Precision Detection of the Cosmic Neutrino Background

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In the standard big bang cosmology the canonical value for the ratio of relic neutrinos to cosmic microwave background (CMB) photons is 9/11. Within the framework of the standard model of particle physics there are small corrections, in sum about 1%, due to slight heating of neutrinos by electron-positron annihilations and finite-temperature QED effects. We show that this leads to changes in the predicted CMB anisotropies that will bias determination of the other cosmological parameters if not correctly taken into account. These changes might be detected by future satellite experiments. [S0031-9007(99)09208-X]

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Neutrinos are almost as abundant as photons in the Universe and contribute almost as much energy density [1]. Under the assumption that neutrinos decoupled completely before electrons and positrons annihilated (at a time of around 1 sec after the big bang), the ratio of the number density of neutrinos to that of photons is

$$\frac{n_\nu}{n_\gamma} = \left(\frac{3N_\nu}{11}\right), \tag{1}$$

where $N_\nu = 3$ is the number of neutrino species. Further, because of the heating of the photons by e^+-e^- annihilations, the ratio of the neutrino temperature to the photon temperature is $(4/11)^{1/3} = 0.714$. It follows that the ratio of the energy density of neutrinos to that of photons is

$$\frac{\rho_\nu}{\rho_\gamma} = \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_\nu = 0.681\,. \tag{2}$$

It has been pointed out that the assumption that neutrinos decoupled completely before e^+-e^- annihilations is not precisely valid [2]. There is now a consensus that the neutrinos share in the heating somewhat, so their number and energy density are slightly larger than the canonical values, Eqs. (1) and (2). The increase is equivalent to having slightly more than three neutrino species and the canonical ratios. (This is just a heuristic device, of course; the actual number of generations in the standard model of particle physics is three.) The change in the effective number of neutrino generations is [2–8]

$$\delta N_\nu^{\rm ID} = 0.03\,. \tag{3}$$

The first calculations [2–4] of this effect were "one-zone" estimates that evolved integrated quantities through the process of neutrino decoupling. More refined "multizone" calculations tracked many energy bins, assumed Boltzmann statistics, and made other approximations [5,6]. The latest refinements have included these small effects as well [7–9].

There is another effect operating at roughly the same time which acts in the same direction; it involves finite-temperature QED corrections to the energy density of the electron, positron, and photon portions of the plasma [10,11]. This effect decreases the energy density of the $e^\pm\gamma$ plasma. Consequently, this reduces the amount of energy converted to photons when electrons and positrons annihilate thereby slightly raising the ratio of the neutrino to photon energy densities. This QED effect can also be expressed as an increase in the number of neutrino species [10,11]

$$\delta N_\nu^{\rm QED} = 0.01\,. \tag{4}$$

Together, incomplete annihilation and QED finite-temperature corrections lead to an increase in the neutrino energy density over the canonical value by slightly more than 1%, corresponding to

$$\delta N_\nu = 0.04\,. \tag{5}$$

These two corrections were initially considered in the context of big bang nucleosynthesis. Their net effect is to increase the predicted ^{4}He abundance by a tiny amount, $\Delta Y_P = +1.5 \times 10^{-4}$, which given the present observational uncertainties, $\sigma_{Y_P} \sim 10^{-2}$, is undetectable. (The quantity Y_P denotes the primordial mass fraction of ^{4}He.)

On the other hand, the small increase in the neutrino energy density can have a significant—and potentially detectable effect—on another remnant of the big bang—the cosmic microwave background (CMB). In particular, the anisotropies in the CMB are very sensitive to the epoch of matter-radiation equality, which depends on the neutrino energy density. The aim of this paper is to address quantitatively the detectability of the small increase in neutrino energy density due to incomplete decoupling and finite-temperature QED effects. For definiteness and simplicity, we assume inflation and cold dark matter with six parameters: cosmological constant (Ω_Λ), baryon density (Ω_B), Hubble constant (H_0), amplitude of primordial perturbations, slope of primordial perturbations (n), and epoch of reionization. We find that if the anisotropy and

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polarization of the CMB anisotropy is measured with the precision anticipated for Planck [12], these small corrections are marginally detectable. Perhaps more importantly, if they are neglected or incorrectly included in parameter fits, other parameters will be biased at a level comparable to the expected statistical errors.

