Relevance of Center Vortices to QCD

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In a numerical experiment, we remove center vortices from an ensemble of lattice SU(2) gauge configurations. This removal adds short-range disorder. Nevertheless, we observe long-range order in the modified ensemble: confinement is lost and chiral symmetry is restored (together with trivial topology), proving that center vortices are responsible for *both* phenomena. As for the Abelian monopoles, they survive but their percolation properties are lost. [S0031-9007(99)09294-7]

The essential nonperturbative properties of quantum chromodynamics (QCD) are confinement and chiral symmetry breaking (χSB). It has been observed through numerical lattice simulations that these two properties persist in the quenched theory up to a critical temperature $T_c \sim$ 220 MeV, where confinement is lost; chiral symmetry appears to be simultaneously restored [1]. The disorder which leads to the area law for the Wilson loop thus seems to be somehow tied to the existence of a chiral condensate. Although effective mechanisms have been proposed to explain confinement or χSB, no successful common explanation is yet available.

Two effective descriptions of QCD have been receiving a lot of attention: one considers instantons as the effective degrees of freedom (d.o.f.), the other chromomagnetic monopoles. Instantons are natural candidates to explain χSB: each instanton is associated with a zero mode of the Dirac operator [2], and there must be an accumulation of zero eigenvalues to obtain a quark condensate [3]. Above T_c, the vanishing of the condensate must correspond to a qualitative change in the instanton ensemble (see, e.g., Ref. [4]). On the other hand, it is unlikely that instantons play a significant role in confinement (see [5] for a recent discussion). Instead, an attractive mechanism for confinement is dual superconductivity of the QCD vacuum [6]. Considerable evidence for this dual Meissner effect has been accumulated on the lattice, including a disorder parameter demonstrating the condensation of chromomagnetic monopoles below T_c [7]. This condensation has been observed directly after gauge fixing to "maximal Abelian gauge" [8], and the idea of "Abelian dominance" has emerged, according to which the Abelian d.o.f. of the Yang-Mills field encode all its long-distance [infrared (IR)] properties. Indeed, χSB and its restoration have been observed in the Abelian sector [9]. The Abelian dominance scenario has some flaws, however: it does not explain the breaking of the adjoint string, and the Abelian string tension differs slightly from the Yang-Mills one [10]. Moreover, d.o.f. more elementary than Abelian monopoles, embedded in them and solely responsible for the physics assigned to them, cannot be ruled out.

The idea of center vortices, which initially failed due to their misidentification [11], has been successfully revived, first as an embedded model inside the Abelian sector [12], then without reference to Abelian projection [13]. Center vortices are exposed by gauge fixing: after a gauge transformation which brings each lattice link as close as possible to a center element of the gauge group, vortices consist of defects in the center-projected gauge field. The idea of center dominance is, again, that the center d.o.f. encode all of the IR physics. The density of center vortices seems to be a well-defined continuum quantity [13,14]; the center string tension appears to more or less match the original one; and an explanation for the behavior of the adjoint potential has been proposed [15]. However, chiral symmetry has not yet been studied in this context.

An additional problem with Abelian and center dominance is that the relevant d.o.f. are identified only after gauge fixing. The gauge-fixing of non-Abelian fields is notoriously ambiguous, and different Gribov copies produce different Abelian monopoles or center vortices, whose properties, such as the string tension, differ slightly. For this reason, we are *a priori* suspicious of effective models which involve gauge fixing, and so we designed a simple numerical experiment to disprove the center-dominance scenario. For simplicity, we consider the gauge group SU(2), with center Z_2. Our starting point is an ensemble of lattice gauge fields representative of the continuum. We identify center vortices in this ensemble, and we construct from it a *modified* ensemble where all center vortices have been removed by flipping the sign of a subset of SU(2) gauge links. This operation introduces a lot of disorder in the gauge field. Nonetheless, these disordered gauge fields now have a trivial, vortex-free center projection, and so, according to the credo of center dominance, they should not confine. That is, our introduction of short-range disorder should at the same time bring long-range order. To our surprise, this is indeed what happens. One may then ask if the spectral properties of the Dirac operator are not also dominated by the center components of the gauge field. In that case, our modified ensemble should show no sign of χSB, since its center projection is the trivial (perturbative)

vacuum. Indeed, this is what we observe: removal of center vortices causes *both* loss of confinement and restoration of chiral symmetry.

