Random Roughness of Boundary Increases the Turbulent Convection Scaling Exponent

S. Ciliberto and C. Laroche
Ecole Normale Supérieure de Lyon, Laboratoire de Physique, C.N.R.S. URA1325, 46, Allée d'Italie, 69364, Lyon Cedex 07, France
(Received 4 September 1998)

The influence of the boundary layer properties on the heat transport in turbulent thermal convection is experimentally investigated in a cell with a rough bottom plate. It is shown that the standard 2/7 exponent of the convective heat flow dependence on the Rayleigh number, usually observed in a cell with smooth boundaries, increases if the roughness has power law distributed asperity heights and the thermal boundary layer thickness is smaller than the maximum asperity size. In contrast a periodic roughness does not influence the heat transport law exponent. [S0031-9007(99)09169-3]

PACS numbers: 47.27.Te, 05.40.–a, 68.35.Ct

In the last few years many models and experiments have been done in order to understand the heat transport properties of turbulent thermal convection in a fluid layer heated from below, that is, Rayleigh Benard convection [1–7]. These properties are characterized by the dependence on the Raleigh number Ra of the nondimensional heat flow, that is, the Nusselt number Nu. In many experiments it has been observed that, for $\mathrm{Ra} > 10^6$, Nu has a power law dependence on Ra; that is, $\mathrm{Nu} = \alpha \mathrm{Ra}^{\gamma}$. Specifically, in fluids with a Prandtl number of about 1, α and γ take the following values for $10^6 < \mathrm{Ra} < 10^{11}$: $\gamma = 2/7$ and $\alpha \simeq 0.2$ [2–4].

In order to construct a reliable model of the heat transport law several aspects of the turbulent flow have to be solved. For example, it is still unclear how the heat transport is influenced by the size distribution of thermal fluctuations in the boundary layer and by the coupling of these fluctuations with the mean circulation flow (MCF). Thermal fluctuations of the boundary layer are associated with thermal plumes [2,8] and the MCF is a large scale convective roll involving all the cells containing the convective fluid [8,9].

The role of MCF on the heat flux has been studied in several experiments, but the values of γ and α are not modified by either the perturbation [10,11] or the suppression [12] of the MCF. No influence on the value of γ has been observed in numerical simulation where no-slip boundaries were used [13]. In two recent experiments [14,15] boundaries with periodic roughness have been used. In this case α is much larger than the value measured in cells with smooth boundaries; that is, the heat flux is enhanced. However, it is important to stress that a periodic roughness does not modify the value of γ which is still 2/7 as in the case of smooth boundaries. Finally, it is worthwhile to mention that in a recent experiment the transition toward the ultimate regime has been observed [4]. This transition, which manifests itself with an increasing of γ for $\mathrm{Ra} > 10^{11}$, has been explained by the change of the dissipation properties in the thermal and viscous boundary layers (see also Ref. [7]). One of the consequences that one may extract from all these experiments and simulations is that the value of γ could be mainly controlled by thermal fluctuations (the thermal plumes) and by their size distribution in the boundary layer. Therefore a perturbation of this distribution may change the value of γ.

The purpose of this Letter is to show that using a very rough boundary, with power law distributed asperity heights, it is possible to strongly modify the value of $\gamma = 2/7$ observed in cells with smooth boundaries. The influence of the boundary roughness on transport properties is a very important topic in turbulence [16,17]. This topic has not been widely studied in turbulent thermal convection. In Refs. [14,15] only a periodic roughness has been considered. As already mentioned, this kind of roughness does not modify the value of γ but only that of α. Therefore, in this Letter we want to stress the difference between a periodic and a random roughness.

