# On Conservation of Helicity and Energy of Reflecting Electron Magnetohydrodynamic Vortices

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The reflection of a magnetic vortex from a conducting boundary is studied experimentally in an electron magnetohydrodynamic plasma. The reflection conserves energy but not helicity, which reverses sign. Field line slippage occurs in a thin boundary layer, creating a divergence in the ac helicity flow vector yet negligible dissipation. The change in self-helicity is accounted for by the volume term $-2 \int \tilde{\mathbf{E}} \cdot \tilde{\mathbf{B}} \, dV$ which, for Hall electric fields, does not produce dissipation. [S0031-9007(99)09171-1]

Helicity, defined by $H = \int \mathbf{A} \cdot (\nabla \times \mathbf{A}) \, dV$, is a fundamental quantity which is used to characterize the topology of a vector field, for example, that of the magnetic field $\mathbf{B} = \nabla \times \mathbf{A}$ in plasmas [1]. Helicity properties are important in space plasma physics (reconnection [2], turbulence [3]), magnetic confinement devices (tokamaks [4], spheromaks [5], reverse field pinches [6]), and in electron magnetohydrodynamic (EMHD) plasmas [7–9]. In ideal (collisionless) plasmas magnetic energy and helicity are conserved quantities [1]. In nonideal plasmas, Taylor has conjectured that magnetic helicity relaxes at a slower rate than energy when the dissipation is highly localized (e.g., in current sheets) [10]. Near conducting boundaries, the conservation of magnetic helicity has been questioned [11]. Likewise, the assumption of line tying at boundaries [12] is debatable [13]. In this Letter we present experimental results showing that magnetic helicity changes drastically while the energy does not, i.e., the opposite of Taylor's conjecture [10], whose generality has also been questioned theoretically [11]. This situation arises in an EMHD plasma when a propagating magnetic vortex $\tilde{\mathbf{B}}(\mathbf{r}, t)$ (a bounded whistler wave packet), superimposed on a uniform background magnetic field $\mathbf{B}_0$, reflects at a highly conducting boundary. Reflection preserves energy but reverses the sign of helicity. The latter is consistent with a property of EMHD vortices that the helicity sign depends on the direction of propagation along a uniform background magnetic field $\mathbf{B}_0$ [14].

One can separate the total magnetic helicity into two terms, $H_{\mathrm{tot}} = H_{\mathrm{self}} + H_{\mathrm{mutual}} = \int (\mathbf{A}_0 + \tilde{\mathbf{A}}) \cdot (\mathbf{B}_0 + \tilde{\mathbf{B}}) \, dV = \int \tilde{\mathbf{A}} \cdot \tilde{\mathbf{B}} \, dV + \int \mathbf{A}_0 \cdot \tilde{\mathbf{B}} \, dV$, where $\mathbf{A}$ is the vector potential and $\mathbf{B}$ is the magnetic induction. The first term describes the topology of the vortex, the latter the linkage of the vortex fields (denoted by a tilde) with the dc field. Analogous to Poyntings theorem for the energy change, the relaxation of the total magnetic helicity can be accounted for by a flow and a volume term, $\partial H / \partial t = -\oint (\mathbf{A} \times \partial \mathbf{A}/\partial t + 2\phi \mathbf{B}) \cdot d\mathbf{a} - 2 \int \mathbf{E} \cdot \mathbf{B} \, dV$, where $\phi$ is the scalar potential. If flows through boundaries are negligible, helicity can change only through the volume term. For $\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{J}$, helicity is dissipated by resistivity, $\eta$, since for $H > 0$ one has $\mathbf{J} \parallel \mathbf{B}$; hence $\partial H / \partial t \propto -\eta \int J^2 \, dV < 0$. No helicity change is produced by the $\mathbf{v} \times \mathbf{B}$ term, where $\mathbf{v}$, for EMHD, is the electron fluid velocity.

