Quantum Projection Noise in an Atomic Fountain: A High Stability Cesium Frequency Standard

G. Santarelli, Ph. Laurent, P. Lemonde, and A. Clairon

BNM-LPTF, Observatoire de Paris, 61 Avenue de l'Observatoire, 75014 Paris, France

A. G. Mann, S. Chang, and A. N. Luiten

Physics Department, University of Western Australia, Nedlands, 6907 Western Australia, Australia

C. Salomon

Laboratoire Kastler Brossel, ENS, 24 rue Lhomond, 75005 Paris, France

We describe the operation of a laser cooled cesium fountain clock in the quantum limited regime. An ultrastable cryogenic sapphire oscillator is used to measure the short-term frequency stability of the fountain as a function of the number of detected atoms N_{at}. For N_{at} varying from 4×10^4 to 6×10^5 the Allan standard deviation of the frequency fluctuations is in excellent agreement with the $N_{at}^{-1/2}$ law of atomic projection noise. With 6×10^5 atoms, the relative frequency stability is $4 \times 10^{-14} \tau^{-1/2}$, where τ is the integration time in seconds. This is the best short-term stability ever reported for primary frequency standards, a factor of 5 improvement over previous results. [S0031-9007(99)09299-6]

Among the new generation of high accuracy laser cooled atomic frequency standards [1–3], neutral atom fountains have been in existence for ten years [4–6]. During this time, the relative frequency stability of fountain standards has improved by 3 orders of magnitude: from $2 \times 10^{-10} \tau^{-1/2}$ [4] down to $2 \times 10^{-13} \tau^{-1/2}$ [1], where τ is the averaging time in seconds. Yet, all these experiments were limited by technical noise rather than by the fundamental quantum noise inherent to the measurement process [7,8]. It was recognized in 1991 that a cesium fountain in the quantum limited regime could have a frequency stability of $2 \times 10^{-14} \tau^{-1/2}$ [5].

A pioneering study of the influence of the quantum measurement noise in frequency standards using two level atoms has been reported by Itano *et al.* [8]. If an atomic system is prepared in a linear superposition $|\psi\rangle = \alpha |g\rangle + \beta |e\rangle$ of the two states $|g\rangle$ and $|e\rangle$ and subject to a measurement indicating whether the system is in $|g\rangle$ or $|e\rangle$, quantum mechanics predicts that the probability of finding the system in $|g\rangle$ is $|\alpha|^2$ with $|\alpha|^2 + |\beta|^2 = 1$. Except when α or $\beta = 0$, the outcome of the measurement cannot be predicted with certainty. Itano *et al.* [8] names this effect *quantum projection noise* and have observed it both in the case of repeated measurements on a single particle prepared successively under identical conditions and in the case of an ensemble average with a number N_{at} of identical trapped particles, up to $N_{at} = 380$. In a frequency standard such as an atomic fountain, the population of $|g\rangle$ (and/or $|e\rangle$) is measured as a function of the frequency of an external interrogation field, and this information is used to lock the standards output frequency to the atomic transition frequency. As shown in [8] for Ramsey's method of separated fields, if the technical noise is much smaller that the quantum projection noise and the field amplitude is optimal, then the frequency fluctuations of the standard are independent of the choice of $|\alpha|^2, |\beta|^2$. For a system containing N_{at} uncorrelated atoms, the standard deviation of frequency fluctuations scales as $N_{at}^{-1/2}$.

In this Letter, we demonstrate the operation of a cold cesium fountain frequency standard at the quantum projection noise level using up to 6×10^5 detected atoms in each fountain cycle. By varying the number of atoms in the fountain, we checked the $N_{at}^{-1/2}$ law at the level of 6% for the exponent. For $N_{at} = 6 \times 10^5$, the measured short-term frequency stability is $4 \times 10^{-14} \tau^{-1/2}$, where τ is the integration time in seconds. Because of the large atom number in a fountain, reaching the quantum projection noise regime corresponds to a record frequency stability: a factor of 5 improvement over our previous fountain results [1] and a factor of 8 over the recent laser cooled Hg^+ ion frequency standard [3]. To our knowledge, this is the first time that the best frequency stability of an atomic clock is obtained in the quantum limited regime. By comparing the cesium fountain to a hydrogen maser, a fractional frequency stability of 6×10^{-16} has been measured for an integration time of 2×10^4 s, most likely limited by the maser frequency stability. Our work makes frequency measurements at the unexplored level of 1×10^{-16} realistic, since only about one day of averaging time is needed. This is a prerequisite for the evaluation of the systematic frequency shifts of an atomic clock at this level. Finally, we propose a new method to perform an absolute measurement of the number of atoms in a cold sample by means of the quantum projection noise.