The experimental apparatus has already been described in Ref. [3] and we recall here only the main features. The cell has horizontal sizes $L_x = 40$ cm and $L_y = 10$ cm and two different heights d equal to 20 and 10 cm. With these two cells filled with water (at an average temperature of 45 °C corresponding to a Prandtl number, of about 3) we are able to cover the interval $10^7 < \mathrm{Ra} < 10^{10}$. The bottom copper plate is heated with an electrical resistor. The top copper plate is cooled by a water circulation and its temperature is stabilized by an electronic controller. All of the apparatus is inside a temperature stabilized box. The temperature of the plates is measured in several locations. Local temperature measurements, of the turbulent flow, are done with two small thermocouples (P1, P2) of 0.04 cm in diameter with a response time of 5 ms. The probes P1 and P2 are located at $(L_x/4, L_y/2)$ and at $(L_x/2, L_y/2)$, respectively. Both probes can be moved along z with micrometric devices in order to measure the mean temperature profile and that of the temperature fluctuations as a function of Ra. To measure the heat flow we first estimate the heat losses of the cell [3]. These heat losses are then subtracted from the heating power to evaluate the fraction of heat effectively transported by the convective water.

 0031-9007/99/82(20)/3998(4)$15.00

We perturb the bottom boundary layer by changing the roughness of the bottom plate. This roughness is made by small glass spheres of controlled diameter glued on the bottom copper plate, with a very thin layer of thermal conductive paint. We use N sets of spheres such that each set j is composed by spheres having the same diameter h_j, with $1 \leq j \leq N$ and $h_1 < h_2 \ldots < h_N$. The sphere number $P(h_j)$ in each set is selected in order to produce a well defined power law distribution; that is, $P(h_j) = Ah_j^{-\xi}$. Here A is a normalization factor such that the ensemble of spheres covers uniformly the copper plate surface; that is, $L_x \times L_y = \pi/4 \sum_{j=1,N} h_j^2 P(h_j)$. The spheres are mixed and randomly glued on the bottom plate. The roughness properties can be changed by modifying N, ξ, the minimum sphere diameter h_1, and the maximum sphere diameter h_N.

The roughness has two important characteristic lengths h_1 and h_N. These lengths have to be compared with one of the main characteristic lengths of turbulent thermal convection, that is, the thermal boundary layer thickness $\lambda = (d/2)/\mathrm{Nu}$. Indeed if $\lambda \gg h_N$ or $\lambda < h_1$ it is reasonable to think that no effect on the convection thermal properies will be observed. In contrast if $h_1 < \lambda < h_N$ several important changes could be produced. The mean roughness height $\tilde{h}$ does not seem to play any important role.

In order to understand the role played by the roughness on the boundary layer, we performed several experiments. In four experiments, labeled I, II, III, IV, respectively, the surface roughness had power law distributed asperities. A periodic roughness, with only one characteristic length, was used in another experiment labeled V. Specifically the roughness parameters, in the different experiments, took the values indicated in Table I. The results of these five experiments have been compared to those obtained in the same cell with smooth boundaries [3,12].

The nondimensional convective heat flow Nu versus Ra measured in a cell with smooth boundaries is compared in Fig. 1 with that measured in the experiments (III and IV). We clearly see that in the three cases Nu is a power law function of Ra; that is, $\mathrm{Nu} \propto \mathrm{Ra}^\gamma$ with $\gamma \simeq 2/7$ in the smooth plate case, $\gamma \simeq 0.45$ in experiment IV ($\xi = 1$), and $\gamma = 0.35$ in experiment III ($\xi = 2$). It is important to notice that in the experiments III and IV the thermal boundary layer thickness λ is always smaller than h_N in all the interval $10^8 < \mathrm{Ra} < 10^{9.5}$, where these two experiments have been performed. Indeed from Fig. 1 one sees that at $\mathrm{Ra} = 10^8$, $\mathrm{Nu} \simeq 16$; therefore $\lambda = 0.63$ cm $< h_N$ in experiments III and IV. These measurements clearly show that, when $\lambda < h_N$ has a value comparable to that of the roughness, the dependence of Nu as a function of Ra is strongly modified and γ is a function of ξ, that is, the exponent of the roughness height distribution $P(h)$. Specifically γ increases when the roughness height distribution becomes flatter.