In the present work we first show that during reflection the magnetic self-helicity of the vortex is not a conserved quantity. Second, the larger mutual helicity is also not conserved. The lack of helicity conservation is connected with the presence of a vacuumlike sheath between plasma and conductor where the field lines are not tied to a conducting medium. Field lines frozen into the plasma and anchored to the conducting boundary cannot be connected uniquely. Slippage of field lines allows the topology to change.

The experiments are performed in a large, uniform, quiescent, magnetized afterglow plasma with parameters indicated in Fig. 1a. The plasma is produced by a pulsed discharge with a large oxide-coated cathode. A current pulse (10 A, $t_{\mathrm{rise}} \simeq 0.1 \ \mu$s, $t_{\mathrm{pulse}} \simeq 5 \ \mu$s), applied to a toroidal antenna ($r_{\mathrm{major}} \simeq r_{\mathrm{minor}} \simeq 4$ cm) excites in the plasma a magnetic perturbation, $\tilde{\mathbf{B}}(\mathbf{r}, t)$. The perturbation is measured with a triple magnetic probe ($\tilde{B}_x, \tilde{B}_y, \tilde{B}_z$ vs $t$), movable in three-dimensional (3D) space, and its spatial evolution is obtained from repeated experiments. The current density is obtained from Ampère's law, $\mathbf{J} = \nabla \times \mathbf{B}/\mu_0$ ($\gg \varepsilon_0 \partial \mathbf{E}/\partial t$). The topology of the perturbed field is that of a 3D vortex, consisting of a linked toroidal field ($\tilde{B}_\theta$ or $\tilde{B}_x, \tilde{B}_y$) and a dipolar or poloidal field ($\tilde{B}_y, \tilde{B}_z$), snapshots of which are shown in two orthogonal planes in Figs. 1b,c. In time, the vortex propagates in the whistler mode guided by the uniform dc field $\mathbf{B}_0$ ($B_0 > \tilde{B}$), which, together with $\omega$ and $\omega_{pe}$, determines the whistler speed. For propagation against $\mathbf{B}_0$, the field linkage is left-handed, i.e., the helicity is negative. The vortex reflects normally from a large conducting plate (30 cm diam, 1 mm thick, high conductivity Al).

Figure 2 demonstrates the propagation, reflection, and field reversal of the vortex. Contours of the field component $\tilde{B}_x$ ($= \tilde{B}_\theta$ for $x = 0, y < 0$) are shown vs axial position and time. The antenna, centered at $z = 0$, launches two vortices, one along $\mathbf{B}_0$, the other opposite to $\mathbf{B}_0$. They have the same sign for $\tilde{B}_\theta$, but the opposite sign for $\tilde{B}_z$, hence opposite self-helicity densities, such that
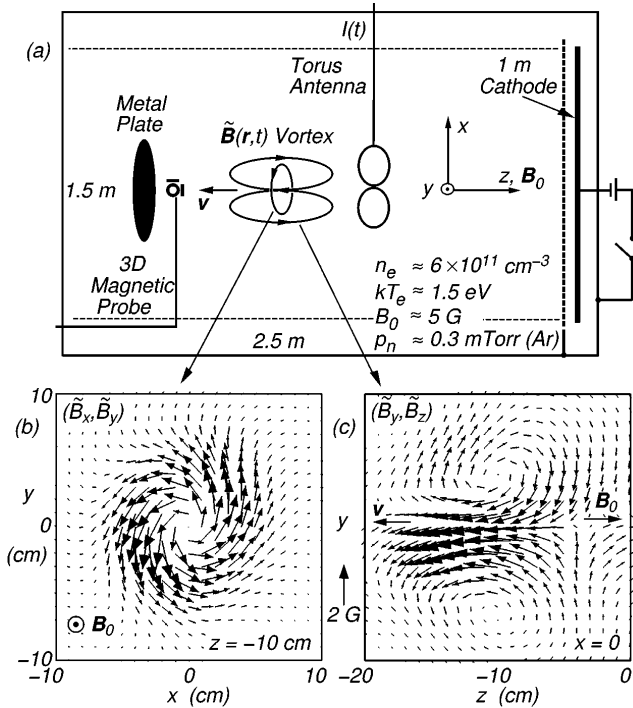
FIG. 1. (a) Experimental setup and basic plasma parameters. (b) Toroidal ($\tilde{B}_x, \tilde{B}_y$) and (c) poloidal ($\tilde{B}_y, \tilde{B}_z$) magnetic field components of a 3D vortex propagating in the whistler mode along a background magnetic field $\mathbf{B}_0$.

