Rayleigh-Bénard Convection Onset in a Compressible Fluid: ^{3}He near T_C

Andrei B. Kogan, Daniel Murphy,* and Horst Meyer
Duke University, Department of Physics, Durham, North Carolina 27708-0305
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We present results on the onset of convective behavior of ^{3}He along the critical isochore where the fluid compressibility diverges as $T \to T_c$, the critical temperature. The aspect ratio of the Rayleigh-Bénard cell was 57. The "adiabatic temperature gradient" that stabilizes the fluid layer was observed and agreed with predictions. This is the first systematic study of the convection onset in the compressibility-dominated stability regime. The initial slope of the convection current versus the reduced Rayleigh number was found to be independent of compressibility and to be the same as in experiments with a similar aspect ratio in liquid ^{3}He, ^{4}He, and water. [S0031-9007(99)09283-2]

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A common feature of many fluids is a very weak dependence of their density on pressure, which allows one to treat them as incompressible in hydrodynamic theories. In this paper, we are interested in the hydrodynamic effects due to a non-negligible compressibility of a fluid. We are studying a classical problem of Rayleigh-Bénard (RB) convection with particular emphasis on the region very close to the convection onset. For this problem, the assumption of fluid incompressibility, which is equivalent to assuming that the layer is sufficiently thin, leads to the so-called Rayleigh condition for the onset of convection [1]. According to this criterion, the mechanical stability of a horizontal layer of a pure fluid heated from below is determined by the strength of dissipative mechanisms characterized by the viscosity and thermal conductivity coefficients. The increase of the fluid compressibility is expected to bring into prominence an additional source of mechanical stability, described by the "adiabatic temperature gradient" (ATG) criterion (often attributed to Schwarzschild), which does not depend on the dissipative coefficients. This criterion is based on the requirement for the fluid stability that the entropy s of a fluid particle increases with height z, $ds/dz > 0$ [1,2]. As a result, a very compressible fluid heated from below does not start convecting until the temperature difference across the layer, ΔT, becomes much larger than the onset value ΔT_R calculated from the Rayleigh condition.

The mechanical stability of a layer of a compressible fluid was first analyzed by Gitterman and Steinberg (GS) [3,4], as reviewed by Gitterman [5], and very recently by Carlés and Ugurtas (CU) [6]. For typical values of the parameters, the expressions derived in Refs. [5,6] for the onset condition $\Delta T_{\mathrm{onset}}$ reduce to the sum of the two limiting expressions: $\Delta T_{\mathrm{onset}} = \Delta T_R + \Delta T_{ad}$, where ΔT_{ad} is the temperature difference from the ATG effect. The term ΔT_R is given by [1]

$$\Delta T_R = \mathrm{Ra}_c \times \frac{\nu D_T}{\alpha_P g h^3} . \qquad (1)$$

where Ra_c is the critical Rayleigh number, while the ATG effect results in

$$\Delta T_{ad} = \frac{hgT\alpha_P}{C_P} = \rho g h \left(1 - \frac{C_V}{C_P}\right)\left(\frac{\partial T}{\partial P}\right)_\rho . \qquad (2)$$

In the expressions above, ν, D_T, α_P, and C_P are, respectively, the kinematic viscosity, thermal diffusivity, isobaric expansion coefficient, and heat capacity at constant pressure of the fluid. Furthermore, g is the gravity, h is the height of the fluid layer, and $\mathrm{Ra}_c \sim 1708$ is the critical Rayleigh number for a cell with an aspect ratio $\Gamma = \infty$, which is quite well approximated by our cell. Note that as $T \to T_c$, $C_V/C_P \to 0$, and the ATG criterion then reduces to the condition $d\rho/dz < 0$. This means that the density gradient due to the temperature inhomogeneity associated with the heat flow needs to be smaller than the density gradient that is due to the compression of the fluid under its own weight (gravity-induced stratification).

