Matrix Elements for Low-Energy *p-d* Radiative Capture

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Measurements of polarization observables are presented for the p-d radiative capture reaction. A new analysis technique, based on Watson's theorem, is used to extract the reaction matrix elements. The new method allows one to fix the phases of the matrix elements by incorporating information from the elastic scattering channel. [S0031-9007(99)09297-2]

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The radiative capture reaction $p + d \rightarrow {}^3\mathrm{He} + \gamma$ has been the focus of many experimental and theoretical studies over the past 20 years, and during this time experiments with polarized protons and deuterons have come to be fairly common. Initially, the interest in polarization experiments arose in part from the observation that the "tensor analyzing powers" for p-d capture are sensitive to D-state components in the ^{3}He wave function [1–4]. Additionally, it was known that measurements of the proton analyzing power and the deuteron vector analyzing power are sensitive to $M1$ transitions [5,6] which can be influenced by meson exchange processes and non-nucleonic degrees of freedom.

Recently, the focus of the work in this field has expanded somewhat as theorists continue to make progress in developing techniques for performing exact quantum calculations in few-body systems. Within just the last few years we have seen publication of the first calculations [7] of p-d radiative capture which incorporate both realistic NN interactions and a correct treatment of Coulomb forces. In view of this new capability, one may now view p-d capture experiments as a means for testing, in a more general way, our understanding of the spin structure of the $A = 3$ system and of the fundamental NN interaction.

The purpose of this Letter is to present a new set of measurements for p-d radiative capture at an energy just below the deuteron breakup threshold, $E_{\mathrm{c.m.}} = 2$ MeV. We also describe a new method for the analysis of subthreshold capture data, which is based on Watson's theorem [8]. This new technique has made it possible to carry out a partial-wave analysis of the data which is significantly more extensive in scope than any previous analysis of this kind for p-d capture.

The value of a partial-wave analysis should be readily apparent. In this kind of analysis one determines (by fitting data) a set of parameters which specify the contributions to the reaction amplitude from the individual angular momentum states. In general, a partial-wave analysis provides insight about the details of the reaction process (see, for example, Ref. [9]), and, in addition, makes it possible to compare theory and experiment at a more fundamental level.

In the present context the fitting parameters in the partial-wave analysis are the reaction matrix elements, and the main difficulty of carrying out the analysis for p-d capture is that the number of independent matrix elements is large. It is known from previous work (see, for example, Ref. [10]) that for energies of a few MeV the reaction is dominated by $E1$ transitions, with small but important contributions from both $M1$ and $E2$. Because the spin structure of the p-d system is moderately complex the number of parameters is fairly large, with 5 $E1$, 5 $M1$, and 6 $E2$ matrix elements. Since the matrix elements are, in general, complex, the number of free parameters in a conventional analysis would be quite large.

It has been pointed out recently [11] that the number of undetermined parameters in a matrix element analysis can be reduced by essentially a factor of 2 in situations where radiative capture is the only open reaction channel. When this condition is met, it is possible to choose a representation in which the capture matrix elements are required to be real as a consequence of time reversal invariance. The phase information needed to construct the reaction amplitudes is then obtained from a separate phase shift analysis of elastic scattering data obtained at the same c.m. energy. This technique, which is a straightforward extension of Watson's theorem [8], is described in detail in Ref. [11]. This new insight now makes it possible for the first time to carry out a thorough and extensive matrix element analysis of p-d radiative capture at energies below the deuteron breakup threshold.

New measurements of the relative differential cross section, the proton analyzing power (A_y), the deuteron vector analyzing power (iT_{11}), and the three deuteron tensor analyzing powers (T_{20}, T_{21}, and T_{22}) have been obtained at $E_{\mathrm{c.m.}} = 2$ MeV. The measurements were carried out at the University of Wisconsin Nuclear Physics Laboratory using a tandem electrostatic accelerator and a crossed-beam polarized ion source [12]. The targets consisted of pure hydrogen or deuterium gas contained in a cell 4.13 cm in length and closed at the beam entrance and exit with thin Ni foils (typically 1–2 μm). The gas cell was made of copper with a wall thickness of 0.8 mm to minimize γ-ray attenuation.