the total self-helicity is zero, consistent with no injection of helical fields. (Note that the applied field $\tilde{B}_\theta$ links with the dc field $\mathbf{B}_0$, hence injects mutual helicity, and both vortices have positive mutual helicity consistent with helicity conservation.) As the pulses propagate, their amplitudes slowly decrease due to (i) slow expansion of the vortex across $\mathbf{B}_0$ and (ii) weak collisional damping. At $t \simeq 0.35\ \mu$s, the left-traveling vortex reflects at the metal plate. No fields are detected at the backside of the metal plate. No currents flow from the metal plate to ground, implying no potential changes of the plate. The fields
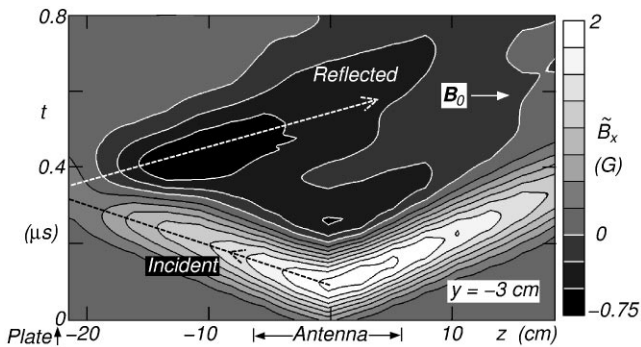


FIG. 2. Toroidal magnetic field component, $\tilde{B}_x = \tilde{B}_\theta$, vs axial position and time, demonstrating the propagation and reflection of a vortex at a conducting plate. Note the sign reversal of $\tilde{B}_\theta$ upon reflection.

inside the antenna ($|z| < 4$ cm) have been extrapolated since the probe cannot scan through the torus.

Figure 3 compares the helicity densities of the incident and reflected vortices as they traverse an $x$-$y$ plane at 12 cm from the reflector plate. The magnetic field components (Fig. 3a) show that only $\tilde{B}_\theta$, but not $\tilde{B}_z$, reverses sign upon reflection, implying reversal of self, mutual, and total magnetic helicities. Figure 3b shows contours of the helicity density of the current density, $\tilde{\mathbf{J}} \cdot \tilde{\mathbf{B}}$, and Fig. 3c shows that of the electron fluid vorticity, $\tilde{\mathbf{v}}_e \cdot \tilde{\boldsymbol{\omega}}_e = \tilde{\mathbf{J}} \cdot (\nabla \times \tilde{\mathbf{J}})/n^2 e^2$, all of which change sign, i.e., are not conserved. However, the magnetic energy density of the reflected pulse is not changed compared to that of a nonreflected vortex propagating over the same distance as the reflected vortex. As pointed out earlier [9], collisional damping of a propagating vortex decreases energy and helicities at the same rate.