The first crucial element in our experiments is the use of an ultralow frequency noise cryogenic sapphire oscillator (SCO) [9] as the interrogation oscillator in the fountain. Until now, only quartz oscillators have been used to generate the $\nu_0 = 9.192$ GHz interrogation field in Cs fountains. Their phase noise severely degraded the frequency stability of the clock with an additional noise independent of N_{at} [1,10,11]. The SCO possesses a mode which is only 1.3 MHz away from ν_0. Oscillating on this mode, the frequency stability is below 10^{-14} from 0.1 to 10 s [12]. The excess noise is then negligible compared to the projection noise in our atomic fountain.

The second key feature of this experiment is an efficient method to measure the populations of the two states of the cesium clock transition. The laser cooled atomic fountain operates in a sequential mode [1]. First, 10^7–10^8 Cs atoms are loaded into a magneto-optical trap. After the magnetic field has been switched off, the atoms are launched upward at ~4 m/s and cooled to 1.6 μK. With laser and microwave pulses, we select only atoms in the $|F = 3, m_F = 0\rangle$ quantum state. Atoms in $m_F \neq 0$ are pushed away by the radiation pressure of a laser beam pulse. A Ramsey interrogation scheme is used. On their way up, the atoms interact with the 9.192 GHz electromagnetic field in a microwave cavity providing a first $\pi/2$ pulse. After a ballistic flight of duration ~500 ms above the cavity, the atoms pass through the microwave cavity for a second time, experience a second $\pi/2$ pulse, and fall back down to the detection region. The width $\Delta\nu$ of the Ramsey resonance is thus 1 Hz. This width has been varied by $\pm20\%$ by changing the launch velocity. The population in each of the $|F = 3, m_F = 0\rangle$ and $|F = 4, m_F = 0\rangle$ hyperfine levels is measured by light induced fluorescence as follows: First the atoms cross a probe beam of height 8 mm which is retroreflected on a mirror and tuned half a natural width below the $6S_{1/2}$ $F = 4$, $6P_{3/2}$ $F' = 5$ cycling transition. This beam is σ^+ polarized and has an intensity of 1 mW/cm^2. By detecting the 5-ms-long pulse of fluorescence light on a low noise photodiode, the population of $|F = 4, m_F = 0\rangle$, proportional to the time integrated fluorescence pulse, is measured. With a collection efficiency of about 0.8%, $n_{ph} = 150$ photons per atom are detected. These atoms are then pushed away by the radiation pressure of a traveling wave. This is accomplished by blocking the lowest 2 mm of the retroreflected probe beam. Four milliseconds after the first fluorescence pulse, atoms in the $|F = 3, m_F = 0\rangle$ state cross two superimposed laser beams. The first one is resonant with the $6S_{1/2}$ $F = 3$, $6P_{3/2}$ $F' = 4$ transition. It quickly pumps the atoms into the $F = 4$ state. The second beam has the same parameters as the upper probe beam. The fluorescence pulse is detected on a second photodiode. Thus in each fountain cycle the total number of atoms, N_{at}, is determined by adding together the number of atoms detected in the two hyperfine levels.

The absolute uncertainty on N_{at} is about 50%. It depends on the probe laser parameters and on the geometry of the detection area. The signal used for the frequency stabilization is the transition probability p which is the ratio of the population of the $|F = 4, m_F = 0\rangle$ state divided by the sum of the $|F = 3, m_F = 0\rangle$ and $|F = 4, m_F = 0\rangle$ populations. In nominal conditions, $p = 1/2$. With this detection method, p is largely independent of shot-to-shot fluctuations in atom number. Similarly, fluctuations of the detection laser intensity and frequency are well rejected because we use the same narrow linewidth laser (~100 kHz) for both detection channels.