Unless h is large, the role of the adiabatic gradient effect in liquids away from the critical point is negligible: for example, in liquid ^{3}He at $T = 2.6$ K, which is well below T_C, and for a layer of 1 mm thickness, we calculate from its known properties $\Delta T_R = 750\ \mu$K, while $\Delta T_{ad} = 1.3\ \mu$K. We mention that the ATG effect is important in geophysical systems with large vertical dimensions, such as the earth's atmosphere, as discussed by Tritton [2], but is usually irrelevant in laboratory-scale experiments.

With SF_6 near the critical point and in a RB cell of $h =$ 10 cm, Ashkenazy and Steinberg [7] recently observed the ATG from the onset and disappearance of vertical velocity, and found agreement with predictions. Also recently, results of an experimental study of convection in ^{4}He over the temperature range between 3.8 and 5.9 K and a wide density range ($0.01 < \rho/\rho_c < 2$) were published [8]. The shift in the convection onset at the lowest densities, as caused by the ATG, was clearly observed and consistent with predictions [9]. This research, carried out with a cell of 20 cm height and $\Gamma = 1/2$, focused on the dynamics of highly developed turbulent states at Rayleigh numbers up to 10^{15}. In that work the plot of the Nusselt number Nu versus the Rayleigh number Ra could be scaled for all the densities, provided that a correction was made for this ATG term and which becomes increasingly important for a given density as Ra decreases. This correction and the scaling are described in detail in the thesis by Chavanne [10].

Our experiments are performed in He^3 close to the critical point ($T_c = 3.317$ K on the T_{90} scale, density $\rho_c = 41.45$ kg/m^3). The unique feature of near-critical fluids is the strong dependence of their isothermal compressibility on the distance from the critical point, as defined by the reduced temperature $\epsilon = (T - T_c)/T_c$. This makes them particularly convenient for our study because the behavior typical both for a very compressible and a weakly compressible fluid can be observed in the same system. In our present work, we are interested in the opposite end of the heat transport from that of Chavanne *et al.* [8], namely close to the onset of convection. The heat dependence of $\Delta T(q)$ as a function of the heat flux q was determined at 22 different values of ϵ, the temperature of the top plate. The heat transport was studied by measuring the temperature difference ΔT across a horizontal fluid layer as a function of heat flux q where q has been corrected for the heat flow through the side walls of the cell. The temperature range was $1 \times 10^{-3} \leq \epsilon \leq 2 \times 10^{-1}$, which corresponds to Prandtl numbers $350 \geq \mathrm{Pr} \geq 2$ and to compressibilities $7 \times 10^{-4} \geq \beta_T \geq 1 \times 10^{-6}$ dyne/cm^2, still much larger than $\beta_T \simeq 2.4 \times 10^{-7}$ dyne/cm^2 for the ideal gas at $T = 3.5$ K and $\rho = \rho_c$. The techniques for density and temperature calibration and measurements used in this work are analogous to those used in the thermal conductivity experiments by Pittman *et al.* [11]. The experimental arrangement, sample preparation, and measurement routine were described in a preliminary note [12] and will be presented in more detail elsewhere.

Matched germanium thermometers, with a resolution of 0.3 μK were used to both control the temperature of the upper plate and measure ΔT. The density of the fluid was determined via its dielectric constant in a separate cell, thermally anchored to the temperature-controlled platform, to which the top of the RB cell was attached. The flat, polished top and bottom endplates of the RB cell with a diameter $D = 57$ mm are made out of OFHC copper and are separated by a thin stainless steel sidewall which is a poor heat conductor. The copper surfaces facing the fluid were made parallel to 0.025 mm or better as measured across the fluid.

The cell height was chosen to make ΔT_{ad} large enough to be observable without introducing excessive fluid relaxation times which diverge as $T \to T_c$. As a compromise, we chose h to be 1.06 mm, for which we calculate $\Delta T_{ad} \approx 3.6\,\mu$K and relaxation times of the order of 10^3 s and 10 s at $\epsilon = 10^{-3}$ and 10^{-1}, respectively. The cell was filled with ^{3}He to a density within 0.5% of ρ_c and then sealed off from the sample handling system. During the experiments, the temperature of the top plate was controlled at a fixed value with a short-term stability of ± 0.8 μK for most experiments and with slow drifts typically within 3–5 μK over tens of hours. In a measurement, the power is applied to the bottom plate and a change in the temperature difference across the fluid was measured as a function of time and its steady-state value $\Delta T(q)$ is recorded. To improve the resolution, we averaged the results of 10–20 measurements for each heat to obtain the average $\Delta T(q)$ with an uncertainty of ± 0.3 μK.