Before addressing the physics of helicity reversal it is useful to review briefly the relation between fields and currents in a vortex propagating in a uniform
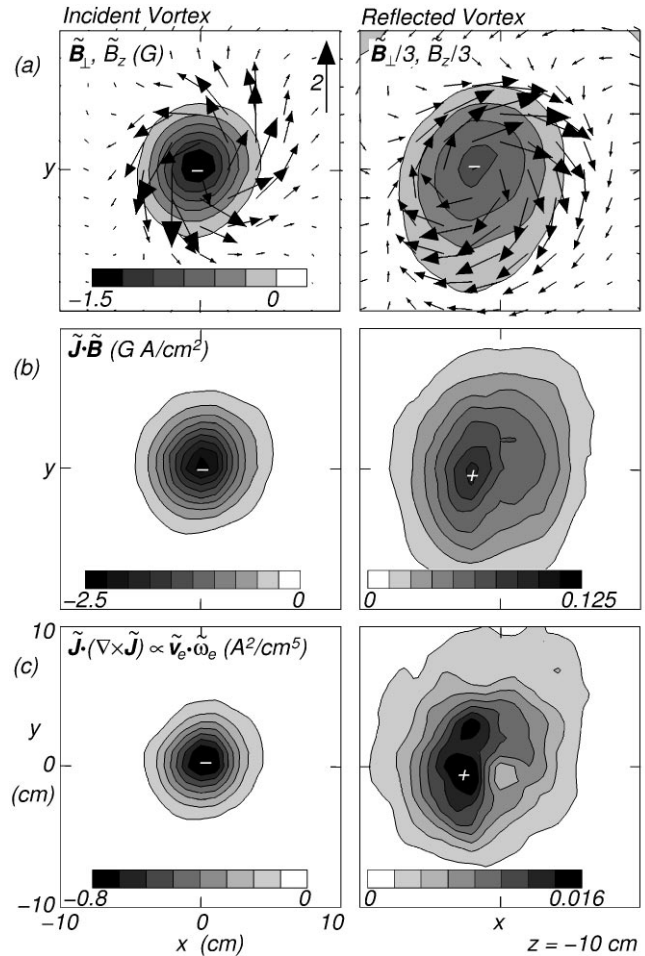


FIG. 3. Helicity properties of incident and reflected vortex for (a) the perturbed magnetic field $\tilde{\mathbf{B}}$, (b) the current density $\tilde{\mathbf{J}}$, and (c) the electron fluid vorticity $\tilde{\boldsymbol{\omega}}_e = \nabla \times \tilde{\mathbf{v}}_e$. All helicities reverse sign upon reflection.

magnetoplasma. In a stationary plane ($z = $ const) the propagating vortex produces a rising and decreasing field $\tilde{B}_z$, hence an inductive field $\tilde{E}_\theta$ of opposite polarity in the first and second half of the vortex. $\tilde{E}_\theta$ produces radial Hall currents $\tilde{J}_r = ne\tilde{E}_\theta/B_0$, which close via field-aligned currents $\tilde{J}_z$ to form the poloidal current system and the toroidal field $\tilde{B}_\theta$. The time variation of $\tilde{B}_\theta$ creates a radial electric field $\tilde{E}_r$, which produces a toroidal Hall current $\tilde{J}_\theta = ne\tilde{E}_r/B_0$ and poloidal magnetic field $(\tilde{B}_r, \tilde{B}_z)$.