Probing neutrino physics with the CMB.—Anisotropies in the CMB are best characterized by expanding the temperature field on the sky in terms of spherical harmonics:

$$T(\theta,\phi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} a_{lm}Y_{lm}(\theta,\phi)\,. \qquad (6)$$

A given theory, specified by the primordial spectrum of perturbations and cosmological parameters, makes predictions about the multipole amplitudes, the a_{lm}'s. The predictions take the form of statements about the distribution of the a_{lm}'s. Inflationary theories typically predict that each of these coefficients is drawn from a Gaussian distribution; as such, the distribution can be defined by its variance. Thus, the fundamental predictions of inflationary models are

$$C_l \equiv \langle a_{lm}a_{lm}^{*}\rangle\,. \qquad (7)$$

Much effort has gone into computing the C_l's over the last few years; they can be calculated very accurately once the cosmological parameters are chosen [13]. Viewed simplistically, the results of a CMB experiment are estimates of the C_l's, with errors given by ΔC_l. Then, by minimizing a χ^2 statistic

$$\chi^2(\{\lambda_i\}) \equiv \sum_{l=2}^{\infty}\frac{[C_l(\{\lambda_i\}) - C_l^{\text{estimate}}]^2}{(\Delta C_l)^2}\,, \qquad (8)$$

the underlying set of unknown cosmological parameters $\{\lambda_i\}$ can be estimated.

Of course, we cannot know in advance the values of C_l's that a given experiment will measure; however, by knowing what we expect for the ΔC_l's, we can estimate how large the uncertainties in the parameters should be ("error forecasting"). To do this, we assume that the measured C_l's will be close to the true C_l's. Then, by expanding χ^2 around its minimum at $\{\lambda_i^{\text{true}}\}$ we can estimate the precision to which a parameter can be determined (for further discussion of error forecasting in parameter estimation, see, e.g., Ref. [14]):

$$\chi^2(\{\lambda_i\}) \simeq \chi^2(\{\lambda_i^{\text{true}}\}) + F_{ij}(\lambda_i - \lambda_i^{\text{true}})(\lambda_j - \lambda_j^{\text{true}})\,. \qquad (9)$$

The second-derivative (Fisher) matrix F_{ij} carries information about how quickly χ^2 increases as the parameters move away from their true values. Therefore, under some reasonable assumptions [15], the uncertainties in the parameters are determined by this matrix. We are interested only in the parameter N_ν. [Jungman *et al.* carried out a parameter estimation including N_ν as a free parameter [14]. Their analysis, performed several years ago, considered only $l \lesssim 1000$ (then considered optimistic). Further they did not consider polarization. Where it is possible to compare with them, our results agree.] If all the other cosmological parameters are allowed to vary, then

$$\sigma_{N_\nu}^2 = (F^{-1})_{N_\nu,N_\nu}\,. \qquad (10)$$

To proceed we need to specify the following:

(i) Cosmological model: For definiteness, we take this to be a cold dark matter model with cosmological constant $\Omega_\Lambda = 0.7$, Hubble constant $H_0 = 50$ km sec^{-1} Mpc^{-1}, baryon density $\Omega_B = 0.08$, COBE-normalized spectrum of scale invariant density perturbations (i.e., power-law index $n = 1$), no reionization, and energy density in cold dark matter particles $\Omega_{\text{CDM}} = 1 - \Omega_\Lambda - \Omega_B = 0.22$. (We assume the simplest inflationary prediction of $\Omega = 1$.)

