

## Skyrmions in Disordered Heterostructures

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We have investigated the effect of weak disorder on the ground state of a two-dimensional electron gas in the quantum Hall regime at filling factors slightly deviating from unity. The Skyrmions and anti-Skyrmions are found to be present even at filling factor  $\nu = 1$ . They may be strongly squeezed by the disorder. We have studied the effect of temperature on Skyrmion density and size. [S0031-9007(98)08190-3]

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Recently it has been proved that the lowest energy charged excitations of a two-dimensional electron gas (2DEG) in a quantum Hall regime at filling factor  $\nu = 1$  are Skyrmions [1]. These excitations involve many electrons and can be adequately described in the framework of a semiclassical  $\sigma$  model as the topologically nontrivial spin textures of an isotropic ferromagnet [2]. Owing to the incompressibility of the quantum Hall ground state the topological charge of a Skyrmion is equal to its electric charge [3].

There is a convincing experimental evidence for these collective excitations [4]. However, the experiments show only qualitative agreement with the theory [5,6]. It has been suggested [5,6] that the agreement will be better if the effects of disorder are taken into account. Despite the importance of the topic, very few theoretical studies [7,8] deal with Skyrmions in disordered heterostructures. The weak disorder limit has received no attention.

In this Letter we show that even a weak disorder gives rise to (anti-)Skyrmions even at filling factor  $\nu = 1$ . In the limit of a small Zeeman energy the Skyrmion radius is decreased by the disorder potential. We present the results for the (anti-)Skyrmion density at filling factors deviating from unity. We have found an interesting temperature effect on Skyrmion density and radius. We attempted to compare our results with experimental data of [6].

In the limit of small  $\tilde{g} \ll 1$ , which stands for the ratio between the Zeeman and exchange energy, Skyrmions can be described with an effective energy functional derived in [1]. The energy of a single Skyrmion as a function of its radius  $a$  reads [1,9]

$$\begin{aligned} E_{\text{sk}} &= E_0 + E_{\text{Zeeman}} + E_{\text{Coulomb}} \\ &= \frac{1}{4} \sqrt{\frac{\pi}{2}} \frac{e^2}{\epsilon l_B} + \frac{g \mu_B B a^2}{l_B^2} \ln\left(\frac{r_s}{a}\right) + \frac{3\pi^2}{64} \frac{e^2}{\epsilon a}, \end{aligned} \quad (1)$$

where  $E_0$  is the Skyrmion gap energy,  $l_B$  is the magnetic length,  $g$  is the electron  $g$  factor, and  $r_s \simeq l_B \tilde{g}^{1/2}$ . It has been assumed that  $a \gg l_B$ . Although the gap energy dominates the total energy, the radius is set by the two smaller terms depending on  $a$ . The equilibrium Skyrmion radius  $a_0$  can be found by minimizing the total energy

with respect to  $a$ . In this Letter we neglect the slowly changing logarithmic factor in the Zeeman energy and incorporate it to  $\tilde{g}$ .

The disorder is brought into a 2DEG mainly by the donors situated in a layer several hundreds of angstroms away from the electron layer. We incorporate the effect of disorder in a fourth term in the energy functional. For a Skyrmion at position  $\mathbf{r}_0$  in a random donor potential  $V(\mathbf{r})$  this energy reads

$$E_{\text{dis}}(\mathbf{r}_0) = \int d^2r \rho(\mathbf{r} - \mathbf{r}_0, a) V(\mathbf{r}), \quad (2)$$

where  $\rho(\mathbf{r}, a) = (a/r^2 + a^2)/\pi$  is the charge density of a Skyrmion with radius  $a$ . The potential  $V(\mathbf{r})$  is not screened by the electrons of the 2DEG, since at integer filling factors the 2DEG is an insulator. We assume the Gaussian distribution of  $V(\mathbf{r})$  with a correlator

$$\langle |V_q|^2 \rangle = \frac{U_0^2}{q^2}, \quad (3)$$

which is calculated by averaging over donor positions. Weak disorder requires  $U_0 \ll E_0$ . If we assume that donor positions do not correlate, we come up with an unrealistically big estimate of  $U_0 \simeq e^2/\epsilon d$ ,  $d$  being the distance between the 2DEG and the donor plane. In realistic heterostructures the observed disorder is known to be much weaker, possibly due to correlations between donor positions. Therefore, we assume that  $U_0 \ll e^2/d, E_0$ .  $U_0$  can be extracted from the experimentally measured mobility at zero magnetic field.