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The measurements of A_y and $\frac{d\sigma}{d\Omega}$ were made with 3 MeV polarized and unpolarized proton beams, respectively. The outgoing γ rays were detected in a 25 cm $\times$ 25 cm NaI detector. Background subtraction was accomplished by alternating between runs with deuterium gas and ordinary hydrogen gas in the cell. With hydrogen there are no capture reactions, but the energy loss and multiple scattering effects of the deuterium gas are still approximately duplicated. Measurements were obtained at 9 c.m. angles ranging from 25° to 155°.

For the measurements of the deuteron analyzing powers we used a 6 MeV polarized deuteron beam and a hydrogen gas target. With the more energetic deuteron beam the backgrounds are greatly increased, and as a result, a more elaborate experimental arrangement was required. For these measurements the background was eliminated by detecting the recoil ^{3}He nuclei in coincidence with the γ rays. Because the γ-ray momentum is small, the ^{3}He nuclei are emitted into a narrow forward cone. The ^{3}He's were separated from the primary beam with a simple dipole magnet. The magnetic field strength was adjusted to deflect the beam by 30°, and the corresponding deflection of the $^3He^{++}$ ions was about 60°. The momentum analyzed ^{3}He's were detected in an array of silicon microstrip detectors located roughly at the focal point of the magnet. The microstrip detectors, which were about 120 cm from the target, covered an area 8 cm high by 14 cm along the focal plane. For this experiment we obtained measurements at the 9 c.m. angles simultaneously by using an array of 7.5 cm diameter NaI and BGO detectors. The use of multiple small detectors was possible for this measurement since the ^{3}He energy together with the timing information from the ^{3}He-γ coincidence was sufficient to identify the capture events, making it unnecessary to collect the full γ-ray energy.

The new measurements are given in Fig. 1. The error bars shown in Fig. 1 include statistical uncertainties and also an estimate of the uncertainty associated with background subtraction. The overall scale of the differential cross section measurements has been chosen to correspond to a total reaction cross section of 7.5 μb. The full experimental details will be presented in a future publication.

Using only the general features of these measurements it is possible to draw a number of qualitative conclusions about the γ-ray multipolarities which contribute to the reaction. First of all, we note that for pure $E1$ capture the cross section must be of the form $C_0 + C_2 \cos^2\theta$. On theoretical grounds one expects the reaction to be dominated by $E1$ capture from the $^2P_{1/2}$ and $^2P_{3/2}$ scattering states, and if the matrix elements for the two states are equal one obtains $\frac{d\sigma}{d\Omega} = C \sin^2\theta$. While the measured cross sections are roughly of this form, some asymmetry about 90° is clearly seen in the data. This asymmetry is of the form expected for $E1$-$E2$ interference and therefore it is clear that $E2$ transitions play a role.

From the measurements of A_y and iT_{11} one concludes that $M1$ radiation is also present. Interference between $M1$ and $E1$ amplitudes will produce nonzero vector analyzing powers with products $\frac{d\sigma}{d\Omega} \times A_y$ and $\frac{d\sigma}{d\Omega} \times iT_{11}$ varying as $\sin\theta$, and this is essentially what we see in the data.

Finally, the measurements of T_{20} and T_{21} indicate the presence of $E1$ capture from states with channel spin $s = \frac{3}{2}$. The tensor analyzing powers must be zero if there are no $s = \frac{3}{2}$ contributions and the observed angular dependences are of the form expected for interference between doublet $E1$ and quartet $E1$.

