Evolution of Axionic Strings and Spectrum of Axions Radiated from Them

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Cosmological evolution of axionic strings is investigated by numerically solving field equations of a complex scalar field in 3 + 1 dimensions. It is shown that the global strings relax to the scaling solution with a significantly smaller number density than the case of local strings. The power spectrum of axions radiated from them is calculated from the simulation data, which is found to be highly peaked around the Hubble scale, and a more accurate constraint on the Peccei-Quinn breaking scale is obtained. [S0031-9007(99)09276-5]

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The global Peccei-Quinn (PQ) U(1) symmetry is introduced into the standard model of particle physics in order to solve the strong CP problem of the quantum chromodynamics (QCD) [1]. The axion appears as a Nambu-Goldstone boson in consequence of the spontaneous breakdown of such a global symmetry, and acquires a mass through the instanton effect at the QCD scale [2].

This breaking scale, f_a, is constrained by the terrestrial and astrophysical experiments as well as by cosmological considerations. The most stringent lower bound has been obtained from SN 1987A as $f_a \gtrsim 10^{10}$ GeV [3]. On the other hand, the upper bound is given by the condition that the present energy density due to the axion should not overclose the universe.

In the very early universe, the $U(1)_{PQ}$ symmetry is expected to be restored due to the high temperature effects. As the universe cools down, it spontaneously breaks down and global strings (axionic strings) are formed [4]. Subsequent evolution of global strings has been less examined than that of local strings, and the result for the latter has been applied to axionic strings without direct numerical verification. In particular, global strings have been assumed to obey a scaling solution like local ones, in which the energy density of infinite strings is given by

$$\rho = \xi \mu / t^2, \tag{1}$$

where μ is the average energy per unit length of strings and ξ is a constant. It has been found that $\xi \sim 13$ for local strings in the radiation dominant era [5].

But there is a crucial difference between global and local strings; that is, the former strongly couples with the associated Nambu-Goldstone fields. As a result, global strings have the following prominent features: (a) These fields carry most of the energy rather than string cores, (b) long-range forces proportional to the inverse separation work between strings, and (c) the dominant energy-loss mechanism of loops is radiation of Nambu-Goldstone bosons [6]. Therefore it is not a trivial problem whether the global string network obeys the scaling solution like local ones. In fact, a deviation from scaling property is observed in 2 + 1 dimensions [7,8], where "strings" cannot intercommute but only pair-annihilate. Of course, since strings intercommute with the probability of order unity [9] in 3 + 1 dimensions, the results in 2 + 1 dimensions do not apply directly. In investigating the property of a global string, the effective action is often used, which is valid when massive modes are sufficiently damped. For a global string we have to use the Kalb-Ramond action [10], which is much harder to deal with than the Nambu-Goto action for a local string. Hence in the present Letter, instead of solving equations of motion derived from the Kalb-Ramond action, we numerically solve the evolution equation of the complex scalar field to trace the evolution of the global string network and calculate the spectrum of axions radiated from them. (If the reheating temperature after inflation is lower than the symmetry breaking scale, global strings and radiated axions are washed out and relic axions come only from coherent oscillations of the zero mode [11]. Here we assume that this is not the case.)

Axions are massless when they are emitted from strings, and hence they behave like radiation. Later around the QCD scale, they acquire a small mass, m_a, through QCD instanton effects. Since at present kinetic energy of axions is much smaller than the rest mass, the present energy density of axions, ρ_a, is given by $\rho_a = m_a n_a$ with n_a being their number density. Thus, in order to estimate the present energy density of axions, we must transform the energy density of radiated axions into the number density by use of the spectrum. However, the spectral shape is still in dispute. Davis, Shellard, and co-workers insist that the spectrum has a sharp peak at the Hubble horizon scale [12–14]. On the other hand, Sikivie and co-workers claim that it is proportional to the inverse momentum [15]. We clarify the shape of this spectrum in realistic situations where global strings evolve according to the scaling solution; that is, we identify kinetic energy of axions emitted from strings and elucidate the spectrum by Fourier transforming them. Hence our approach is free from the ambiguities associated with the *ad hoc* choice of the initial configuration of strings.