At a very qualitative level, our results are straightforward. At zero temperature it is energetically favorable for a Skyrmion to exist in the potential wells which are sufficiently deep (Fig. 1), so that  $E_{\text{dis}} < E_0$ . Since the disorder is weak, such wells are infrequent, so that the Skyrmion density will be much smaller than the electron density. These deep wells are expected to be very steep, which compresses the Skyrmion to make it fit into the well (see Fig. 1). This agrees with the preliminary results of computer simulations reported in [8].

At temperatures of the order of  $U_0$  the Skyrmions may appear in the wells which are less deep (left side of Fig. 1), which increases their density. Since the potential

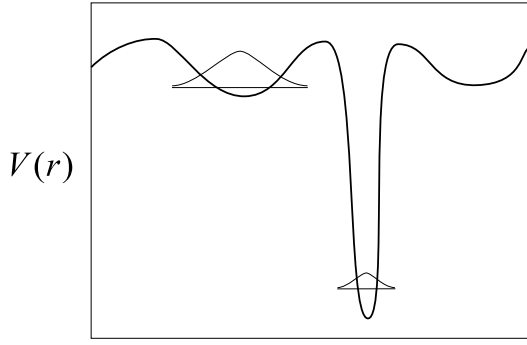


FIG. 1. Skyrmions in disorder potential. Their size must be small to fit into a deep well.

minima become less steep, the Skyrmion radius becomes bigger. The same consideration is valid also for anti-Skyrmions which are situated in the maxima of the random potential.

Let us make these simple ideas quantitative. To calculate the Skyrmion density at filling factor  $\nu = 1$  and zero temperature, we consider the following integral:

$$n_{\text{sk}}(E, a) = \int d^2r \sum_m \delta(\mathbf{r} - \mathbf{r}_m) \delta(a - a_m) \times \delta[E_{\text{dis}}(\mathbf{r}, a) - E] \int d^2r, \quad (4)$$

where  $n_{\text{sk}}(E, a)$  is the density of Skyrmions having potential  $E_{\text{dis}} = E$  with radius  $a$  per interval of energy and radius (which we can call Skyrmion *density of states*) and  $\{\mathbf{r}_m, a_m\}$  is a set of Skyrmion positions and radii for a certain  $V(\mathbf{r})$  configuration. These  $\{\mathbf{r}_m, a_m\}$  are not arbitrary but determined from the fact that the Skyrmions are

in equilibrium. Since we assume that Skyrmion density is small, we can consider isolated Skyrmions. It means that we disregard interactions between Skyrmions when they are in different wells, but due to strong repulsive interaction within the well there is only *one* Skyrmion in each well. In this case a Skyrmion interacts only with the disorder potential. This results in three conditions, two for Skyrmion position:  $\partial_x E_{\text{dis}} = \partial_y E_{\text{dis}} = 0$  and one for Skyrmion radius:  $\partial_a E_{\text{dis}} = -\partial_a E_{\text{sk}}$ . The actual density of Skyrmions is to be found by integrating  $n_{\text{sk}}(E, a)$  over all radii and  $E < E_{\text{sk}} \approx E_0$ .

To proceed we rewrite this integral as follows. To incorporate the equilibrium conditions we mentioned above, we transform  $\delta$  functions in terms of the three equilibrium conditions. The Jacobian of this transformation enters the integral:  $|\partial_{\alpha\beta}(E_{\text{sk}} + E_{\text{dis}})|$ , with  $\alpha, \beta = \{x, y, a\}$ . Then we average Eq. (4) at  $\mathbf{r} = 0$  over all possible  $V_q$  configurations. We have to take into account that Skyrmions are situated in minima, so that all eigenvalues of the Jacobian are positive. We express  $E_{\text{dis}}$  in terms of Fourier components:  $E_{\text{dis}}(0) = \sum_q E_{\text{dis}}(q)$ , with  $E_{\text{dis}}(q) = V_q \rho_q$  and  $\rho_q = aqK_1(aq)$  [1,2]. By adding extra variables we can rewrite all  $\delta$  and  $\theta$  functions in the form of exponents. Then integration over the random potential appears to be Gaussian and can be performed. In the course of averaging the terms like  $\sum_q \rho_q^2 \langle V_q^2 \rangle$  will appear. Formally they diverge at  $q \rightarrow 0$ . To deal with this divergence we have to recall that the interaction between Skyrmions is a long range one so that it becomes effectively important at large distances and Skyrmions screen out components of disorder potential having  $q < \sqrt{n_{\text{sk}}}$ , inverse distance between Skyrmions. So we cut off integration at  $q \approx \sqrt{n_{\text{sk}}}$ .