Figure 1 shows the central Ramsey resonance in our apparatus. To lock the output signal of the fountain to the hyperfine transition frequency, the microwave interrogation frequency is alternated between $\nu_0 - \Delta\nu/2$ and $\nu_0 + \Delta\nu/2$ on each launch sequence so that $p \sim 1/2$. The difference between two successive measurements is integrated to provide a correction to the output signal frequency. The time constant of this servo is about three fountain cycles. The Allan standard deviation [13] of the relative frequency fluctuations $y(t)$ of an atomic fountain can be expressed as

$$\sigma_y(\tau) = \frac{1}{\pi Q_{at}} \sqrt{\frac{T_c}{\tau} \left(\frac{1}{N_{at}} + \frac{1}{N_{at} n_{ph}} + \frac{2\sigma_{\delta N}^2}{N_{at}^2} + \gamma \right)^{1/2}}.$$

(1)

In (1), τ is the measurement time in seconds, T_c is the fountain cycle duration (~1 s) and $\tau > T_c$. $Q_{at} = \nu_0/\Delta\nu$ is the atomic quality factor. The first term in the brackets is the atomic projection noise $\propto N_{at}^{-1/2}$ [8]. The second term is due to the photon shot-noise of the detection fluorescence pulses. It scales as $N_{at}^{-1/2}$ as the projection noise, and in our fountain this term is less than 1% of the projection noise. The third term represents
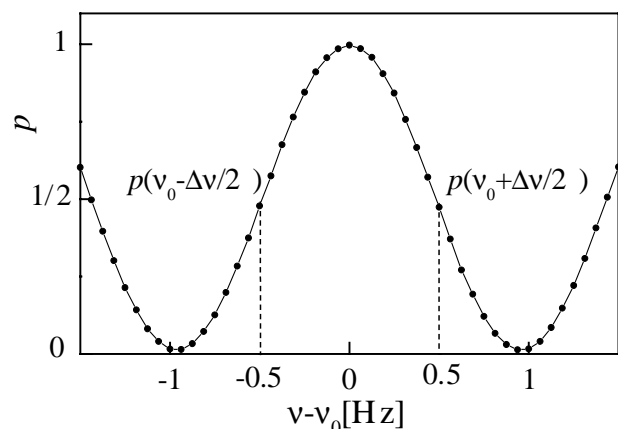

FIG. 1. Center of the Ramsey resonance in the fountain and principle of frequency locking. For a duration of 0.5 s between the two microwave interactions, the fringes have a width $\Delta\nu = 1$ Hz. The error signal is obtained from successive measurements of the transition probability p at $\nu_0 - \Delta\nu/2$ and $\nu_0 + \Delta\nu/2$.

the effect of the noise of the detection system. $\sigma_{\delta N}$, the uncorrelated rms fluctuations of the atom number per detection channel, is about 85 atoms per fountain cycle. This noise contribution becomes less than the projection noise when $N_{at} > 2 \times 10^4$. γ is the contribution of the frequency noise of the interrogation oscillator [10,11]. With the SCO, this contribution is at most $10^{-14}\tau^{-1/2}$ and can be neglected. As an example, for $N_{at} \sim 6 \times 10^5$ detected atoms, $\nu = 0.8$ Hz, $Q_{at} = 1.2 \times 10^{10}$, and $T_c = 1.1$ s, the expected frequency stability is $\sigma_y(\tau) = 4 \times 10^{-14}\tau^{-1/2}$.