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We consider the following Lagrangian density for a complex scalar field $\Phi(x)$,

$$\mathcal{L}[\Phi] = \partial_\mu \Phi \partial^\mu \Phi^\dagger - V_{\rm eff}[\Phi], \tag{2}$$

with the potential $V_{\rm eff}[\Phi]$ given by

$$V_{\rm eff}[\Phi] = \frac{\lambda}{2}(\Phi\Phi^\dagger - \eta^2)^2 + \frac{\lambda}{3}T^2\Phi\Phi^\dagger. \tag{3}$$

Hereafter we set $\lambda = 1.0$ for brevity. For $T > T_c = \sqrt{3}\,\eta$, the potential $V_{\rm eff}$ has a minimum at $\Phi = 0$, and the U(1) symmetry is restored. On the other hand, new minima $|\Phi|_{\rm min} = \eta\sqrt{1-(T/T_c)^2}$ appear and the symmetry is broken for $T < T_c$. In this case the phase transition is of second order.

In the flat Friedmann universe, the equation of motion is given by

$$\ddot{\Phi}(x) + 3H\dot{\Phi}(x) - \frac{1}{a(t)^2}\nabla^2\Phi(x) = -V'_{\rm eff}[\Phi], \tag{4}$$

where the prime represents the derivative $\partial/\partial\Phi^\dagger$ and $a(t)$ is the scale factor. The Hubble parameter $H = \dot{a}(t)/a(t)$ and the cosmic time t are given by

$$H^2 = \frac{8\pi}{3M_{\rm Pl}^2}\frac{\pi^2}{30}g_* T^4, \qquad t = \frac{1}{2H}, \tag{5}$$

where $M_{\rm Pl}$ is the Planck mass, g_* is the total effective number of relativistic degrees of freedom, and radiation domination is assumed. We define the dimensionless parameter ζ as

$$\zeta \equiv \left(\frac{45M_{\rm Pl}^2}{16\pi^3 g_* \eta^2}\right)^{1/2}. \tag{6}$$

In our simulation, we take $\zeta = 10$ and $g_* = 1000$, which corresponds to $\eta \sim 10^{16}$ GeV, but the essential result is independent of this choice.

We take the initial time $t_i = t_c/4$ and the final time $t_f = 75\,t_i = 18.75\,t_c$, where t_c is the epoch $T = T_c$. Since the U(1) symmetry is restored at the initial time $t = t_i$, we adopt as the initial condition a thermal equilibrium state with Φ's mass equal to the inverse curvature of the potential at the origin. The scale factor $a(t)$ is normalized as $a(t_i) = 1$.

We perform the simulations in 256^3 lattices with the physical lattice spacing $\delta x_{\rm phys} = \sqrt{3}\,t_i a(t)/25$. The time step is taken as $\delta t = 0.01 t_i$. Hereafter the subscript "phys" is omitted. The box size is nearly equal to the Hubble horizon H^{-1} and a typical width $d \sim 1.0/\sqrt{2}\,\eta$ of a string is twice as large as the lattice spacing at the final time t_f. We simulate the system from 10 different thermal initial conditions using the second order leapfrog method and the Crank-Nicholson scheme. We use the same method to identify a string core as in our previous work [8]. More details will be published elsewhere [16]. The zero temperature potential $V_{\rm eff}[\Phi, T = 0]$ is used after $t = 20t_i$ so that the axion is identified as in Eq. (8) below. We find that after some relaxation period the energy density of strings ρ is given by

$$\rho = \xi\mu/t^2, \tag{7}$$

where $\xi \simeq (1.00 \pm 0.08)$ irrespective of time and $\mu \equiv 2\pi\eta^2 \ln(t/d\xi^{1/2})$ is the average energy per unit length of strings. Therefore, we can conclude that global strings network relax into scaling regime. In Fig. 1, we show time development of ξ.