The next intriguing question regards the fate of Abelian monopoles as center vortices are removed. They do not disappear; on the contrary, the introduction of short-range disorder increases their number. However, we observe the complete disappearance of monopole current loops winding around the periodic lattice: we can thus identify these as the fundamental objects associated with confinement in the Abelian sector, apparently influenced by the underlying center d.o.f.

Finally, we investigated the effect of Gribov copies which caused our initial skepticism. We repeated our experiment on the same SU(2) ensemble, but introduced a systematic tolerance in the gauge condition to be satisfied before identifying the center vortices to be removed. Although the location and number of center vortices we removed varied appreciably, the modified SU(2) ensemble was always nonconfining and chirally symmetric.

The numerical experiment.—We start from an ensemble of SU(2) lattice gauge fields obtained by Monte Carlo simulation using the standard Wilson plaquette action. To identify center vortices, we gauge fix our configurations in order to bring each SU(2) gauge link $U_\mu(x)$ as close as possible to an element of the center $Z_2 = \{+\mathbf{1}, -\mathbf{1}\}$. We therefore try to iteratively maximize

$$Q(\{U_\mu\}) \equiv \sum_{x,\mu} [\mathrm{Tr}U_\mu(x)]^2, \tag{1}$$

as in [13], where this gauge is called the "direct maximal center gauge." The gauge-fixed SU(2) links, denoted $U_\mu^{\mathrm{GF}}(x)$, are then projected to Z_2 elements, $Z_\mu(x)$, using

$$Z_\mu(x) = \mathrm{sgn}[\mathrm{Tr}U_\mu^{\mathrm{GF}}(x)]. \tag{2}$$

Plaquettes in the Z_2-projected theory with value -1 represent defects of the Z_2 gauge field called *P*-vortices [12]. Numerical evidence has been presented [12,13] showing that plaquettelike *P*-vortices signal the presence of macroscopic, physical excitations, called center vortices, in the unprojected original SU(2) configuration.

Consider then the modified SU(2) configuration made of gauge links $U'_\mu(x)$ constructed as

$$U'_\mu(x) \equiv Z_\mu(x)U_\mu(x). \tag{3}$$

The gauge transformation which maximizes $Q(\{U_\mu\})$ in (1) also gives the same maximum to $Q(\{U'_\mu\})$, so that the modified gauge-fixed links $U'^{\mathrm{GF}}_\mu(x)$ are simply $Z_\mu(x)U_\mu^{\mathrm{GF}}(x)$. Therefore, we instantly know the center projection $Z'_\mu(x)$ of $U'_\mu(x)$:

$$Z'_\mu(x) = \mathrm{sgn}[\mathrm{Tr}U'^{\mathrm{GF}}_\mu(x)] = \mathrm{sgn}[\mathrm{Tr}Z_\mu(x)U_\mu^{\mathrm{GF}}(x)]$$
$$= \{\mathrm{sgn}[\mathrm{Tr}U_\mu^{\mathrm{GF}}(x)]\}^2 = +\mathbf{1}. \tag{4}$$

Every modified configuration U' thus projects onto the trivial Z_2 vacuum: all center vortices have been removed.

Our ensemble consists of about 1000 SU(2) configurations, on a 16^4 lattice at $\beta = 2.4$. To maximize $Q(\{U_\mu\})$ [Eq. (1)] we use standard overrelaxation, stopping when

$$\epsilon \equiv \sum_{x,\mu} \Delta[\mathrm{Tr}U_\mu(x)]^2 < 10^{-6} \tag{5}$$

from one gauge-fixing sweep to the next.

In Fig. 1 we show the distribution of SU(2) plaquette values on the original and modified ensembles. It is apparent that, under the sign flip Eq. (3), many SU(2) plaquettes acquire a negative value. The modified ensemble has an increased action, i.e., more short-range disorder.