In order to show that for $\lambda > h_N/2$ no effect on Nu is observed we describe the results of the experiments (I, II). These two experiments have the same ξ of experiment III but the important difference is that λ becomes comparable to $h_N/2$ in the middle of the Ra spanning range. Specifically from the Nu measured in these experiments, plotted as a function of Ra in Figs. 2(a) and 2(b), we find $\lambda \simeq 0.2$ cm at $\mathrm{Ra} \simeq 1.5 \times 10^9$ for experiment I and $\lambda = 0.2$ cm at $\mathrm{Ra} \simeq 3 \times 10^8$ for experiment II. In Fig. 2(a) we clearly see that when $\lambda < h_N/2$, that is, for $\mathrm{Ra} \geq 10^9$, the dependence of Nu changes and we find $\gamma = 0.35$ as in experiment III. The value of γ in this figure and in the next is certainly not very precise because of the very limited scaling range. However, the exact value of γ is not very important for the discussion. What we want to show here is just the clear change of trend for $\mathrm{Ra} \geq 10^9$. In contrast for $\mathrm{Ra} < 10^9$, that is, for $\lambda > h_N/2$, we see that the experimental points are parallel to those corresponding to the smooth plate. In Fig. 2(b) the results of experiment II are directly compared with those of experiment I. Experiment II has the same roughness of experiment I and a smaller d; therefore, $\lambda < h_N/2$ at $\mathrm{Ra} \simeq 3 \times 10^8$. Indeed we see that Nu begins to increase faster for $\mathrm{Ra} > 3 \times 10^8$. At the same time we notice that in both experiments I and II all the points for which $\lambda > h_N/2$ are aligned on the same straight line parallel to that of the smooth case. Thus we see that in order to have an interaction of the thermal boundary layer with the roughness λ should be smaller than $h_N/2$.

To show that this is actually the case we have measured the profile of the temperature and of the temperature fluctuations as a function of z in experiment I for two values of Ra. These profiles are plotted in Fig. 3 as a function of $0.5z/\lambda = z\mathrm{Nu}/d$. For $\lambda > h_N/2$ the profile with roughness is very close to that with a smooth plate. In contrast for $\lambda < h_N/2$ the dependence as a function of z of the temperature fluctuation rms and of temperature

TABLE I. Roughness parameters: experiments I, II, III, and IV have power law distributed asperity heights, whereas experiment V has a periodic roughness.

Experiment	$P(h_j)$	N	h_1 (cm)	h_N (cm)	$\tilde{h}$ (cm)	d (cm)
I	h^{-2}	3	0.06	0.4	0.08	20
II	h^{-2}	3	0.06	0.4	0.08	10
III	h^{-2}	5	0.06	1	0.08	20
IV	h^{-1}	5	0.06	1	0.18	20
V	$\delta(h - h_N)$	1	0.2	0.2	0.2	20

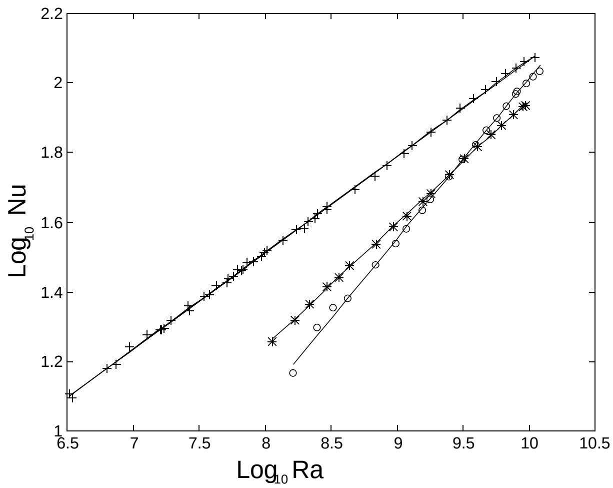

FIG. 1. Dependence of Nu on Ra: with smooth bottom plate (+) and with a rough bottom plate in experiment III (*) with $\xi = 2$ and in experiment IV (○) with $\xi = 1$. In experiments III and IV λ was always smaller than h_N.