Now we turn to the spectrum of axions emitted from axionic strings under the situation where they follow the scaling solution. If we represent the complex field $\Phi(t, \boldsymbol{x})$ in terms of the radial mode $\chi(t, \boldsymbol{x})$ and the axion field $\alpha(t, \boldsymbol{x})$ as

$$\Phi(t, \boldsymbol{x}) = \left[\eta + \frac{\chi(t, \boldsymbol{x})}{\sqrt{2}}\right]\exp\left(\frac{i\alpha(t, \boldsymbol{x})}{\sqrt{2}\,\eta}\right), \tag{8}$$

the kinetic energy density of axions is given by

$$\frac{1}{2}\dot{\alpha}(t, \boldsymbol{x})^2 = \frac{\eta^2}{|\Phi(t, \boldsymbol{x})|^4} \times [-{\rm Im}\Phi(t, \boldsymbol{x})\,{\rm Re}\dot{\Phi}(t, \boldsymbol{x}) + {\rm Re}\Phi(t, \boldsymbol{x})\,{\rm Im}\dot{\Phi}(t, \boldsymbol{x})]^2. \tag{9}$$

Since emitted axions damp like radiation, the average energy density of axions radiated in the period between t_1 and t_2, $\bar{\rho}[t_1, t_2]$, is given by

$$\begin{aligned}\bar{\rho}[t_1, t_2] &= \frac{1}{V}\int d^3\boldsymbol{x}\,\rho[t_1, t_2; \boldsymbol{x}] = \frac{1}{V}\int d^3\boldsymbol{x}\left[\frac{1}{2}\dot{\alpha}(t_2, \boldsymbol{x})^2 - \frac{1}{2}\dot{\alpha}(t_1, \boldsymbol{x})^2\left(\frac{t_1}{t_2}\right)^2\right] \\ &= \frac{1}{V}\int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\left[\frac{1}{2}|\dot{\alpha}_{\boldsymbol{k}}(t_2)|^2 - \frac{1}{2}|\dot{\alpha}_{\boldsymbol{k}}(t_1)|^2\left(\frac{t_1}{t_2}\right)^2\right] \equiv \int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\tilde{\rho}_{\boldsymbol{k}}[t_1, t_2] \equiv \int_0^\infty \frac{dk}{2\pi^2}\rho_k[t_1, t_2],\end{aligned} \tag{10}$$

where V is the simulation volume and $\dot{\alpha}(t, \boldsymbol{x}) = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\dot{\alpha}_{\boldsymbol{k}}(t)\exp(i\boldsymbol{k}\cdot\boldsymbol{x})$.

To avoid contamination of string cores to the spectrum of emitted axions, we divide the simulation box into 8 cells and stock the field data of a cell if there are no string cores in that cell between t_1 and t_2. Over all such cells, we average power spectra of kinetic energy of axions obtained through Fourier transformation. We follow the above procedure between $t_1 = 65t_i$ and $t_2 = 75t_i$, whose result is depicted in Fig. 2.

As is seen there, the spectrum, ρ_k, is peaked around the horizon scale and decays exponentially for higher momenta. Note that it is contributed from both infinite and loop strings. For comparison, we have also given the power spectrum averaged over all cells, some of which include string cores. Note that effects of string

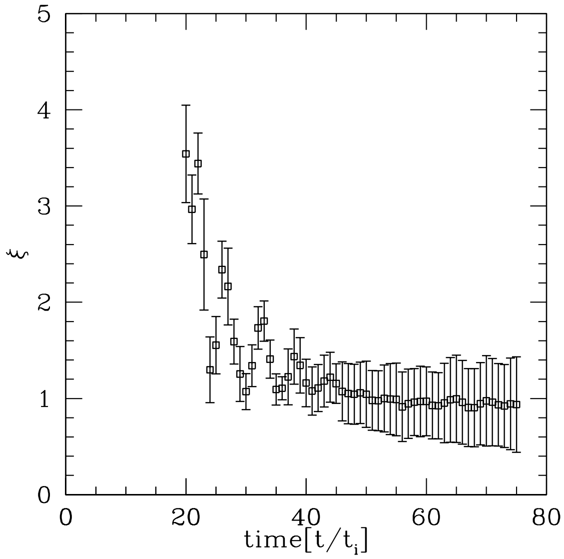

FIG. 1. Symbols (□) represent time development of ξ. The vertical lines denote a standard deviation over ten different initial conditions.