(ii) Experimental errors: Instead of tying ourselves to a particular experiment, we assume that the experimental uncertainty is given by

$$\Delta C_l = \begin{cases} \sqrt{\frac{2}{2l+1}}\,C_l & l \le l_{\max}\,. \\ \infty & l > l_{\max}\,. \end{cases} \qquad (11)$$

The error $[2/(2l + 1)]^{1/2}C_l$ is the smallest possible given that each multipole amplitude a_{lm} can be sampled only $2l + 1$ times; it is the irreducible sampling or *cosmic* variance. Equation (11) is obviously a simplification, but we have found it to be a reasonable approximation to the more realistic formula [16] which also accounts for detector noise. Further, it allows us to display our results as a function of $l_{\max}$, which will give a clear sense of what angular scales need to be probed. We use a similar formula for polarization (with different $l_{\max}$). For orientation, MAP is characterized by $l_{\max} \simeq 1000$ and PLANCK by $l_{\max} \simeq 2500$.

(iii) Model parameters: We allow for variation in six parameters besides the neutrino energy density: overall amplitude of the spectrum of density perturbations, epoch of reionization parametrized by the optical depth back to last scattering τ, H_0, Ω_Λ, Ω_B, and n.

Figure 1 shows the 7×7 Fisher matrix. Strong correlations exist among N_ν, h, and Ω_Λ, and to a lesser extent with Ω_B as well. The correlations between N_ν, h, and Ω_Λ are expected: these are the only parameters we have chosen which affect the epoch of matter-radiation equality. Using the Fisher matrix, we compute the expected error in N_ν. Our results are summarized in Fig. 2. Shown are the 1σ errors on N_ν emerging from an experiment characterized by $l_{\max}$. The limits would truly be impressive if N_ν was the only free parameter. Unfortunately the more realistic assessment comes from allowing the other parameters to vary as well. In that case, the degeneracy between N_ν, h, and Ω_Λ leads to much larger errors in N_ν, as depicted by the top curves in Fig. 2. The upper dashed curve corresponds to the limit from temperature anisotropy alone, while the upper solid curve includes polarization.

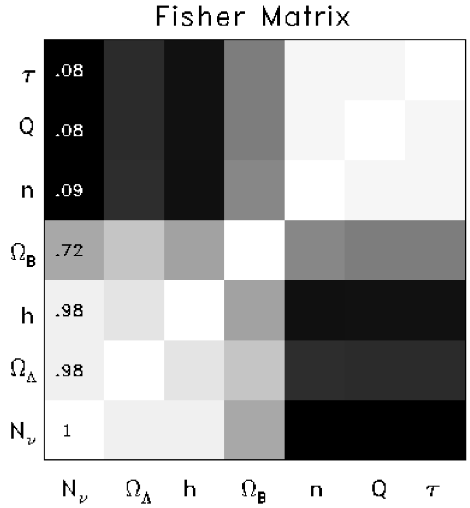

FIG. 1. The Fisher matrix normalized by its diagonal elements, $|F_{ij}|/F_{ii}^{1/2}F_{jj}^{1/2}$. Strong correlations (lighter squares) exist among n, Q, τ and among N_ν, Ω_Λ, h. Information is taken from both polarization and temperature maps out to $l = 3000$.