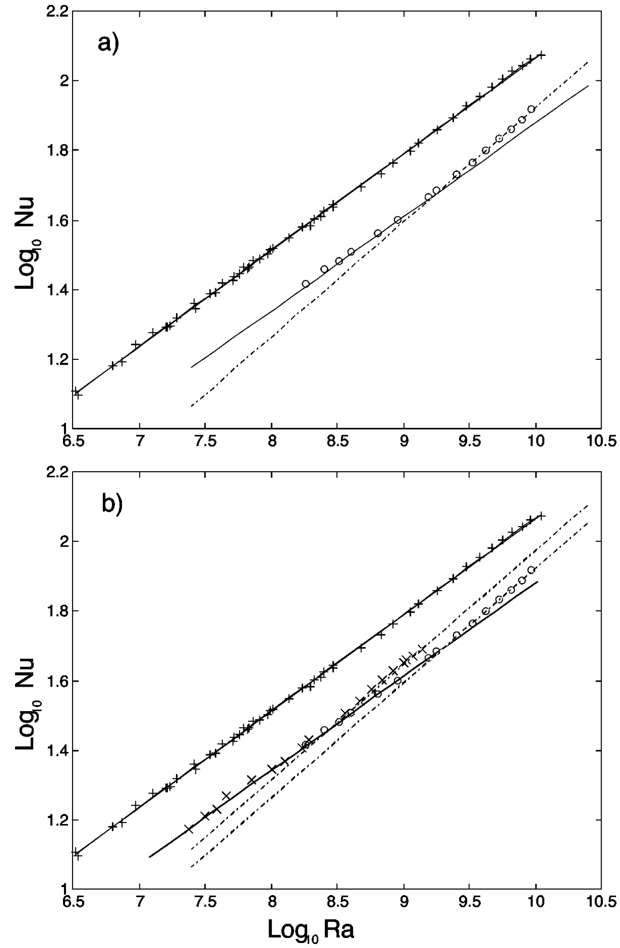

FIG. 2. Dependence of Nu on Ra: with smooth bottom plate (+) and with a rough bottom plate in experiment I (○) and in experiment II (×). The roughness exponent was $\xi = 2$. (a) The boundary layer thickness is smaller than h_N at $\mathrm{Ra} > 10^9$. There is a clear change of slope when $\lambda < h_N/2$. (b) The results of experiment I are compared with those of experiment II. For experiment II, $\lambda < h_N/2$ for $\mathrm{Ra} > 3 \times 10^8$ which corresponds to the transition point.

is strongly perturbed by the presence of the roughness, which induces the appearance of a second maximum in the rms profile. The position of this maximum is close to h_N. Thus the profile shape confirms that the dependence of Nu as a function of Ra is modified by the roughness only when $h_N > \lambda$.

Finally, we compare the results of the experiments I, ..., IV with those of experiment V which has a periodic roughness. The curve Nu versus Ra measured in the experiment (V) is plotted in Fig. 4. The presence of a jump in the curve is clearly observed. The location of the jump corresponds to the value of Ra where $\lambda \simeq h_N = h_1$. However, above and below this jump the slope of the curve is very close to that with smooth plates. These results agree with those of Refs. [14,15], where a periodic rough plate was used. They also tell us that when $\lambda < h_1$ the roughness does not influence the value of γ. From the comparison of the results of experiment V with those of experiments III and IV one deduces that an important ingredient for modifying the exponent γ is the presence of a roughness with a power law distributed height and that a periodic roughness does not influence the value of γ.

