# Sharpening the Predictions of Big-Bang Nucleosynthesis

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We have reexamined the nuclear inputs to big-bang nucleosynthesis using Monte Carlo realization of the cross-section data to directly estimate theoretical uncertainties for the yields of D, $^3$He, and $^7$Li. Our results indicate that previous estimates of the uncertainties were too large by a factor of 2. Using the Burles–Tytler deuterium measurement, we infer a baryon density $\Omega_B h^2 = 0.019 \pm 0.0024$, predict a primeval $^4$He mass fraction $Y_P = 0.246 \pm 0.0014$, and obtain a limit to the equivalent number of neutrino species $N_\nu < 3.20$ (all at 95% C.L.). We also identify key reactions and the energies, where improved data would allow further progress. [S0031-9007(99)09188-7]

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*Motivation.*—Big-bang nucleosynthesis (BBN) is an observational cornerstone of the hot big-bang cosmology. For more than two decades the predicted abundances of the light elements D, $^3$He, $^4$He, and $^7$Li have been used to test the consistency of the hot big-bang model at very early times ($t \sim 0.01$–$200$ s) [1,2]. The state of affairs in 1995 was summarized by a concordance interval for the baryon density, $\Omega_B h^2 = 0.007$–$0.024$, for which the predicted abundances for all four light elements were consistent with the observational data [1]. In addition to testing the standard cosmology, BBN also gave the best determination of the baryon density and was the linchpin in the case for nonbaryonic dark matter.

The big-bang abundance of deuterium is most sensitive to the baryon density [3], making it the "baryometer." However, deuterium is fragile and is destroyed by stars even before they reach the main sequence. Thus, local measurements, where probably about 50% of the material has been through stars, do not directly reflect its primeval abundance. Recently, the situation has changed dramatically. Burles and Tytler measured the deuterium abundance in high-redshift hydrogen clouds, where it is expected that almost none of the material has been processed through stars, and they have made a strong case for a primeval deuterium number density, $(\mathrm{D/H})_P = (3.4 \pm 0.25) \times 10^{-5}$ [4,5]. Their measurement has opened the door to a precision era for BBN [2].

From this 10% measurement of $(\mathrm{D/H})_P$, the baryon density can be inferred to about 10%, at $\Omega_B h^2 = 0.019$, or in terms of baryon-to-photon ratio, $\eta = 5.1 \times 10^{-10}$. With the baryon density in hand, one can predict the abundances of the other three light elements. Then, $^4$He and $^7$Li can test the consistency of BBN, D and $^3$He can probe stellar processing since BBN, and $^7$Li can test stellar models. Furthermore, a precise determination of the baryon density can make BBN an even sharper probe of particle physics (e.g., the limit to the number of light particle species).

To take full advantage of BBN in the precision era requires accurate predictions. The uncertainty in the deuterium-inferred baryon density comes in almost equal parts from the (D/H) measurement and theoretical error in predicting the deuterium abundance. The BBN yields depend upon the neutron lifetime and eleven nuclear cross sections (see Table I). In 1993, Smith, Kawano and Malaney (SKM) estimated the theoretical uncertainties [6]. While their work has set the standard since, it is not without its shortcomings: Treatment of systematic effects and correlated errors was neither uniform nor explicit. More importantly, data sets were not simply weighted by their reported errors; rather, subjective uncertainties were attached to *ad hoc* theoretical fits on the basis of scatter among the experiments. Finally, there have been new measurements [7–9].

After a careful analysis and updating of the microphysics for small but important effects, the theoretical uncertainty in the predicted $^4$He abundance has been reduced essentially to that in the neutron lifetime, $\Delta Y_P = \pm 0.001$ (95% C.L.) [10]. Motivated by the primeval deuterium measurement, we decided to refine the error estimates for the other light elements, using the nuclear data themselves and