As the vortex interacts with the conducting boundary the field topology changes. Figure 4a shows the field components in an $x$-$y$ plane at $\Delta z \simeq 1$ cm in front of the plate. Throughout the pulse, the transverse field $\tilde{\mathbf{B}}_\perp = (\tilde{B}_x, \tilde{B}_y)$ points essentially radially outward. This is different from the predominantly toroidal field inside a propagating vortex (see Fig. 1b). The axial field, $\tilde{B}_z$, is weaker than without the plate because the boundary condition implies $\tilde{B}_z \to 0$ at the conductor surface. This is due to induced surface currents in the plate, $\mathbf{K} = \hat{\mathbf{n}} \times \tilde{\mathbf{B}}/\mu_0$, shown in Fig. 4b. Thus, since the normal dipolar field of the vortex, $\hat{\mathbf{n}} \cdot \tilde{\mathbf{B}}$, cannot penetrate the conductor, the field lines flare out tangentially to the surface as shown in the schematic picture of Fig. 4c. The absence of $\tilde{B}_\theta$ near the surface implies that no current flows in or out of the plate. The axial plasma current closes radially in front of the plate. This radial Hall current is driven by a toroidal inductive electric field $\tilde{E}_\theta \propto -\partial \tilde{B}_z/\partial t$. The temporal rise and fall of
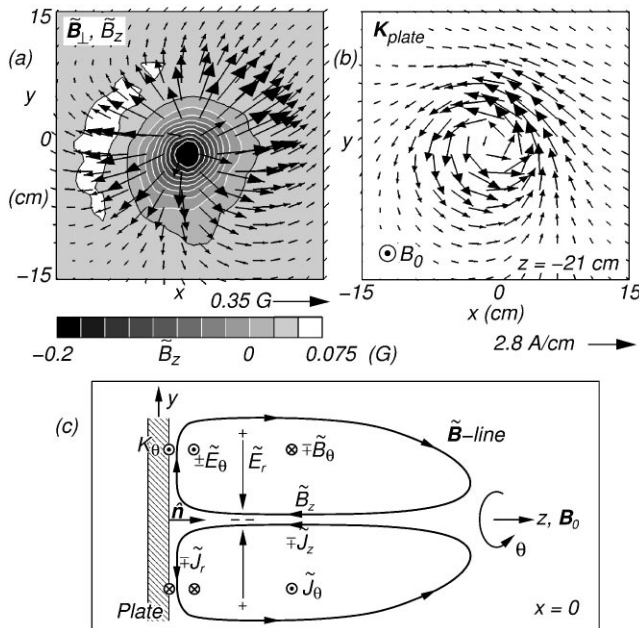


FIG. 4. (a) Magnetic field components in an $x$-$y$ plane at $\Delta z \simeq 1$ cm in front of the reflector plate. The field is tangential to the plate and lost its helicity. (b) Induced surface current in the conducting plate. (c) Schematic picture of field components during vortex reflection.

$\tilde{B}_z(t)$ of the approaching/receding vortex creates a sign reversal in $\tilde{E}_\theta$, hence also in the poloidal current $(\tilde{J}_r, \tilde{J}_z)$, and toroidal field $\tilde{B}_\theta$, and thereby in the helicity. The time dependence $\tilde{B}_{\text{tor}} \propto -\partial \tilde{B}_{\text{pol}}/\partial t$ occurs only because of the stagnation of the vortex in front of the plate where the front of the incident vortex turns into the tail of the reflected vortex, and vice versa. Helicity reversal is a self-consistent outcome of the electrodynamics of vortex reflection. Also, continuity of the toroidal electron drift satisfies angular momentum conservation, while reversal of the axial drift is consistent with linear momentum change during a reflection.