After integration over extra variables the expression for density of states reads

$$n_{\text{sk}}(E, a) = \frac{2^{15} \pi^{10} a^3}{\sqrt{3}(4\alpha - 3)^{7/2} U_0^4} \left( -\frac{4}{3} \frac{E}{a^2} + \frac{\partial_a E_{\text{sk}}}{a} (4\alpha - 1) + \partial_a^2 E_{\text{sk}} (4\alpha - 3) \right) \left( \frac{\partial_a E_{\text{sk}}}{a} (3 - 6\alpha) + \frac{E}{a^2} \right)^2 \times \exp\left( -24\pi \frac{\frac{1}{3}E^2 - E\partial_a E_{\text{sk}}a + (\partial_a E_{\text{sk}})^2 a^2 \alpha}{U_0^2(4\alpha - 3)} \right), \quad (5)$$

where

$$\alpha = 2\pi \sum_q \frac{\rho_q^2}{q^2} \approx -\ln(a\sqrt{n_{\text{sk}}}). \quad (6)$$

Here we take into account that the sum in Eq. (6) diverges and must be cut off at  $q \approx \sqrt{n_{\text{sk}}}$ . We assume that  $a\sqrt{n_{\text{sk}}} \ll 1$ . So that  $\alpha$  depends on Skyrmion concentration and Eqs. (5) and (6) form together a self-consistency problem.

Now we evaluate the radius  $a$  which minimizes the exponent of Eq. (5). We substitute the Skyrmion energy of Eq. (1) in the exponent of Eq. (5). It appears that there are two limits corresponding to the strength of the Zeeman energy

$$a_{\text{opt}} = \begin{cases} \left(\frac{3\pi^2}{2^7 \tilde{g}}\right)^{-1/3} l_B = a_0, & \tilde{g}^{1/3} \frac{E_0}{U_0} \gg 1, \\ \frac{3\pi^{3/2}}{2^{5/2}} \alpha l_B \ll a_0, & \tilde{g}^{1/3} \frac{E_0}{U_0} \ll 1, \end{cases} \quad (7)$$

where  $a_0$  is the Skyrmion radius in the absence of a random potential field.

Now we can proceed with calculation of the total density. We expand the exponent around  $a = a_{\text{max}}$  and  $E = -E_{\text{sk}}$ . We restrict ourselves to the more interesting limit of small Zeeman energy:  $\tilde{g}^{1/3} \frac{E_0}{U_0} \ll 1$ . In this limit we encounter again two different limits depending on the strength of the Zeeman energy. The point is that the prefactor, which is in fact the determinant  $|\partial_{\alpha\beta}(E_{\text{sk}} + E_{\text{dis}})|$ , vanishes at  $a = a_{\text{opt}}$  in the first order approximation. In both cases the exponent is the same,  $n_{\text{sk}} \propto \exp(2\pi E_0^2/\alpha)$ . This is sufficient to solve the self-consistency problem. The leading approximation to  $\alpha$  is  $\alpha_0 = \sqrt{\pi} \frac{E_0}{U_0}$ ; the next order approximation is necessary to evaluate the prefactor. Finally we obtain for the density

$$n_0 = \begin{cases} \gamma_1 (\frac{U_0}{E_0})^{9/4} n_e \exp(-2\sqrt{\pi} \frac{E_0}{U_0}), & \tilde{g}^{1/3} \frac{E_0}{U_0} \ll (\frac{U_0}{E_0})^{1/3}, \\ \gamma_2 \tilde{g} (\frac{E_0}{U_0})^4 (\frac{U_0}{E_0})^{9/4} n_e \exp(-2\sqrt{\pi} \frac{E_0}{U_0}), & \tilde{g}^{1/3} \frac{E_0}{U_0} \gg (\frac{U_0}{E_0})^{1/3}, \end{cases} \quad (8)$$

where  $n_e$  is the electron density in the 2DEG,  $\gamma_1 \approx 9 \times 10^{-4}$ , and  $\gamma_2 \approx 6 \times 10^3$  are numerical factors. Here we have taken into account that the number of Skyrmions at  $\nu = 1$  is equal to the number of anti-Skyrmions, and those two equally contribute to  $n_{\text{sk}}$ .