To observe the quantum projection noise, we vary the number of atoms in the fountain and measure the frequency stability $\sigma_y(\tau)$ by comparison to the free running sapphire oscillator which is used as a very stable reference up to 10–20 s. In Fig. 2 is plotted a typical measurement of the Allan standard deviation of the frequency corrections fed to a synthesizer (frequency resolution 5×10^{-5} Hz), which is controlled to bridge the gap between the Cs frequency and the SCO. The frequency stability for times longer than the response time of the servo loop, $\tau > 3$ s, and shorter than ~10 s, represents the fountain short-term frequency stability, here $6 \times 10^{-14}\tau^{-1/2}$. For $\tau > 10$ s, the frequency stability departs from the $\tau^{-1/2}$ line because of the drift of the SCO. A plot of the normalized Allan standard deviation as a function of atom number is presented in Fig. 3. Since we explored several values for Q_{at} and for the cycle duration T_c, we plot the quantity $\sigma_y(\tau)\pi Q_{at}\sqrt{\tau/T_c}$ for $\tau = 4$ s. This quantity should simply be equal to $N_{at}^{-1/2}$ when the detection noise is negligible. At low atom numbers, the $1/N_{at}$ slope indicates that the stability is limited by the noise of the detection system [third term in Eq. (1)]. Around 4×10^4 atoms the slope changes. In order to test the atomic projection noise prediction, we perform a least squares fit of the experimental points
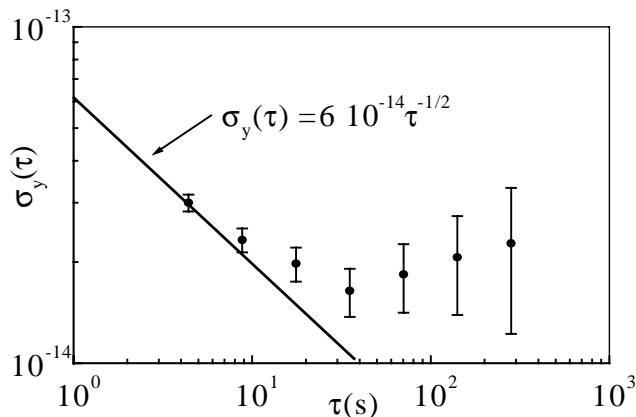
for $N_{at} > 4 \times 10^4$ with the exponent b of N_{at} as a free parameter. We subtract the small contribution of the detection noise from the experimental points and the fit gives $b = 0.47(0.03)$. Thus, over more than 1 order of magnitude in atom number, this result is in very good agreement with the $N_{at}^{-1/2}$ law.

Since the atomic projection noise is the dominant contribution, we can in turn use these frequency stability measurements and Eq. (1) to evaluate the number of detected atoms. If we set $b = -1/2$ and fit to $y = aN_{at}^{-1/2}$ then we get $a = 0.91(0.1)$ close to the expected value of 1 (Fig. 3). The 10% uncertainty on a indicates that this method for measuring the atom number has an accuracy of 20%, 2.5 times better than that from the time-of-flight signal. This accuracy could be improved to better than 5% by a careful evaluation of all terms in Eq. (1) [14].

At the largest number of detected atoms, $N_{at} = 6 \times 10^5$, the stability is $4 \times 10^{-14}\tau^{-1/2}$ for $Q_{at} = 1.2 \times 10^{10}$, $T_c = 1.1$ s. This is an improvement by a factor of 5 for primary atomic frequency standards [1]. It is comparable to the best short-term stability achieved with microwave ion clocks using uncooled ^{199}Hg$^+$ and ^{171}Yb$^+$ samples [15,16]. In a second experiment, we have locked the SCO to the fountain signal for $N_{at} = 5 \times 10^5$ and compared it to a hydrogen maser. Figure 4 shows the Allan standard deviation of this frequency comparison. The stability is $7 \times 10^{-14}\tau^{-1/2}$ and reaches 6×10^{-16} at $\tau \sim 2 \times 10^4$ s, a value close to the flicker floor of the H-maser. Also shown is the estimated stability of the H-maser alone ($5 \times 10^{-14}\tau^{-1/2}$ for $\tau > 10$ s). The comparison confirms that both the H-maser and the cesium fountain have equal medium-term frequency stabilities. Under these conditions, the fountain relative
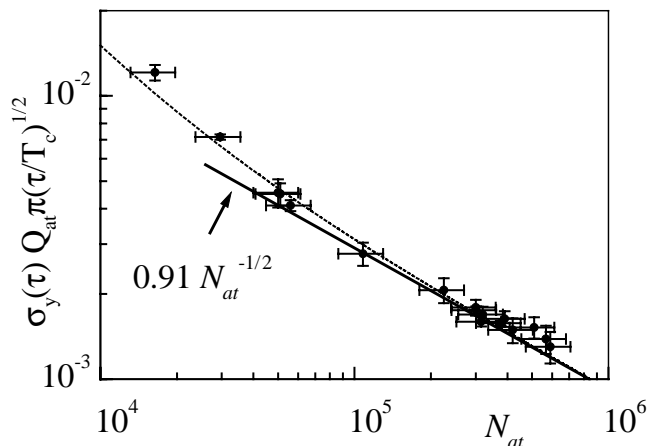
FIG. 2. The measured Allan standard deviation of the Cs fountain for $N_{at} = 2.7 \times 10^5$ compared to a free running sapphire clock oscillator (SCO). Between 3 and 10 s this represents the fountain performance. Above 10 s the drift of the SCO becomes apparent.