We now analyze the change in self-helicity in terms of flows and volume terms. The vortex lies within a volume bounded by the conducting plate and other distant surfaces on which the vortex fields are negligibly small. Helicity injection from the conducting plate can be ruled out since both the normal magnetic field and the tangential electric field vanish at its surface, $(\tilde{\mathbf{A}} \times \tilde{\mathbf{E}}_{\text{ind}}) \cdot d\mathbf{a} = 0$, $\tilde{\mathbf{B}} \cdot d\mathbf{a} = 0$. However, the volume term, $-2 \int \tilde{\mathbf{E}} \cdot \tilde{\mathbf{B}} \, dV$, is positive throughout the reflection process since $\tilde{B}_r > 0$ (see Fig. 4a) and $\tilde{E}_r < 0$. The latter drives toroidal Hall currents $\tilde{J}_\theta = ne\tilde{E}_r/B_0 < 0$, which produce the axial field $\tilde{B}_z < 0$. The contribution from $-2\tilde{E}_z\tilde{B}_z > 0$ is negligibly small, $|\tilde{E}_z\tilde{B}_z/\tilde{E}_r\tilde{B}_r| \simeq |\tilde{B}_z/B_0| \ll 1$. Note that $\tilde{E}_z > 0$ is necessary to balance the magnetic force $\tilde{\mathbf{J}} \times \tilde{\mathbf{B}} \simeq \tilde{J}_\theta\tilde{B}_r\hat{\mathbf{z}}$ since at the moment of reflection $\tilde{J}_z \to 0$. With $\partial H_{\text{self}}/\partial t > 0$, the initially negative self-helicity can reverse sign as demonstrated quantitatively in Fig. 5. Contours of $\partial^2 H_{\text{self}}/\partial z \partial t = -\int \tilde{E}_r\tilde{B}_r 2\pi r \, dr$, displayed in a $z$-$t$ diagram (Fig. 5a), show that the traveling incident vortex has alternating signs in its first and second half; hence axial integration produces no helicity change. Here, $\tilde{E}_r$ is calculated from the Hall current $\tilde{J}_\theta$. However, in the stagnation region in front of the plate the volume term is large and positive throughout the reflection process. After integrating the helicity change over $z$ and $t$ one obtains $H_{\text{self}}(t) - H_{\text{self}}(0) = -2 \int_0^t dt' \int \tilde{E}_r\tilde{B}_r 2\pi r \, dr \, dz$, displayed in Fig. 5b. The initial helicity is that of the incident vortex which is found to be $H_{\text{self}}(0) \simeq 2\Phi_{\text{tor}}\Phi_{\text{pol}} \simeq 2 \int \tilde{B}_\theta \, dr \, dz \int \tilde{B}_z 2\pi r \, dr \simeq -300$ G$^2$ cm$^4$. The volume term changes the helicity by $\Delta H \simeq +550$ G$^2$ cm$^4$ during the reflection process, which reasonably accounts for the final positive self-helicity of the reflected pulse, $H_{\text{self}}(t > t_{\text{refl}}) \simeq +250$ G$^2$ cm$^4$. Energy dissipation is negligible because $\tilde{E}_r \perp \tilde{J}_\theta$ and $\langle \tilde{E}_r\tilde{J}_r(t) \rangle \simeq 0$ due to the sign reversal of $\tilde{J}_r$.

The reversal of the larger mutual helicity can be explained by the divergence of the ac helicity flow vector, $\mathbf{A}_0 \times \partial \tilde{\mathbf{B}}/\partial t = -\mathbf{A}_0 \cdot (\nabla \times \tilde{\mathbf{E}}_{\text{ind}}) = \nabla \cdot (\mathbf{A}_0 \times \tilde{\mathbf{E}}_{\text{ind}}) - \tilde{\mathbf{E}}_{\text{ind}} \cdot \mathbf{B}_0$. The last term integrates to zero and the first term, which accounts for the change in toroidal field $\tilde{B}_\theta$, leads to a radial and axial outflow of mutual helicity, $\partial H_{\text{mut}}/\partial t = \oint (\mathbf{A}_0 \times \tilde{\mathbf{E}}_{\text{ind}}) \cdot d\mathbf{a} < 0$, near the plate, hence a decrease of the incident positive
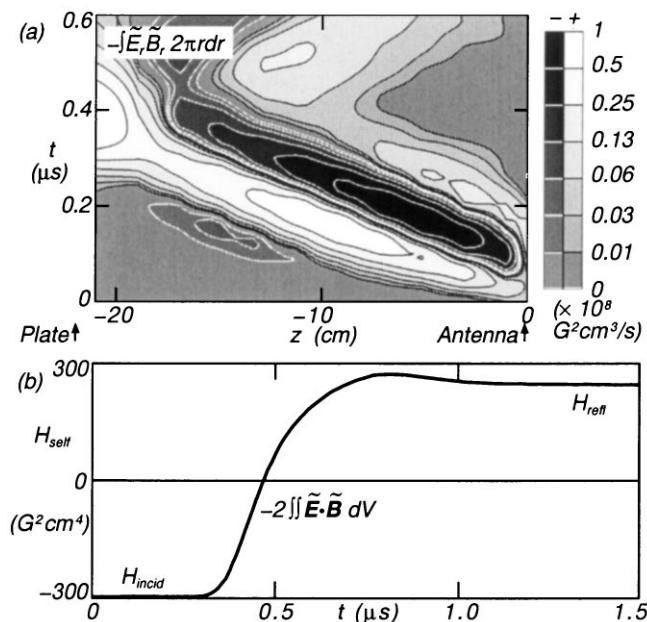
FIG. 5. (a) Change of the magnetic self-helicity, $\partial^2 H_{\text{self}}/\partial z \partial t$ vs $z$, $t$. While propagating the positive and negative changes average to zero, during reflection the large positive change near the plate causes helicity reversal. (b) Magnetic self-helicity vs time. The initial value is that of the incident vortex, the final value that of the reflected vortex.