Therefore we find three regimes in dependence of the relative strength of the Zeeman energy with respect to disorder. The most probable Skyrmion radius is equal to its disorderless value at  $\tilde{g}^{1/3} \frac{E_0}{U_0} \gg 1$  and to  $a_{\text{opt}} \approx \frac{9\pi^2}{4} \sqrt{\frac{E_0}{2U_0}} l_B \ll a_0$  in the opposite limit. At  $\tilde{g}^{1/3} \frac{E_0}{U_0} \approx (\frac{U_0}{E_0})^{1/3}$  the prefactor changes. Let us stress that in both cases  $a \gg l_B$  and  $\tilde{g} \ll 1$  so that the conditions of validity of Eq. (1) are satisfied.

Let us evaluate Skyrmion densities at filling factors slightly deviating from unity. In the absence of disorder, Skyrmion and anti-Skyrmion densities are proportional to the filling factor deviation (Fig. 2),  $n_{\text{sk,ask}} = \pm \delta f \theta(\pm \delta f)$ , where  $\delta f = n_e(\nu - 1)$ . This is no longer correct in the presence of disorder potential. We note that at unity filling factor the chemical potential lies precisely in the middle of the gap between Skyrmion and anti-Skyrmion states. If the filling factor deviates from unity, we have to recalculate the densities for a shifted chemical potential  $\mu$ . Fortunately, this is simple. The density of states exhibits exponential dependence on energy,  $n_{\text{sk}}(E) \propto \exp(-E/\Omega)$ ,  $\Omega \equiv 2\alpha U_0^2/E_0$ . This is why the Skyrmion density is changed by a factor  $\exp(-\mu/\Omega)$ , whereas the anti-Skyrmion one by  $\exp(\mu/\Omega)$ . Attention should be paid to the fact that the change of the chemical potential also affects  $\alpha$ . We note that the total charge per unit area must correspond to the filling factor, that is,  $n_{\text{sk}} - n_{\text{ask}} = \delta f$ . This allows us to express  $\mu$  in terms of  $\delta f$  and obtain for the densities

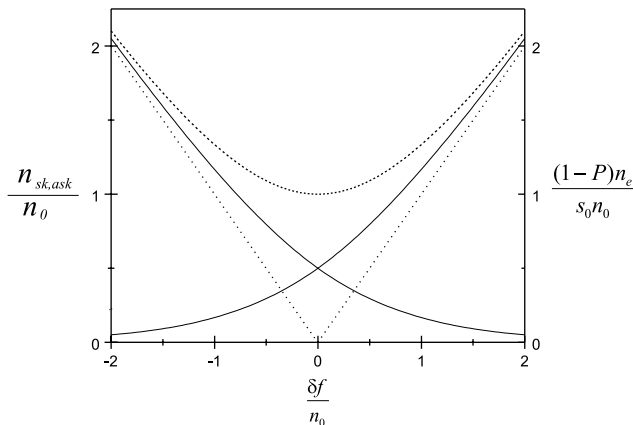


FIG. 2. Skyrmion densities versus filling factor.

$$\begin{cases} n_{\text{sk}} \\ n_{\text{ask}} \end{cases} = \sqrt{\frac{\delta f^2}{4} + \frac{n_0^4}{\delta f^2 + \sqrt{\delta f^4 + 16n_0^4}}} \pm \frac{\delta f}{2}. \quad (9)$$

These densities are plotted in Fig. 2, together with the ratio of the total number of spin flips  $n_{\text{sk}} + n_{\text{ask}}$  and the number of spin flips at  $\delta f = 0$ . The latter quantity can be observed experimentally since it determines the rounding of the spin polarization peak [4].

Let us consider the effect of a finite temperature. Still we have to assume that the Skyrmion density is much smaller than the electron density, that is,  $T \ll E_0$ . Also the temperature should be low enough for Coulomb interaction to make it impossible for two Skyrmions to occupy the same potential well. Under these circumstances the Skyrmions effectively behave as fermions, so we can make use of Fermi statistics [10]. To calculate the total density at  $\mu = 0$  we integrate the density of states of Eq. (5) multiplied by the Fermi distribution function,  $n_{\text{sk}}(E, a) \times f(E - E_0)$  (see Figs. 3a and 3b). If we expand the exponent in  $n_{\text{sk}}(E)$  near  $E = E_{\text{sk}}$  yielding a term linear in  $E - E_{\text{sk}}$ , we see that the integral converges only at  $T < T_c, T_c = U_0/4\sqrt{\pi}$ .