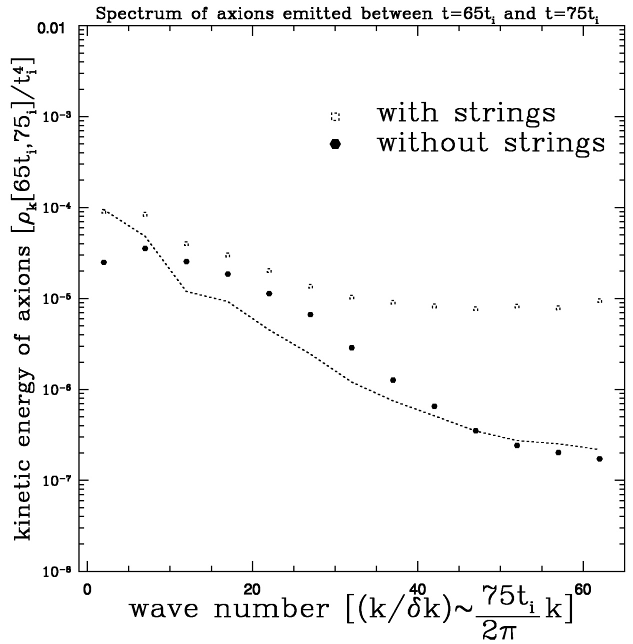

FIG. 2. Filled dots represent the power spectrum, ρ_k, which is already averaged over the direction of $\boldsymbol{k}$ and multiplied by the phase-space factor as defined in Eq. (10). It is obtained from cells with no string cores. The dotted line denotes its standard deviation. Open dots represent the spectrum obtained by averaging over all cells including string cores. Bins are cut every $5\delta k$. $k = 64\delta k$ corresponds to string cores.

cores lift the spectrum at high momenta and make it flatter.

Defining the average inverse momentum by

$$\overline{k^{-1}} = \frac{\int_0^\infty (dk/2\pi^2)\,(\rho_k/k)}{\int_0^\infty (dk/2\pi^2)\rho_k}\,, \tag{11}$$

we find $\overline{k^{-1}} \simeq (0.25 \pm 0.18) \times (t/2\pi)$.

Finally, we want to estimate energy density of relic axions radiated from axionic strings. Though dynamic range of our simulation is limited to $\ln(t/d) \sim 5$, we extrapolate our results up to the cosmological scale with $\ln(t/d) \sim 75$. From the scaling property, we can estimate the energy density of axions radiated from strings from $\tau = t_*$ to $\tau = t$,

$$\rho_a(t) = 2\pi f_a^2 \frac{\xi}{t^2} \int_{t_*}^{t} \frac{1}{\tau}\left[\ln\left(\frac{\tau}{d\xi^{1/2}}\right) - 1\right] d\tau\,, \tag{12}$$

where t_* is the cosmic time when axionic strings began radiating axions and we denote the breaking scale, η, by f_a. Multiplying it by the average inverse momentum $\overline{k^{-1}(\tau)} \equiv \tau/2\pi\epsilon$, the number density of relic axions is given by

$$n_a(t) = 2\pi f_a^2 \frac{\xi}{t^2} \int_{t_*}^{t} \frac{1}{\tau}\left[\ln\left(\frac{\tau}{d\xi^{1/2}}\right) - 1\right] \overline{k^{-1}(\tau)}\, d\tau$$
$$\sim \frac{f_a^2}{t}\,\frac{\xi}{\epsilon}\,\ln\left(\frac{t}{d\xi^{1/2}}\right). \tag{13}$$