TABLE I. For each reaction and nuclide, the energies (in keV, center of mass) at which the sensitivity functions for D and $^7$Li attain half their maximum value; these intervals indicate the energies relevant for BBN ($\Omega_B h^2 = 0.019$).–

| Reaction | D | $^7$Li |
|---|---|---|
| $p(n,\gamma)d$ | 25–200 | 17–153 |
| $d(p,\gamma)^3$He | 53–252 | 65–270 |
| $d(d,p)^3$H | 55–242 | 134–348 |
| $d(d,n)^3$He | 62–258 | 79–282 |
| $^3$He$(\alpha,\gamma)^7$Be | No effect | 157–376 |
| $^3$He$(d,p)^4$He | 187–325 | 107–283 |
| $^3$He$(n,p)^3$H | 52–228 | 24–188 |
| $^7$Li$(p,\alpha)^4$He | No effect | 57–208 |
| $^7$Li$(p,n)^7$Be | No effect | 1649–1690 |
| $^3$H$(\alpha,\gamma)^7$Li | No effect | 62–162 |
| $^3$H$(d,n)^4$He | 176–338 | 167–285 |

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Monte Carlo realization to make our error estimates. This method also allowed us to identify where improvements in the nuclear data would be most useful. We assume the standard BBN model (standard hot big-bang model plus standard model of particle physics). One might expect different cross-section dependences for other (e.g., inhomogeneous) models.

*Method and results.*—The details of our method are described in a longer paper [11]; here we outline the salient points. The nuclear inputs come in the form of measurements of cross sections, $\sigma(E)$, or, equivalently, the astrophysical $S$ factor, $S(E) = E\sigma(E)e^{2\pi\zeta}$, where $e^{-2\pi\zeta}$ is the Coulomb-barrier tunneling probability. From these, the needed thermally averaged reaction rates per particle follow

$$\langle \sigma v \rangle = \sqrt{\frac{8}{\pi \mu (kT)^3}} \int \sigma(E) E e^{-E/kT}\, dE\,, \qquad (1)$$

where $\mu$ is the reduced mass.

We use Monte Carlo realizations of all of the experimental data sets to determine thermal reaction rates and final yields. For each realization, we proceed as follows. For every data point from every data set, we draw a value from a Gaussian distribution whose mean is the central value and whose variance is the standard error reported for that point. We account for correlated normalization error in a data set by similarly drawing a value for the overall normalization. For each reaction, a smooth representation of $S(E)$ is obtained by fitting a piecewise spline to all of the data, with individual points weighted by their standard errors in the usual way. Using the spline fits, we evolve light-element abundances with a standard BBN code. From 25 000 such realizations, we produce distributions of the light-element yields and compute means and 95% C.L. intervals. Our results, as a function of the baryon density, are shown in Fig. 1.

Data points and uncertainties were extracted from a comprehensive review of the experimental literature from approximately the year 1945 onward, beginning with a careful reading of the original sources. We excluded a small number of data sets for which insufficient information for our technique was provided. Our source data remain almost identical to those of SKM, although we handle these data very differently.

As always, there is the sticky problem of systematic error, especially for cross sections represented by only a few measurements. A case in point is the reaction $^3$He$(\alpha,\gamma)^7$Be, which produces nearly all of the $^7$Li for $\eta = 5.1 \times 10^{-10}$. Activation measurements [12–14] show an apparent disagreement with prompt-photon measurements (see Fig. 2 and Ref. [15]). Because these measurements are not in the energy range of relevance for BBN, they have little influence on our results. (SKM omitted activation measurements from their analysis altogether.) We take them into account by performing a second Monte Carlo, where the prompt-photon measurements are renormalized by the weighted mean (and uncertainty) of the three activation measurements. This shifts the $^7$Li/H 95% C.L. interval upward by 11% (see Fig. 4 below).