Of course, it does not mean that the density actually diverges at  $T = T_c$ . Rather, it indicates an exponential temperature dependence of the density at  $T > T_c$ . To reveal this one, we go back to the expression (5) and consider the most important term in the exponent which is quadratic in energy. This term competes with the exponent of the Fermi distribution tail. The minimum is achieved at  $E = E_{\text{min}} = \alpha T_c^2 T$ . Solving the self-consistency problem gives  $\alpha(T) = E_0/(2T + 2T_c^2/T)$ .

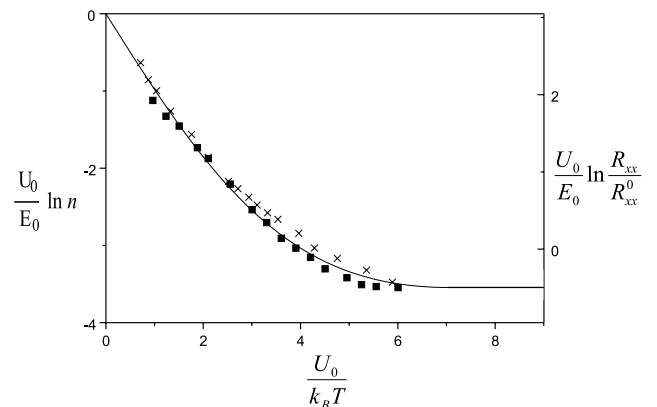


FIG. 3. Temperature dependence of the Skyrmion density. Solid line presents our theoretical results; the squares and crosses present the fitted data of Ref. [6].

At the critical temperature  $E_{\min} = E_0$ , as it should be. The density at  $T > T_c$  thus reads

$$n(T) \propto \exp\left(-E_0 \frac{T}{T^2 + T_c^2}\right) \quad (10)$$

and exhibits a non-Arrhenius behavior. The logarithm of the density is plotted in Fig. 3 versus inverse temperature. The optimal size of a (anti-)Skyrmion can be extracted from the exponent (5). At  $T > T_c$  and it grows with temperature,  $a(T) = a(0)T/T_c$ , the latter is valid till  $a(T) \ll a_0$ . This is because the potential minima become less steep at higher energies.

With our method, we can also obtain the finite temperature results for  $\nu \neq 1$ .

Our results can be checked with spin polarization measurements. Indeed, the finite density of (anti-)Skyrmions at  $\nu = 1$  would manifest itself as a rounding of the spin polarization peak. The reduction of their size  $a$  due to disorder and its restoration at  $T > T_c$  can be detected as a change of Skyrmion spin  $\propto a^2$ . However, this check requires a very accurate measurement of the spin polarization peak in the close vicinity of  $\nu = 1$  at different temperatures, that has not yet been performed.

As to transport measurements, we attempted to compare our results with experimental data of Ref. [6]. We assume the longitudinal resistivity to be proportional to the Skyrmion density:  $\rho_{xx} \sim n(T)$  and the temperature dependence of  $n(T)$  to be dominated by the exponent (10). In Fig. 3 the experimental data are fit to the theoretical curve, with fit parameters  $U_0 \approx 15$  K and  $E_0 \approx 12$  K. Although  $U_0 \approx E_0$ , we expect our theory to be qualitatively true since the numerical factors provide big exponents. Indeed, in this case  $T_c \approx 2$  K  $\ll E_0$ . From the given mobility we extract  $U_0 \approx 10$  K.

However, most of the transport measurements [5] exhibit no saturation of  $\rho_{xx}$  down to 1 K. Although this can be explained by better quality of the heterostructures ( $U_0 < 3$  K), we hesitate to make a point out of our fit. The point is that all transport measurements, whatever interpreted, give an estimation of  $E_0$  which is an order of magnitude smaller than the theoretical value. Possibly this indicates that the assumption  $\rho_{xx} \sim n(T)$  is no good. Our results explain how this may happen. Indeed, even at zero temperature and  $\nu = 1$  we have a finite concentration of (anti-)Skyrmions. What may be thermally acti-

vated is their *mobility* rather than concentration. It is no surprise since they have to overcome high potential barriers when moving between potential minima.

We stress that we have considered a *microscopic* disorder assuming that the sample is homogeneous at *macroscopic* scale. Long range inhomogeneities of the filling factor and disorder that might be present in a sample would significantly complicate the interpretation of the experimental results.

In conclusion, we have developed the theory of Skyrmions in weakly disordered heterostructures. The Skyrmions appeared to be present at filling factor  $\nu = 1$ . The disorder and temperature strongly affect their density and size.

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