We now move on to the quantitative matrix element analysis of the measurements. As noted earlier, the analysis we will present here makes use of elastic scattering data as well as the radiative capture data. As described in Ref. [11], what one requires from the elastic channel is the elastic S-matrix in the low angular momentum states. For each j^π of interest, there are either two or three angular momentum states that may mix in the elastic scattering. From Ref. [13] we know that the off-diagonal S matrix elements are small in magnitude (typically 0.1 or less), but nevertheless of significant importance. As we shall see below, the mixing of the angular momentum states also plays an important role in the capture reaction.

For elastic scattering with no open reaction channels the S matrix is unitary and symmetric, and it follows that S can be diagonalized by a matrix transformation. More specifically, S may be written in the form

$$S = u^\dagger S_0 u\,, \tag{1}$$

where the mixing matrix u is both real and orthonormal and where S_0 is diagonal and unitary. For three-state mixing we have

$$S_0 = \begin{bmatrix} e^{2i\delta_1} & 0 & 0 \\ 0 & e^{2i\delta_2} & 0 \\ 0 & 0 & e^{2i\delta_3} \end{bmatrix}. \tag{2}$$

The phase shifts, δ_α, that appear in Eq. (2) are real parameters, commonly referred to as the eigen-phase-shifts.

In Ref. [11] it is shown that the matrix elements of the multipole operators will be real provided that one chooses the proper scattering wave functions. Specifically, the wave functions we use are the "eigenstates of the S matrix" [14], and the resulting matrix elements are referred to as the eigenchannel matrix elements.

The eigenchannel matrix elements, P_α, are related to the conventional matrix elements, R_α, in a simple way. The conventional matrix elements correspond to scattering states in which the ingoing wave is a pure angular momentum eigenstate (i.e., an eigenstate of both L^2 and S^2). In contrast, the eigenstates of the S matrix have the asymptotic form

$$|\alpha; jm\rangle \rightarrow e^{-i\delta_\alpha} \sum_\beta u_{\alpha\beta}[\chi_\beta^{\text{in}} + e^{2i\delta_\alpha}\chi_\beta^{\text{out}}], \tag{3}$$

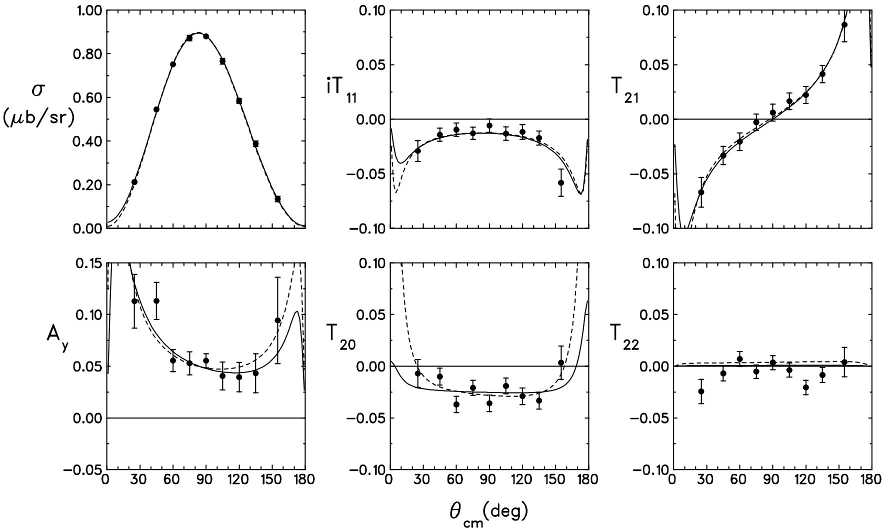

FIG. 1. Measurements of the differential cross section, the proton analyzing power, and the four deuteron analyzing powers for p-d radiative capture at $E_{\mathrm{c.m.}} = 2$ MeV. The curves show the two matrix element fits described in the text. The dashed curves correspond to Set 1 of Table I and the solid curves correspond to Set 2.

where χ_β^{in} and $\chi_\beta^{\mathrm{out}}$ are ingoing and outgoing waves for angular momentum state β. The relation between the matrix elements is [11]

$$R_\beta = \sum_\alpha u_{\alpha\beta}\, e^{i\delta_\alpha} P_\alpha \,. \tag{4}$$

In our matrix element analysis the real quantities P_α are treated as free parameters, while the mixing matrix elements, $u_{\alpha\beta}$, and the eigenphases δ_α are taken from the elastic phase shift analysis.