helicity and the creation of a negative helicity for the reflected vortex. If one reverses the applied antenna field, the incident vortex would have a negative mutual helicity, $\partial H_{\text{mut}}/\partial t > 0$, and the reflected pulse would have positive mutual, self, and total magnetic helicity.

Finally, we address the helicity aspect in terms of the "frozen-in" concept of field lines. Since the vortex fields are smaller than the dc field, the net field lines $\tilde{\mathbf{B}} + \mathbf{B}_0$ exhibit a small bulge and twist in the region of the vortex. Ahead of the propagating vortex a typical field line is the straight, unperturbed $\mathbf{B}_0$ which is thought to be frozen into the stationary plasma and conductor. Inside the vortex, the field line is twisted by the fluid rotation $\tilde{v}_\theta = -\tilde{J}_\theta/ne$. Downstream of the vortex, the field line is again straight ($\mathbf{B}_0$) but now offset by a small angle $\theta$ with respect to the upstream line. As the incident vortex reflects from the conductor, the twist ($\tilde{B}_\theta$) is observed to vanish and reverse sign; i.e., the field line straightens out and then twists in the opposite direction. Since downstream of the vortex the field lines are frozen into the stationary plasma and at the boundary they are frozen into the stationary conductor, the reversal of the twist implies that the footpoint of the field line slips near the plasma-boundary interface. The field lines in the

plasma are obviously not tied to the boundary. They are swept azimuthally by the electron drift $\tilde{v}_\theta$ which does not reverse while $\tilde{B}_\theta$ does. Thus, helicity reversal in the plasma is consistent with the frozen-in concept which holds up to a boundary layer. If the field line was tied to the conductor, a twist opposite to that in the vortex would have to remain which is unphysical for a propagating perturbation and not observed.

Slippage can occur (i) in the electron-depleted Debye sheath ($5\lambda_D \simeq 0.1$ mm) where field lines cannot be associated with a moving conductor, (ii) by magnetic diffusion on a scale length $L$ where the magnetic Reynolds number $R_m = \mu_0 v_\parallel \sigma_\parallel L$ is of order unity, which for the present parameters ($v_\parallel \simeq 7 \times 10^7$ cm/s, $\sigma_\parallel \simeq 10\ \Omega^{-1}\,\text{cm}^{-1}$) yields $L \simeq 1.1$ mm, and (iii) by inertial effects on a scale length of $c/\omega_{pe} \simeq 7$ mm. Diffusion would produce energy losses, which are not observed. Inertial effects should conserve the generalized vorticity, which is not the case. Thus, field line tying and helicity conservation breaks down due to the vacuumlike sheath at the plasma-conductor boundary.

In summary, it is an observed fact that the reflection of an EMHD vortex changes the magnetic helicity while preserving energy. Helicity can change since magnetic lines are not continuous in a boundary layer. These observations are of interest to basic helicity conservation laws and wave reflection processes in plamas.

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