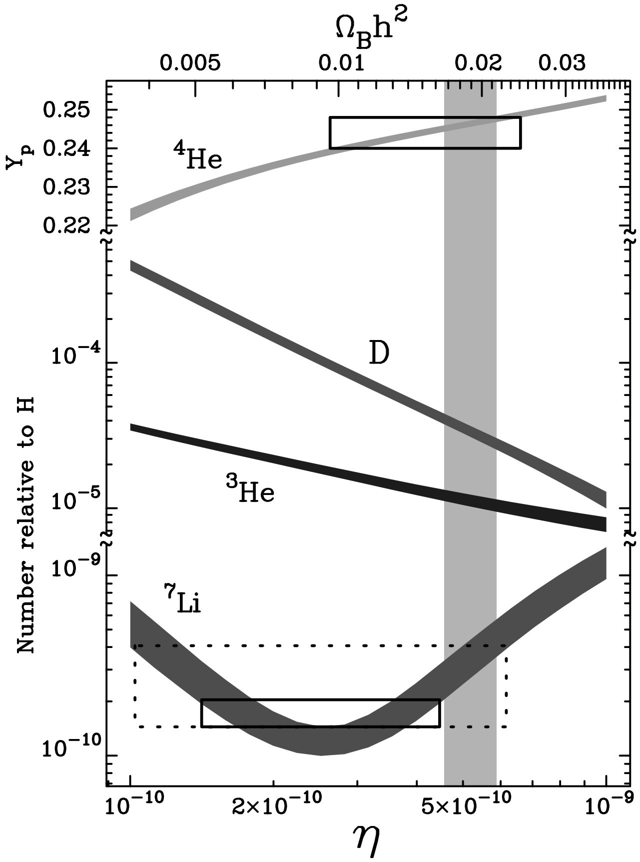


FIG. 1. Summary of the 95% confidence intervals for the BBN predictions for D, $^3$He, $^4$He, and $^7$Li. The $^4$He uncertainty comes from Ref. [10]. Solid boxes indicate 95% C.L. abundances from observation, as discussed in the text. The vertical band indicates the deuterium-inferred baryon density. The dashed box reflects possible $^7$Li depletion, discussed in the text.

Our method breaks down for the process $p + n \to d + \gamma$. This is because of a near-complete lack of data at the energies relevant for BBN. The approach used for this reaction is a constrained theoretical model that is normalized to high-precision thermal neutron capture cross-section measurements. In particular, we use the most recent evaluation, from ENDF-B/VI [16]. This evaluation was performed around 1970 (with a minor update in 1989), and it fitted a capture model to data of similar vintage for the neutron-proton system. No documentation survives, and the uncertainty is difficult to quantify—especially in light of known systematic problems with the likely input data [17]. (Efforts are underway to construct a new model for this reaction [18].) For consistency, we follow SKM and assign a 5% $1\sigma$ uncertainty in the overall normalization (also consistent with an estimate from the evaluation's authors [17]), and we use this value for our Monte Carlo calculations.