We now present the results. In a nonconvecting layer, we expect the measured $\Delta T(q)$ to represent diffusive heat transport and to be given by $\Delta T_{\mathrm{diff}}(q) = qh/\lambda$ where λ is the thermal conductivity of the fluid from Ref. [11] and from present data. When the fluid is driven into convection, the slope $d\Delta T/dq$ suddenly decreases with increasing q.

Figure 1 shows representative curves of ΔT versus q that evidence both the diffusive and convective states, and where the transition is marked only on the top curve. As T_c is approached, the transition becomes less sharp, and the rounding prevents its unambiguous determination for $\epsilon < 5 \times 10^{-3}$. This suggests that the mechanical stability of the fluid might possibly be getting more sensitive to the temperature control quality as the fluid becomes more compressible. It then appears reasonable to determine $\Delta T_{\mathrm{onset}}$ as a crossing point of a linear extrapolation of the $\Delta T(q)$ curve from above the onset and the $\Delta T_{\mathrm{diff}}(q)$ line, which is based on the conductivity data of Ref. [11]. We performed this analysis and compared the results shown in Fig. 2 with theory. Here the solid line is obtained from the expressions for the $\Delta T_{\mathrm{onset}}$ derived in Refs. [3,4,6], where static and transport properties of ^{3}He obtained in this laboratory over the experimental range were used. For comparison, we also show curves representing the individual $\Delta T_R(\epsilon)$ and $\Delta T_{ad}(\epsilon)$ from Eqs. (1) and (2). The agreement is very

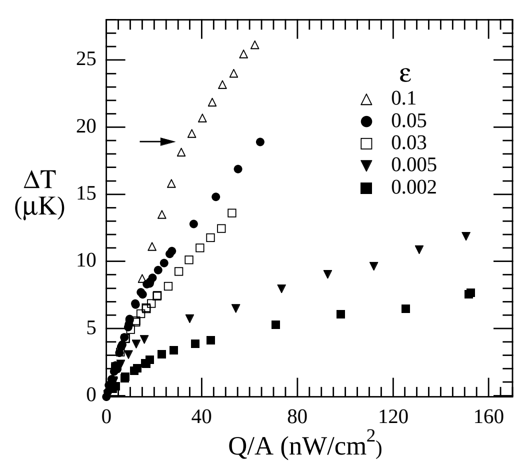

FIG. 1. Temperature difference across the fluid as a function of heat flux at several representative values of ϵ. The transition from the mechanically stable to the convective regime, shown by a sudden change in slope, is marked by an arrow for the top curve alone.

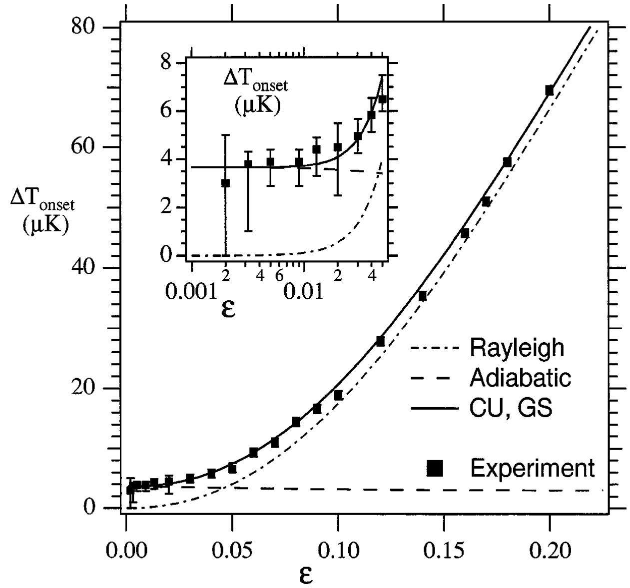

FIG. 2. Comparison of the experimentally determined $\Delta T_{\mathrm{onset}}$ versus ϵ (symbols) with theory (lines). Main figure: Linear plot. Inset: Semilogarithmic plot in the region where the ATG is dominant. The "error bars" represent the approximate width of the transition. For explanations, see text.

good, and therefore constitutes the first systematic study of the convection onset into the regime where the ATG dominates. Furthermore, the agreement implies that if the "potential temperature difference" θ defined below is used in the Rayleigh number calculations, the critical Rayleigh number is close to the predicted one, $\sim$1708, and is independent of compressibility.