In spite of these important changes in the behavior of Nu versus Ra in experiments I, II, III, and IV, the bulk properties are not modified by the presence of the roughness. The histograms and spectra of the local temperature fluctuations measured in the center of the cell by probe P2 and on the side by probe P1 are the same with and without roughness. Furthermore, the frequency f_c of the slow oscillation, which is related to the MCF period, has the same dependence on Ra observed in experiments with smooth plates and $\mathrm{Pr} \simeq O(1)$ [4,9,12]. We find that our data are compatible with a law $f_c \simeq \chi/d^2 A_f \mathrm{Ra}^{0.49}$ with $A_f \simeq 0.06$ without roughness and $A_f \simeq 0.05$ with roughness. This result is in agreement with those of Ref. [4], where it is shown that f_c is always proportional to $\mathrm{Ra}^{0.49}$ even for $\mathrm{Ra} > 10^{11}$, where $\gamma > 2/7$ as in our experiment [4]. Therefore our experiment and that of Ref. [4] seems to indicate that there is a negligible influence of the value of γ on the dependence of f_c on Ra and on the statistical properties.

At the moment we are unable to construct a model which explains the dependence of γ on ξ. The prediction of a recently proposed model does not agree with our observations [18] because this model predicts an increasing of γ for increasing ξ and we observe just the contrary. Nevertheless, one can try to understand why the presence of a random roughness is so important. From Refs. [10,12] we know that any perturbation of the MCF does not change the heat transport. Furthermore, several models do not need to rely upon the MCF in order to explain the $\mathrm{Nu} \propto \mathrm{Ra}^{2/7}$ law [2,7]. Therefore if one assumes that the heat transport is mainly due to plumes, whose characteristic size is close to λ, then if $\lambda \gg h_N$ it is

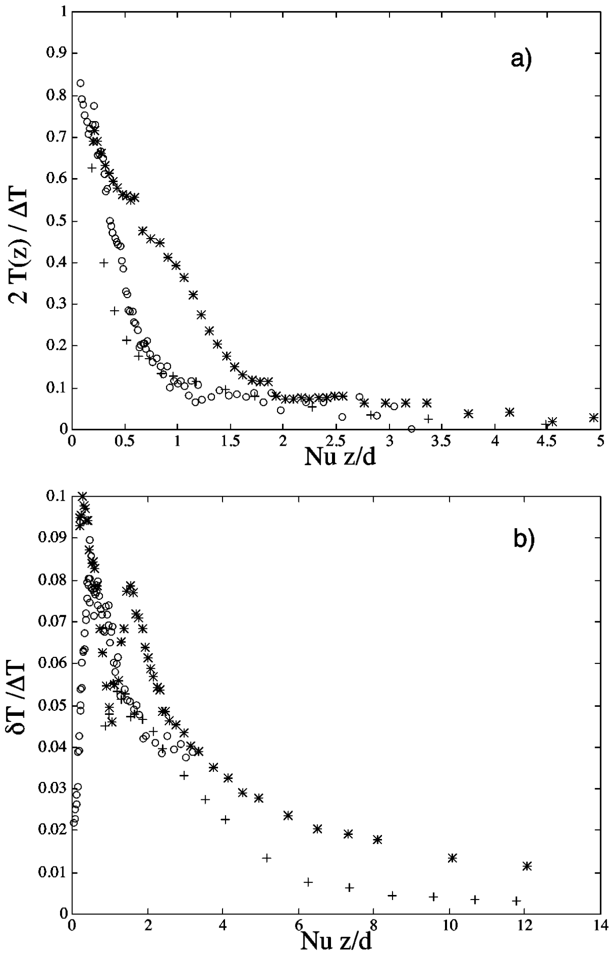

FIG. 3. Vertical profiles of the mean temperature (a) and of the rms of the temperature fluctuations (b). The symbol (+) corresponds to measurements done with a smooth plate whereas (○) and (*) correspond to the rough case with $\lambda \simeq h_N$ (○) ($\mathrm{Ra} = 3 \times 10^8$) and $\lambda < h_N$ (*) ($\mathrm{Ra} = 10^{10}$).

obvious that no perturbation of the heat transport is observed and everything goes as in the smooth case. In contrast when $h_1 < \lambda < h_N$ and the roughness has a power law distribution, the plumes and the boundary layer cannot construct their characteristic length because the roughness contains many different lengths. As a consequence the modified λ changes the dependence of Nu versus Ra. Finally if $\lambda < h_1$ no influence on γ is observed because the system is locally equivalent to a smooth one. It is also important to recall that in Ref. [4] the increase of γ for $\mathrm{Ra} > 10^{11}$ has been justified by the change of the dissipation properties in the boundary layer. From all of these observations one may extract the very important conclusion that the turbulent heat transport is dominated by the thermal fluctuations close to the boundary layer and by their size distribution. If this distribution is perturbed by the presence of a random roughness the transport properties are modified too. Any realistic model of thermal convection should take into account these results, which could be very useful in the study of convection in geophysical flows where the presence of smooth plates is certainly a very idealized case.

