eq. (1). Figure 4(b) shows data for device M5 over a wide range of B . The RTFs at low B can be seen to evolve into the $p = 0$ QHFs at high B , with no qualitative change in their properties. The RTFs may effectively be identified with the $p = 0$ QHFs.

In MOSFETs with dimensions smaller than about $0.2 \mu\text{m}$, the RTFs are often highly periodic [18,20]. This is a signature of Coulomb blockade. The following scenario therefore seems likely: near threshold, conduction is limited by bottlenecks separating puddles of electrons [18]. In devices not much larger than the typical puddle size, one puddle often dominates the conductance, resulting in Coulomb blockade oscillations. Consistent with this, the minimum peak spacing of $\Delta V_g \geq 1$ mV in our devices corresponds to adding one electron to a maximum puddle area of $e/C\Delta V_g \lesssim (0.3 \mu\text{m})^2$, which is one quarter of the area of our smallest device and contains of order a thousand electrons. The insensitivity of the RTFs (the $p = 0$ lines) to B reflects the B -independent charging spectrum of these large puddles. The aperiodicity in larger devices results from charging of many puddles simultaneously.

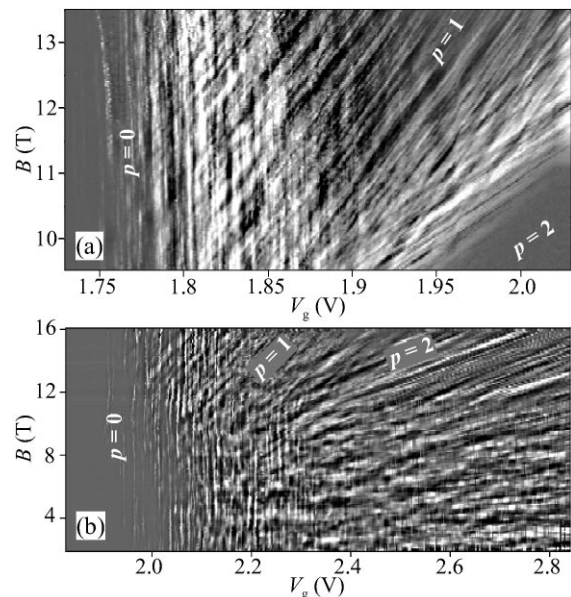


FIG. 4. Grey-scale plots showing fluctuations near threshold. (a) First two QH transitions, device M4 ($L = 1.4 \mu\text{m}$, $W = 3.4 \mu\text{m}$). (b) Wide range of B , device M5 ($L = 0.4 \mu\text{m}$, $W = 0.6 \mu\text{m}$).

The above evidence that the RTFs are strongly influenced by charging, together with the link demonstrated between the QHFs and the RTFs, firmly supports the inference from Eq. (1) that the QHFs are dominated by charging effects. We emphasize however that we do not propose a detailed model for the QHFs. The picture of smooth disorder illustrated in Fig. 3 may not accurately describe MOSFETs, where the disorder is produced by fixed charges near the interface [13,21]. In its defense we note that the effective potential may be softened by electrons bound to the fixed charges [22] and that the results of arguments based on a smooth potential often have a surprisingly general validity. Nevertheless, other difficult questions also arise in this model. Particularly challenging is the coexistence of $p = i$ and $p = i + 1$ extrema on the i th transition. This requires density variations larger than eB/h , to allow different incompressible strips to be present simultaneously, as in Fig. 3. (For lower disorder the situation would presumably be different.) It also requires the simultaneous presence of $p = i$ peaks and $p = i + 1$ dips, which is hard to realize if these features are Coulomb blockade oscillations and if wide plateaus (where one insulating strip percolates) are to be reproduced. Another interesting issue is that the unity amplitude of the QHFs in the smallest devices [9] seems to require phase coherence over the entire system, even as charging takes place.

Finally, we note that our results may have a bearing on the $B = 0$ metal-insulator transition in larger MOSFETs [23] and its relation to the integer QH effect [24]. Further, they support the findings of other recent experiments that single-particle scaling theory may not always be appropriate for describing QH transitions [25].

In summary, our analysis of mesoscopic fluctuations has led us to conclude that interactions, in the form of charging effects, have a profound influence on transport in the integer quantum Hall effect.

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