Our second purpose was to study the initial convective state and determine the convection current j^{conv}, which is the ratio of the convective portion of the heat current to that conducted through the fluid at the transition. This definition leads to the relation [13]

$$j^{\mathrm{conv}} \equiv (\mathrm{Nu} - 1)(\mathrm{ra}^* + 1) \tag{3}$$

versus the reduced Rayleigh number

$$\mathrm{ra}^* \equiv (\mathrm{Ra} - \mathrm{Ra}_c^{\mathrm{obs}})/\mathrm{Ra}_c^{\mathrm{obs}}, \tag{4}$$

where $\mathrm{Nu} \equiv \Delta T_{\mathrm{diff}}/\Delta T_{\mathrm{obs}}$ with ΔT_{obs} the observed temperature change across the fluid layer. Furthermore, $\mathrm{Ra}_c^{\mathrm{obs}}$ is the observed Rayleigh number at the convection onset. For $\mathrm{ra}^* < 0$, $\Delta T_{\mathrm{obs}} = \Delta T_{\mathrm{diff}}$. It was of particular interest to check whether the initial slope $dj^{\mathrm{conv}}/d(\mathrm{ra}^*)$ of the data in the convective state is independent of compressibility, and how it compares with results of other fluids in cells with a similar aspect ratio.

In Fig. 3(a), a plot of j^{conv} versus ra^* is presented for various reduced temperatures, where again ϵ is the temperature of the top plate. For $\epsilon < 1 \times 10^{-2}$ the rounding of the transition to the convective state did not allow us to determine the slope near $\mathrm{ra}^* = 0$. In his discussion of the ATG, Tritton [2] has suggested