To carry out the analysis we make use of the elastic scattering results from Ref. [13]. This paper presents elastic scattering data at $E_{\mathrm{c.m.}} = 2$ MeV and also a phase shift analysis of the elastic measurements. We use these experimentally determined phase shift parameters to construct the elastic S-matrix. In Table I we list the resulting eigenphases for each of the relevant angular momentum states.

With the elastic S-matrix elements in hand we now proceed to the analysis of the capture data. In this analysis, one could choose to treat all of the $E1$, $M1$, and $E2$ matrix elements as freely variable parameters, but as it turns out, this is not the best approach. In particular, effects of the $E2$ transitions are essentially seen only in the shape of the differential cross section, and since there are six $E2$ matrix elements these parameters are not all well determined in a fit. Since the ^{3}He bound state is predominantly total spin $\frac{1}{2}$, one expects that the quartet $E2$ matrix elements will be quite small (for the $E1$ transitions the quartet matrix elements are about an order of magnitude smaller than the doublets) and consequently we set the matrix elements for these states to zero. One also expects the $E2$ matrix elements for the $^2D_{3/2}$ and $^2D_{5/2}$ states to be approximately equal and we impose this constraint in our fits.

For the $M1$ matrix elements we find that the data set is not quite sufficient to determine all of the parameters well. In particular, the data have relatively little sensitivity to the doublet-$M1$ transitions. Some doublet-$M1$ strength is required, but reasonable fits can be obtained if either the $^2S_{1/2}$ or the $^2D_{3/2}$ matrix element is included. For this reason we shall present results for two matrix element fits, one with $^2D_{3/2}$ constrained to be zero and the second with $^2S_{1/2}$ constrained to zero.

The two fits are shown in Fig. 1, and the corresponding matrix element parameters are given in Table I. Both of these fits reproduce the measurements reasonably well, with reduced chi squares of 1.12 and 1.28. It may be noted that the fits to the T_{22} data are not very good; in fact, nearly half of the total chi square is from T_{22}.

One of the interesting features of the present analysis is that one obtains information about the effect that angular momentum mixing in the elastic channel has on the capture observables. This effect can be seen by comparing the eigenchannel matrix elements with the conventional matrix elements. This comparison is shown in Table I for one of the two fits. Notice that for the majority of the matrix elements (the dominant $E1$'s and

TABLE I. Matrix element parameters and eigenphase shifts for p-d radiative capture at $E_{\mathrm{c.m.}} = 2$ MeV.

Channel		δ_α (deg)	$P_\alpha(\times 10^3)$ Set 1	Set 2	$R_\alpha(\times 10^3)$ Set 1
$^2P_{1/2}$	$E1$	-1.82	2.721	2.434	$2.717 - 0.081i$
$^2P_{3/2}$	$E1$	-1.95	2.742	2.837	$2.741 - 0.091i$
$^4P_{1/2}$	$E1$	26.92	-0.122	-0.118	$0.163 - 0.064i$
$^4P_{3/2}$	$E1$	29.42	0.080	0.061	$-0.046 + 0.044i$
$^4F_{3/2}$	$E1$	10.39	0.061	0.085	$-0.038 + 0.013i$
$^2S_{1/2}$	$M1$	-24.86	-0.221	⋯	$-0.207 + 0.092i$
$^2D_{3/2}$	$M1$	9.83	⋯	-0.355	$-0.006 - 0.001i$
$^4S_{3/2}$	$M1$	116.06	-0.221	-0.220	$0.101 - 0.198i$
$^4D_{1/2}$	$M1$	4.27	0.272	0.476	$0.266 + 0.023i$
$^4D_{3/2}$	$M1$	4.12	0.184	0.324	$0.182 + 0.017i$
^2D_j	$E2$	9.83	0.127	0.116	$0.125 + 0.022i$

TABLE II. Eigenchannel matrix element parameters. The parameter uncertainties are given in parentheses.