Results.—In Fig. 2 we present our results for the Creutz ratios $\chi_{R,R} \equiv -\ln[\langle W_{R,R}\rangle\langle W_{R-1,R-1}\rangle/\langle W_{R,R-1}\rangle^2]$ constructed from averages $\langle W_{R,T}\rangle$ of R by T Wilson loops on the original and modified ensembles. For large R, $\chi_{R,R}$ tends to the string tension σ. On the modified ensemble, the Creutz ratios clearly decrease and tend to zero. Despite the increased short-range disorder, long-range order has been created and confinement has been lost.

This is even clearer if one looks directly at the Wilson loop values. In Fig. 3 we show $-\ln\langle W_{R,T}\rangle$ as a function of T. For a fixed R, points at successively larger T form a line whose asymptotic slope is the value of the static potential $V(R)$. The lines corresponding to the modified ensemble are parallel, indicating that $V(R)$ does not grow with R; the string tension has vanished.

Figure 4 illustrates our study of chiral symmetry on the original and modified ensembles. As is well known, χSB cannot occur on a finite lattice. Therefore, we measure $\langle\bar\psi\psi\rangle(m_q) = \langle\mathrm{Tr}(\not{D} + m_q)^{-1}\rangle$ for a range of quark masses m_q where finite-size effects are small, and extrapolate to $m_q \to 0$. In the original ensemble, $\langle\bar\psi\psi\rangle$ clearly extrapolates to a nonzero value which signals χSB. In the modified ensemble, the extrapolated value is zero within errors: center-vortex removal restores chiral symmetry. We then expect the instanton content of the Yang-Mills field to be modified also. To check this, we use improved cooling [17] to measure the topological charge of the modified field: the striking result is that the removal of center vortices always leads to the trivial topological sector.

We therefore have clear evidence for "center dominance": in our modified ensemble, where the center-projected field is the trivial vacuum (all links equal to $\mathbf{1}$),
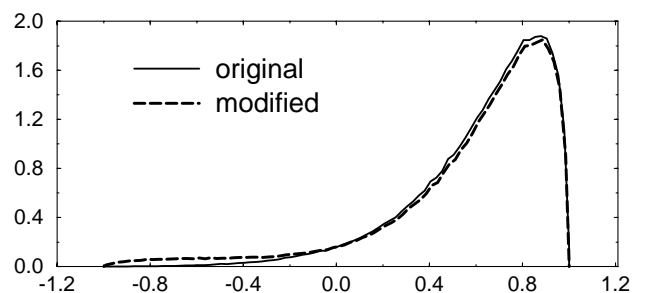

FIG. 1. Normalized plaquette distribution on the original and modified ensembles [SU(2), $\beta = 2.40$]. Center-vortex removal increases short-range disorder.
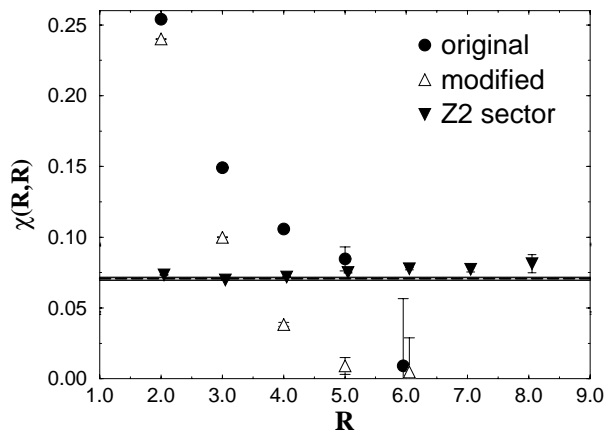
FIG. 2. Creutz ratios on the original and modified ensembles. The dashed band is the string tension result of [16].

FIG. 4. Quark condensate $\langle\bar{\psi}\psi\rangle(m_q)$ on the original and modified ensembles. The dashed line corresponds to "poor" center-vortex identification (see text). In all cases, center-vortex removal restores chiral symmetry.

the Yang-Mills field shows the IR properties of the trivial vacuum, i.e., no confinement, no χSB and no topology. The IR properties of the Yang-Mills field appear to be determined by its center projection.