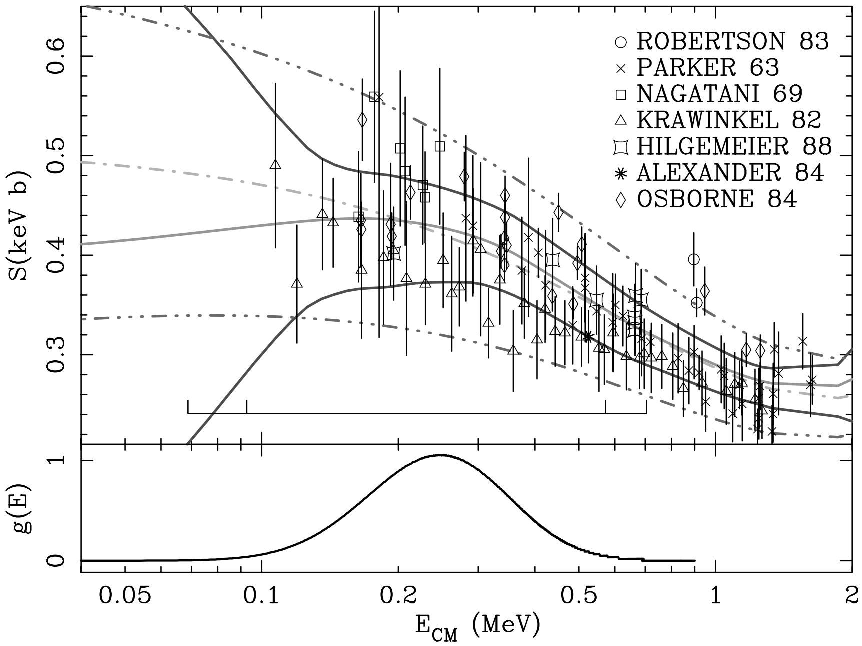


FIG. 2. The data for $^3He(\alpha,\gamma)^7Be$ are shown with our best fit and 95% C.L. interval for $S(E)$ (solid lines). The SKM fit and 95% C.L. interval are shown as dashed-dotted lines. The integration limits on the thermal averages needed for an accuracy of $0.1\sigma$ (inner tick marks) and one part in $10^5$ (outer tick marks) in the final yields are shown. $g(E)$ quantifies the sensitivity of the abundance of $^7Li$ to $S(E)$; $D$ is insensitive to this reaction. See Refs. [11,15] for detailed discussions of these data.

To investigate the role of each reaction independently, we ran the BBN code using the SKM rates for all but one reaction, studying that reaction alone with our Monte Carlo method. This produced, for each of the eleven key reactions, a best fit to the cross-section data, 95% C.L. intervals for the cross sections (Fig. 2), and 95% C.L. uncertainties for D and $^7Li$ yields for each reaction (see Figs. 3 and 4).

Our most important result is apparent: The uncertainty estimate from our method is a factor of 2 smaller than the SKM estimate. Not only have we reduced the theoretical error estimate, but we have also put it on a firmer footing.

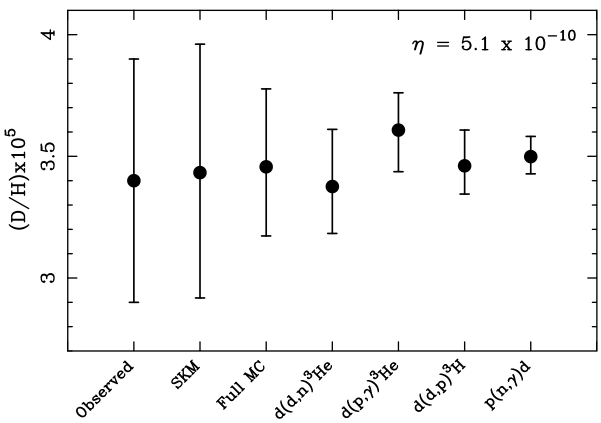


FIG. 3. Uncertainties in the predicted deuterium abundance from SKM, our full Monte Carlo, and individual reactions, compared with the Burles and Tytler [5] measurement. The uncertainties due to reactions not shown are much less important.

Our "most probable" yields also differ slightly (less than $1\sigma$) from the corresponding results of SKM. This reflects both differences in weighting the nuclear data and the inclusion of new data.

We computed "sensitivity functions" for the yields of D and $^7Li$ for each reaction. These functions measure the fractional changes in yield caused by a delta-function change in cross section at a given reaction energy (see Fig. 2 and Table I). The sensitivity functions quantify where precise cross-section measurements are required.

*Discussion and conclusions.*—We have reduced the theoretical error estimate for BBN deuterium production by a factor of 2. The deuterium determination of the

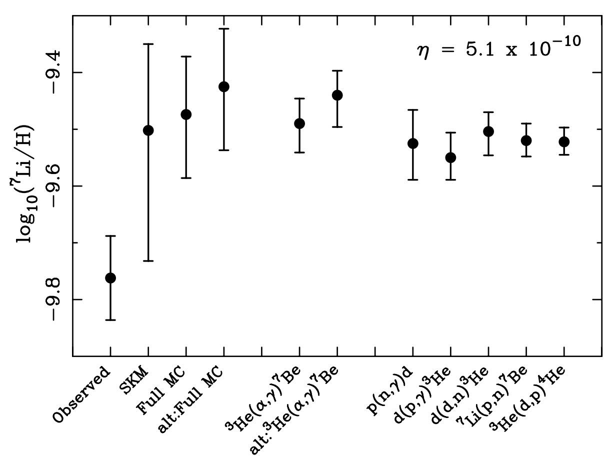


FIG. 4. The same as Fig. 3, but for $^7Li$. The results for the alternative normalization of $^3He(\alpha,\gamma)^7Be$ are also shown.