Parameter		Value
$E1$:		
	${}^2P_{1/2}$	$2.58\ (0.19) \times 10^{-3}$
	${}^2P_{3/2}$	$2.79\ (0.08) \times 10^{-3}$
	${}^4P_{1/2}$	$-0.120\ (0.028) \times 10^{-3}$
	${}^4P_{3/2}$	$0.070\ (0.031) \times 10^{-3}$
	${}^4F_{3/2}$	$0.073\ (0.020) \times 10^{-3}$
$M1$:		
	${}^4S_{3/2}$	$-0.221\ (0.021) \times 10^{-3}$
	${}^4D_{1/2}/{}^4D_{3/2}$	1.47 (0.09)
$E2$:		
	$\frac{2}{5}[{}^2D_{3/2}] + \frac{3}{5}[{}^2D_{5/2}]$	$0.121\ (0.011) \times 10^{-3}$

all of the $M1$'s and $E2$'s) one has

$$R_\alpha \simeq P_\alpha e^{i\delta_\alpha}, \tag{5}$$

which says that the largest contribution to R_α is from the diagonal term in the sum of Eq. (4). On the other hand, Eq. (5) is not even approximately correct for the quartet $E1$ matrix elements. For these parameters the contributions from the off-diagonal elements of the u matrix are comparable in magnitude to the diagonal ones. This suggests that the tensor analyzing powers, which arise mainly from the quartet $E1$ transitions, are strongly influenced by mixing in the elastic channel.

Since the main focus of this paper is the determination of the capture matrix elements, a discussion of the parameter uncertainties is of relevance. While it is clear from Table I that some of the $M1$ matrix elements have large uncertainties, it is our conclusion that many of the other parameters are determined quite well. Our results for several of these well-determined parameters are given in Table II.

To obtain the uncertainties listed in Table II we look both at the statistical errors obtained in individual fits (which are extracted in the usual way from the error matrix) and also at the amount of variation in the parameter values as one imposes different constraints in the fits. For this latter contribution we use a total of ten different fits (including the two from Table I) and find the standard deviation in each parameter. The quoted uncertainty is then obtained by adding the standard deviation in quadrature with the typical single-fit statistical error.

There are several points to note in Table II. For the $M1$ transitions, the ${}^4S_{3/2}$ matrix element is determined with a relative uncertainty of about 10%. The ${}^4D$ $M1$ parameters have comparatively large uncertainties, but the two parameters are strongly correlated, and the fits (mainly to T_{21}) begin to deteriorate if the ratio ${}^4D_{1/2}/{}^4D_{3/2}$ differs from substantially 1.5. The overall $E2$ strength, or more specifically the combination $\frac{2}{5}P({}^2D_{3/2}) + \frac{3}{5}P({}^2D_{5/2})$ is determined fairly well, mainly from the shape of the differential cross section. Finally we note that all five $E1$ matrix elements are moderately well determined. These quantities are of particular interest since the doublet $E1$'s are the dominant transitions, while the quartet $E1$'s are sensitive to the D-state components of the ^{3}He wave function as well as other effects such as channel-spin mixing which arise from the NN tensor interaction.

In summary, we have presented a new set of analyzing power measurements for p-d radiative capture, and have carried out (for the first time) a matrix element analysis which incorporates phase information from the elastic scattering channel with the aid of Watson's theorem. We hope that this work will stimulate further exact quantum calculations of radiative capture, and that workers in related areas will identify additional applications of the Watson theorem concept in the analysis of nuclear reaction data.

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