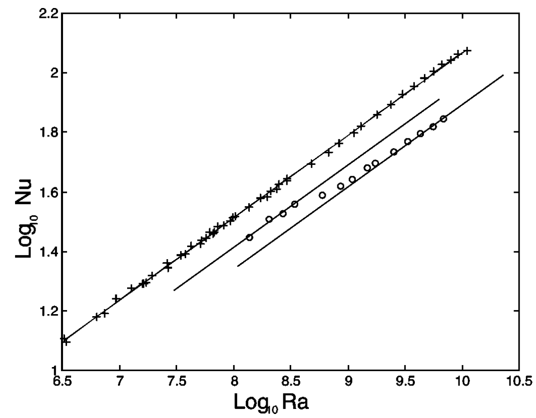

FIG. 4. Dependence of Nu on Ra: with smooth bottom plate (+) and with a rough bottom plate in experiment V (○). The roughness has just one size $h_N = h_1 = 2$ mm in this case.

We acknowledge very useful discussions with R. Benzi, D. Lohse, J. F. Pinton, and E. Villermaux.

[1] E. Siggia, Annu. Rev. Fluid Mech. **26**, 137 (1994).
[2] B. Castaing, G. Gunaratne, F. Heslot, L. Kadanoff, A. Libchaber, S. Thomae, X. Wu, S. Zalesky, and G. Zanetti, J. Fluid Mech. **204**, 1 (1989).
[3] F. Chillá, S. Ciliberto, C. Innocenti, and E. Pampaloni, Nuovo Cimento Soc. Ital. Fis. **15D**, 1229 (1993); F. Chillá, S. Ciliberto, and C. Innocenti, Europhys. Lett. **22**, 681 (1993).
[4] X. Chavanne, F. Chillá, B. Castaing, B. Hebral, B. Chabaud, and J. Chaussy, Phys. Rev. Lett. **79**, 3648 (1997).
[5] B. Shraiman and E. Siggia, Phys. Rev. A **42**, 3650 (1990).
[6] S. Grossmann and D. Lohse, report.
[7] R. Benzi, F. Toschi, and L. Tripiccione, chao-dyn/9808020.
[8] L. N. Howart and R. Krishnamurty, J. Fluid Mech. **170**, 385 (1986); R. Krishnamurty and L. N. Howart, Proc. Natl. Acad. Sci. U.S.A. **78**, 1981 (1981).
[9] M. Sano, X. Z. Wu, and A. Libchaber, Phys. Rev. A **40**, 6421 (1989).
[10] K. Q. Xia and S. L. Lui, Phys. Rev. Lett. **79**, 5006 (1997).
[11] T. H. Solomon and J. P. Gollub, Phys. Rev. A **43**, 6683 (1991).
[12] S. Ciliberto, S. Cioni, and C. Laroche, Phys. Rev. E **54**, R5901 (1996).
[13] F. Toschi and R. Benzi (to be published).
[14] X. Shen, P. Tong, and K. Q. Xia, Phys. Rev. Lett. **76**, 908 (1996).
[15] Y. B. Du and P. Tong, Phys. Rev. Lett. **81**, 987 (1998).
[16] H. Schlichting, *Boundary Layer Theory* (McGraw-Hill, New York, 1968).
[17] N. Mordant, J. F. Pinton, and F. Chillá, J. Phys. VII (France) **2**, 1729 (1997), and references therein.
[18] E. Villermaux, Phys. Rev. Lett. **81**, 4859 (1998).