baryon density is thus sharpened, from 8% to 6% (at $1\sigma$), or $\Omega_B h^2 = 0.019 \pm 0.0024$ (95% C.L.), and the deuterium abundance itself dominates the uncertainty in baryon density. In the next five years, the precision of the primeval deuterium measurement should improve significantly because the Sloan Digital Sky Survey will increase the number of quasars with measured redshifts by a factor of almost 100, with a similar increase in the number of deuterium systems expected. Further improvement in the theoretical prediction is possible; the key reactions in this regard are $d(p,\gamma)^3$He, $d(d,p)^3$H above 100 keV, $d(d,n)^3$He above 100 keV, and $p(n,\gamma)d$ at 30–130 keV (see Fig. 3). Turning the deuterium determination of the baryon density into a few percent measurement will make possible a beautiful consistency test [2]: comparison with a similarly accurate measurement of the baryon density from microwave background anisotropy.

The deuterium-inferred baryon density leads to a prediction for the big-bang $^4$He mass fraction [10]: $Y_P = 0.246 \pm 0.001(\mathrm{D/H}) \pm 0.001(\tau_n) = 0.246 \pm 0.0014$ (all 95% C.L.). (More precisely, $Y_P$ is the helium baryon-number fraction, which differs from actual mass fraction by 0.5% here due to nuclear binding.) When the primeval $^4$He abundance is determined to three significant figures, this will be a powerful consistency test. At the moment, systematic effects dominate the error budget, in particular, underlying stellar absorption in the most metal-poor HII regions. Izotov and Thuan's sample [19,20] excludes the tainted or suspected-to-be-tainted systems, and they find $Y_P = 0.244 \pm 0.002$. This is consistent with the deuterium prediction. A less homogeneous sample [21], which includes some of the tainted systems, indicates a lower value, $Y_P = 0.234 \pm 0.002$, which is not consistent with the deuterium prediction.

Additional light particle species present around the time of BBN led to increased $^4$He production, and an upper limit to the primeval $^4$He abundance can be used to constrain their existence [22]. Using $Y_P = 0.244 \pm 0.002$, the deuterium-determined baryon density, and the prior $N_\nu \geq 3.0$, we derive the 95% C.L. limit, $N_\nu < 3.20$. One should be mindful that systematic error in $Y_P$ could change the limit, and that it will become more secure with better $^4$He measurements.

Finally, we turn to $^7$Li, the light element for which the uncertainty in the predicted abundance is largest. Our analysis has reduced the theoretical uncertainty by a factor of 2, though a small systematic uncertainty remains. Using our full Monte Carlo with the deuterium observations, we predict $(^7\mathrm{Li/H})_P = [3.5^{+1.1}_{-0.9} + 0.4(\mathrm{syst})] \times 10^{-10}$. The abundance derived from old, pop II halo stars is $(^7\mathrm{Li/H})_{\mathrm{pop\,II}} = [1.73 \pm 0.1(\mathrm{stat}) \pm 0.2(\mathrm{syst})] \times 10^{-10} = (1.73 \pm 0.3) \times 10^{-10}$ [23] (all at 95% C.L.). The discrepancy could represent a real inconsistency or merely a depletion of $^7$Li by a factor of about 2 in these stars (predicted by some models of stellar evolution [24,25]). A nuclear solution for the discrepancy is unlikely—a 25% (or $5\sigma$) change in the $p(n,\gamma)d$ rate would be required, and the unresolved systematics of $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ can only make the problem worse. There is still much room to improve the $^7$Li prediction; the key reactions are $p(n,\gamma)d$, $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$, $d(d,n)^3$He, and $d(p,\gamma)^3$He.

Perhaps the most rewarding result of this work is that we have verified what Schramm many times proclaimed, "the predictions of BBN are very robust because the key cross sections are measured at the energies where they are needed." In particular, if all eleven critical cross sections were set to zero outside the intervals where they are measured, the final light-element abundances would change by less than 10% of their current theoretical uncertainty.

This project was initiated with and inspired by David N. Schramm.

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