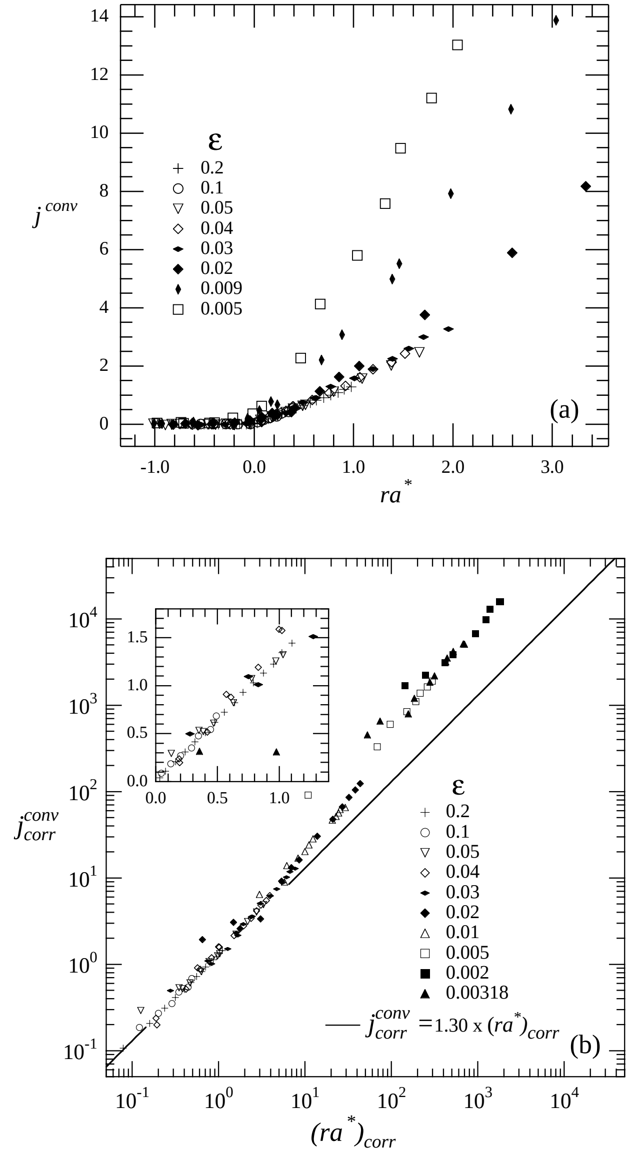

FIG. 3. The convection current j^{conv} versus the reduced Rayleigh number ra^*. (a) Obtained directly from the $\Delta T(q)$ curves, such as shown Fig. 1. (b) Corrected to take into account the presence of the ATG, as explained in the text. The inset enlarges a portion of this plot at low values of $\mathrm{ra}^*_{\mathrm{corr}}$.

introducing a "potential temperature difference" θ which we write as $\theta = \Delta T - \Delta T_{ad}$, and Chavanne [10] has used this concept to propose relations that correct both Nu and Ra for the effect from the ATG. With our variables, these relations take the following simple form:

$$j^{\mathrm{conv}}_{\mathrm{corr}} = [1 + C(\epsilon)]j^{\mathrm{conv}} \quad \text{and}$$
$$\mathrm{ra}^*_{\mathrm{corr}} = [1 + C(\epsilon)]\mathrm{ra}^*. \tag{5}$$

The factor $C(\epsilon) = (\mathrm{Ra}/1708)\,(\Delta T_{ad}/\Delta T)$ expresses the relative strength of adiabatic effects compared to dissipative forces and diverges as $T \rightarrow T_c$. The result is that both j^{conv} and ra^* are rapidly "stretched out" by the corrective transformation as T_c is approached.

The transformed data are shown in Fig. 3(b) with an insert for $0 < \mathrm{ra}^* < 1.2$ that emphasizes the initial slope, S, in the convecting state. This slope remains unchanged for $0.03 \leq \epsilon \leq 0.2$ with the average value 1.3 ± 0.1, as an examination of individual plots for a given value of ϵ shows. This is to be compared with $S = 1.23$, 1.25, and 1.20 for liquid ^{3}He between 2.1 and 2.6 K [14], liquid ^{4}He at 2.4 K [15], and distilled water [16] in RB cells with similar aspect ratios. It is known that the initial slope of $j^{\mathrm{conv}}(\mathrm{ra}^*)$ is strongly dependent on the pattern formed at the convection onset. The experiment with water was the only one provided with a shadowgraph optical arrangement that clearly showed the pattern to be one of concentric rolls (see Fig. 2 of Ref. [16]). For such a pattern with velocity amplitudes at the center of zero, respectively maximum, the predicted slopes are 0.86 and 0.45 [13,17]. Although none of the experiments agree quantitatively with either prediction, it is noteworthy that they are all consistent with one another, regardless of the compressibility that varies over a factor of $\sim 10^5$ between water and ^{3}He near T_c. (The effect from the different Prandtl numbers in these fluids on the predicted slope [13,17] is estimated to be negligible.) This result is intriguing, but is not sufficient by itself for a conjecture that the convection pattern near the onset is all the same.

Within the experimental uncertainty, we find that all the plots of $j^{\mathrm{conv}}_{\mathrm{corr}}(\mathrm{ra}^*_{\mathrm{corr}})$ for various values of ϵ collapse on a single line. Further results, not shown here, extend the range to $\mathrm{ra}^*_{\mathrm{corr}}$ higher than 10^5. The complete plot and further results will be presented in a more detailed paper.

In conclusion, we have studied in a Rayleigh-Bénard configuration the transition to the convective state, when the fluid compressibility diverges, and which is determined by the classically predicted adiabatic temperature gradient. Furthermore, the heat transfer experiments in the convective state have shown that the initial slope of the convective heat current is independent of the compressibility, and is the same as measured for other fluids. The data are found to scale on a $j^{\mathrm{conv}}_{\mathrm{corr}}(\mathrm{ra}^*_{\mathrm{corr}})$ plot.

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*Present address: The Quantum Institute, University of California, Santa Barbara, California 93106.

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