As the temperature cools down to the QCD scale and the axion acquires a nonvanishing mass, a network of domain walls bounded by strings is created and walls start to dominate the dynamics of the system at $t = t_w$ given by [14]

$$t_w \sim 1.7 \times 10^{-6} \Delta^2 \left(\frac{m_a}{6 \times 10^{-6}\ \mathrm{eV}}\right)^{-2} \left(\frac{f_a}{10^{12}\ \mathrm{GeV}}\right)^{-1.64} \times \left(\frac{\mathcal{N}_{\mathrm{QCD}}}{60}\right)^{0.5} \mathrm{sec}\,, \tag{14}$$

where Δ is a constant of order unity which describes uncertainties at the QCD phase transition,

$$\Delta = 10^{\pm 0.5} \left(\frac{m_a}{6 \times 10^{-6}\ \mathrm{eV}}\right)^{0.82} \left(\frac{f_a}{10^{12}\ \mathrm{GeV}}\right)^{0.82} \times \left(\frac{\Lambda_{\mathrm{QCD}}}{200\ \mathrm{MeV}}\right)^{-0.65} \left(\frac{\mathcal{N}_{\mathrm{QCD}}}{60}\right)^{-0.41}, \tag{15}$$

with m_a being the axion mass at zero temperature, Λ_{QCD} the energy scale of the QCD phase transition, and $\mathcal{N}_{\mathrm{QCD}}$ an effective number of massless degrees of freedom at that time. These domain walls bounded by strings also decay by emitting axions if the QCD anomaly factor is unity [17]. (Otherwise, these domain walls would rapidly overdominate the universe.)

As is seen from Eq. (13), the dominant contribution to the present axion density is those radiated just before wall domination. The present density parameter, Ω_s, of relic axions due to emission from strings is given by

$$\Omega_s \sim 2.7 \left(\frac{\xi}{\epsilon}\right) h^{-2} \Delta \left(\frac{f_a}{10^{12}\ \mathrm{GeV}}\right)^{1.18}, \tag{16}$$

where h is the Hubble constant in units of 100 km/sec/Mpc.

With $\xi \sim (1.00 \pm 0.08)$ and $\epsilon^{-1} \sim (0.25 \pm 0.18)$, Ω_s is given by

$$\Omega_s \sim (0.68 \pm 0.46) h^{-2} \Delta \left(\frac{f_a}{10^{12}\ \mathrm{GeV}} \right)^{1.18}. \qquad (17)$$

The condition that $\Omega_s \lesssim 1.0$ constrains the breaking scale of PQ-symmetry f_a as

$$f_a \lesssim (1.39 \pm 0.79) h^{1.7} \times 10^{12}\ \mathrm{GeV}, \qquad (18)$$

which reads $f_a \lesssim (0.19\text{–}1.5) \times 10^{12}$ GeV for $h = 0.5\text{–}0.8$. Our constraint lies in between those found in [13] and in [15], because the spectral shape agrees with the former but $\xi = 1$ is substantially smaller than the case of local strings, $\xi = \mathcal{O}(10)$, which has been adopted in [13].

There are two additional contributions to the present energy density of axions besides that from strings considered above. One is that from decay of a domain wall connected by strings [18] and estimated as

$$\Omega_w = \gamma h^{-2} \left(\frac{f_a}{10^{12}\ \mathrm{GeV}} \right)^{1.18}, \qquad (19)$$

from which we obtain $f_a \lesssim \gamma^{0.85} h^{1.7} \times 10^{12}$ GeV with γ a factor of the order unity. Note that γ also becomes smaller of the order 10 than the previous estimation. The other comes from the coherent oscillation of the axion field [19] and is estimated as

$$\Omega_{co} = 1.5 \times 10^{\pm 0.4} \left(\frac{f_a}{10^{12}\ \mathrm{GeV}} \right)^{1.18}, \qquad (20)$$

which leads to $f_a \lesssim (0.33\text{–}1.5) \times 10^{12}$ GeV. These contributions may be comparable with or larger than axions radiated from axionic strings. Taking all these into account, it is safe to say that $f_a \lesssim 10^{11-12}$ GeV.

In summary, we have solved equations of motion of a complex scalar field to clarify cosmological evolution of axionic strings. As a result we have confirmed these global strings relax into the scaling solution but with a number density much smaller than local strings would occupy. We have also calculated the energy spectrum of axions generated from these axionic strings, which agreed with the result of Davis and Shellard [13]. The constraint on f_a turned out to be between those obtained in [13] and in [15].

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