FIG. 3. The normalized frequency fluctuations as a function of the number of detected atoms N_{at}. The expected quantum projection noise law is $y = N_{at}^{-1/2}$. The thick line $y = 0.91(0.1)N_{at}^{-1/2}$ is a least square fit to the experimental points for $N_{at} > 4 \times 10^4$. The dashed line is the quadratic sum of the detection noise and quantum projection noise.
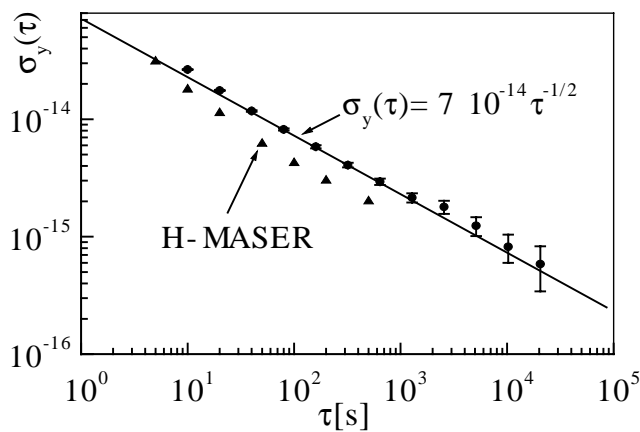
FIG. 4. Allan standard deviation of the Cs fountain using the cryogenic sapphire oscillator as interrogation oscillator versus the H-maser (circles). At $\tau = 2 \times 10^4$ s the frequency stability reaches 6×10^{-16}. Triangles correspond to the estimated frequency stability of the H-maser alone. The comparison between the two plots shows that both clocks have the same frequency stability of $\sim 5 \times 10^{-14} \tau^{-1/2}$ for $\tau > 10$ s.

frequency shift due to interactions between the cold atoms is about -2×10^{-14} [6]. N_{at} is stabilized at the percent level by varying the loading time of the magneto-optical trap. If the other experimental parameters such as the atomic temperature are kept constant, this servo system stabilizes the atomic density and the corresponding collisional shift [17]. It is clear that operating with optimized optical molasses (large laser beam diameter and enough optical power) can provide the same number of detected atoms ($\sim 10^6$) and reduce the collisional shift by about 1 order of magnitude.

In summary, we have observed the quantum projection noise in a cold atom fountain up to $N_{at} = 6 \times 10^5$. This opens the way to a frequency stability of $(1-2) \times 10^{-16}$ for just one day of integration time and an accuracy of 10^{-16}. This accuracy would represent about an order of magnitude gain over the current 2×10^{-15} accuracy of an atomic fountain [2] and 3.4×10^{-15} accuracy of a laser cooled Hg^+ ion clock [3]. Future improvements may result from the use of quantum correlated atoms [18–20].

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[12] The SCO has previously demonstrated a stability of the order of 10^{-15} between 1 to 100 s, oscillating on a mode with had a loaded quality factor of 10^9 at 11.92 GHz.

[13] The two sample Allan deviation is defined as $\sigma_y(\tau) = 1/\sqrt{2(N-1)} \left[\sum_{k=1}^{N-1} (\bar{y}_k - \bar{y}_{k-1})^2 \right]^{1/2}$, $\bar{y}_k = \frac{1}{\tau} \int_{t_k}^{t_k+\tau} y(t) \, dt$.

[14] The determination of N_{at} can be made insensitive to the phase noise of the interrogation oscillator by driving $\pi/4$ pulses in each Ramsey interaction on resonance, measuring then the noise on p. For instance, at $N_{at} = 6 \times 10^5$ the noise on p measured in this way and the noise deduced from the frequency stability measurement agree within 10%. This confirms that the frequency noise of the sapphire oscillator has a negligible contribution in all the stability data. For $\pi/4$ pulses the noise on p is insensitive to the phase noise of the microwave field but is sensitive to its amplitude noise. However, with current microwave oscillators, this noise contribution is negligible.

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