On the other hand, a large number of studies now support the alternative scenario of Abelian dominance. We use our approach of center-vortex removal to directly assess the relationship between these two scenarios.

In a first experiment, we construct the Abelian projection of our original SU(2) ensemble by gauge fixing to maximal Abelian gauge in the usual way [8], and then identify and remove center vortices from the *Abelian* sector. While the original Abelian-projected ensemble shows confinement, with a string tension similar to the non-Abelian one, the modified Abelian-projected configurations do not confine. Therefore, we find no contradiction between Abelian domi-
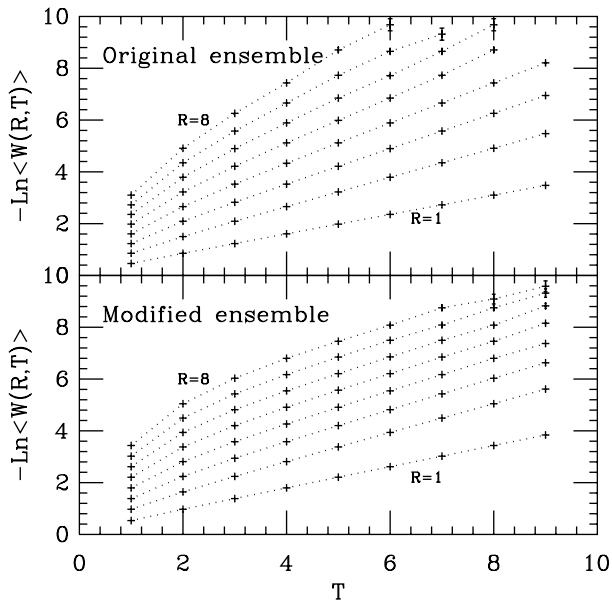
nance and center dominance. The latter simply appears more fundamental because of the greater reduction of the number of d.o.f.

In a second experiment, we look at clusters of Abelian monopole currents, whose percolation has been identified as the signal for confinement [18], obtained from the original and modified ensembles. We find that the removal of center vortices changes the distribution of monopole cluster sizes in a crucial way (see Fig. 5): whereas in the original ensemble each configuration contains typically one very large percolating monopole cluster and many very small ones, the modified ensemble gives a more homogeneous size distribution, with a handful of large clusters per configuration; these are the remnants of the very large one, broken into pieces by the vortex removal. Some of them still percolate, even though confinement has disappeared. Therefore, we are led to associate confinement with a more specific feature of the monopole clusters: monopole current loops which wind around the periodic lattice. Such loops can be found frequently on the original, confining ensemble, but never on the modified, nonconfining one. We conclude that (i) on a finite lattice confinement manifests itself in the Abelian sector by the presence of monopole current loops with nontrivial topology, and (ii) center-vortex removal, which destroys confinement, always finds the "weak links" of these nontrivial loops and breaks them into trivial pieces.

FIG. 3. Wilson loop values $\langle W_{R,T}\rangle$ on the original and modified ensembles. Note the parallel lines for successive R in the latter: Upon center-vortex removal, confinement is lost.
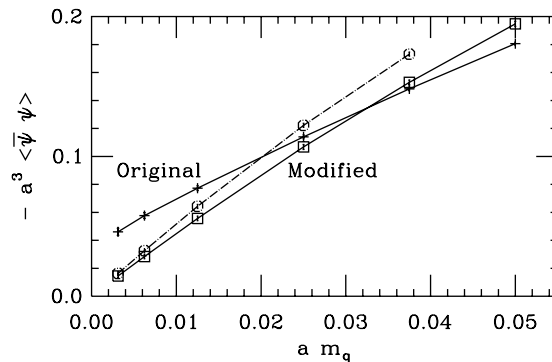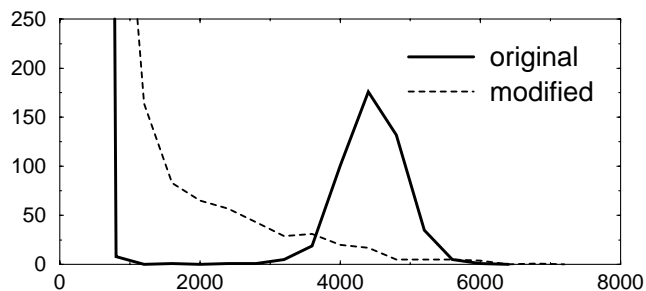

FIG. 5. Size distribution of monopole clusters on the original and modified ensembles.

Now, let us consider the issue of gauge-fixing ambiguities, which was the reason for our initial skepticism about the center-vortex idea. These ambiguities come from the structure of Q [Eq. (1)], which has many local maxima, any of which can be selected by a local iterative maximization algorithm. Each local maximum, or Gribov copy, will have its own set of P-vortices, differing in number and location. The proposal of [12] is that, no matter which Gribov copy one chooses, P-vortices are the traces of physical center vortices and are roughly located at their center. This argument may account for P-vortices differing in location but not in number. To study this question in more detail, we magnified the effect of gauge-fixing ambiguities, by stopping our iterative algorithm early, as soon as ϵ [Eq. (5)] $<10^2$. Thus we not only explore a different basin of attraction of Q, but we do not even stop at a local maximum. One effect of this partial gauge fixing is expected: the density of P-vortices increases from $\rho \approx 5.5\%$ to $\approx 7.4\%$, i.e., shows an increase $\delta\rho \approx 1.9\%$. The string tension measured in the Z_2-projected ensemble increases accordingly: whereas the Z_2 string tension after "complete" gauge fixing ($\sigma a^2 \sim 0.075$) is a little larger than, but compatible with, the non-Abelian string tension [0.0708(11) [16], see Fig. 2], it jumps to ≈ 0.12 after partial gauge fixing. What is remarkable is that this increase $\delta\sigma \approx 0.045$ is similar to that obtained by placing the surplus $\delta\rho$ of P-vortices at *random*, uncorrelated locations: $\delta\sigma \approx -\ln(1 - 2\delta\rho)$. This makes it plausible that the center projection always captures the core d.o.f. relevant for IR properties, plus a varying amount of unrelated noise [19]. Indeed, it has been argued that gauge fixing is not even necessary for center projection [20]. In our description, center gauge fixing acts as a UV noise-filtering device, with different Gribov copies letting through different noise components. Further evidence for this is obtained by removing from the original SU(2) ensemble the center vortices identified after partial gauge fixing only. Just as for complete gauge fixing, we observe that confinement is lost, chiral symmetry is restored, and the topology is trivial. The only difference is that the modified ensemble now has much more short-range disorder.

In conclusion, we have shown that the removal of center vortices from an SU(2) Yang-Mills ensemble causes the loss of confinement *and* the restoration of chiral symmetry. One may ask about the connection of the modified ensemble $\{U'\}$ to the physics of the original SU(2) theory. Note that only plaquettes of $\{U'\}$ at the locations of P-vortices differ from those of $\{U\}$: hence, as $a \to 0$, their proportion goes to zero as a^2, since the density of P-vortices is physical [13,14]. Therefore, rewriting Eq. (3) as $U_\mu(x) \equiv Z_\mu(x) \times U'_\mu(x)$, we see that the original field $\{U\}$ has been factorized into a (maximally) central part $\{Z\}$ and a quotient $\{U'\}$ whose field strength differs from the original only on defects of codimension 2. Nevertheless, this small difference alters the physics dramatically: $\{U'\}$ has perturbative properties, so that all the nonperturbative IR physics *must* be carried by $\{Z\}$, which by definition encodes the center vortices. It would be desirable, of course, to formulate an effective action for the center-projected theory. Reference [21] considers an extension of the Nambu-Goto action, where the fundamental d.o.f. are the two-dimensional random surfaces dual to the P-vortices. Reference [22] instead proposes to consider center monopoles and their world lines. We suggest identifying a "minimum spanning tree" of negative Z_2 links responsible for the P-vortices: perhaps only a subset of them form the essential d.o.f. governing the IR properties.

Finally, our vortex-removal procedure can be used to study properties of nonconfining non-Abelian fields and effects of center-symmetry breaking. For instance, removing timelike center disorder only would be similar to raising the temperature above T_c.

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