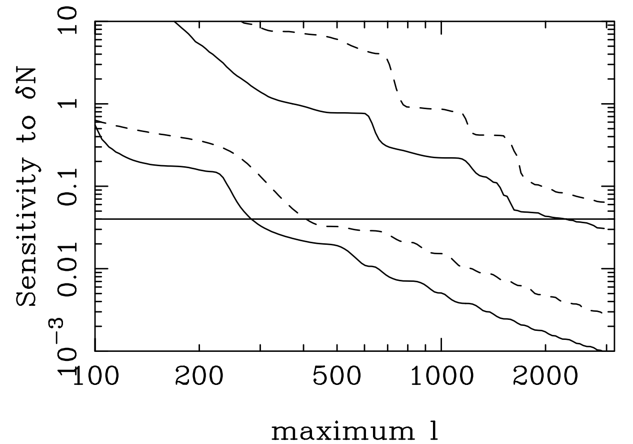

FIG. 2. 1σ sensitivity to δN_ν, for an experiment cosmic-variance limited up to some maximum multipole moment. The horizontal line, $\delta N_\nu = 0.04$, is the change in effective number of neutrino families due to neutrino heating and the QED effect. The bottom two curves are for the case where all cosmological parameters except N_ν are fixed, while the top curves represent the case where all parameters are determined simultaneously. For each group, the dashed line shows the results using only temperature anisotropy data, while the solid line shows the improvement obtained by including polarization data in the analysis.

Polarization information would produce a modest detection of the neutrino excess. Going out to larger l would firm up the detection. Finally, if Ω_Λ is held fixed, e.g., if we assume that $\Omega_\Lambda = 0$, then the sensitivity at high l improves by about 50% without polarization information and 130% with polarization information.

Another way of viewing the effect of neutrino excess on the CMB is to treat it as a source of bias in the estimation of the other cosmological parameters. If the parameter extraction is done neglecting the extra neutrino density, then each parameter will be incorrectly estimated by an amount [17]

$$\Delta\lambda_i = F_{ij}^{-1} \sum_l \frac{\delta C_l^\nu}{(\Delta C_l)^2} \frac{\partial C_l}{\partial \lambda_j}, \qquad (12)$$

where δC_l^ν is the change in C_l due to the excess neutrino energy density. Figure 3 shows the resultant bias to the parameters. For some parameters, the bias exceeds the statistical error predicted by Eq. (11). Again as expected, the largest biases are on h and Ω since they correlate significantly with N_ν.

Concluding remarks.—As our analysis shows, future, high-precision CMB anisotropy measurements have the potential to measure the cosmic energy density in neutrinos to a precision of 1%. Such a measurement would have significant implications:

(i) If $N_\nu = 3$, this would provide further evidence for the existence of the tau neutrino. Note, the tau neutrino has yet to be directly detected in the laboratory.

(ii) Determination that "$N_\nu = 3$" by CMB anisotropy would confirm the canonical assumption for the energy density in relativistic particles at the epoch of big-bang nucleosynthesis (BBN), which is an important input parameter for these calculations.

(iii) Confirmation of the standard cosmology prediction that $T_\nu/T_\gamma = (4/11)^{1/3}$ to 1%. This would test the physics of e^+-e^- annihilation and neutrino decoupling in the early Universe.

(iv) Confirmation of two small physics effects that together increase the cosmic neutrino energy density by about 1%. In particular, this would be the first evidence for finite-temperature QED corrections and a constraint to the strength of neutrino interactions in the early Universe.

(v) If a deviation from the expected $N_\nu = 3.04$ is found, this would provide evidence for additional relativistic particle species present in the early Universe or new physics in the neutrino sector (e.g., neutrino mass or decay) [18]. This would have significant implications for BBN, structure formation in the Universe, and elementary-particle physics.

(vi) Note that the presence of primordial magnetic fields or gravitational waves would effectively increase δN_ν. In fact, finding a limit on δN_ν may prove to put better limits on these fields than BBN. However, other measurements such as Faraday rotation of the CMB [19] and the advanced LIGO configuration will more directly measure these fields and remove any ambiguity with the measurement of N_ν.

Realizing the full potential of the CMB as a probe of the cosmic neutrino backgrounds will require precision polarization and anisotropy maps out to multipole number 3000. This seems very ambitious and perhaps even unattainable. Nonetheless, the potential payoff discussed here makes the goal worth striving for. If we have learned nothing else in the years since COBE, we have certainly learned that the experimenters have consistently managed

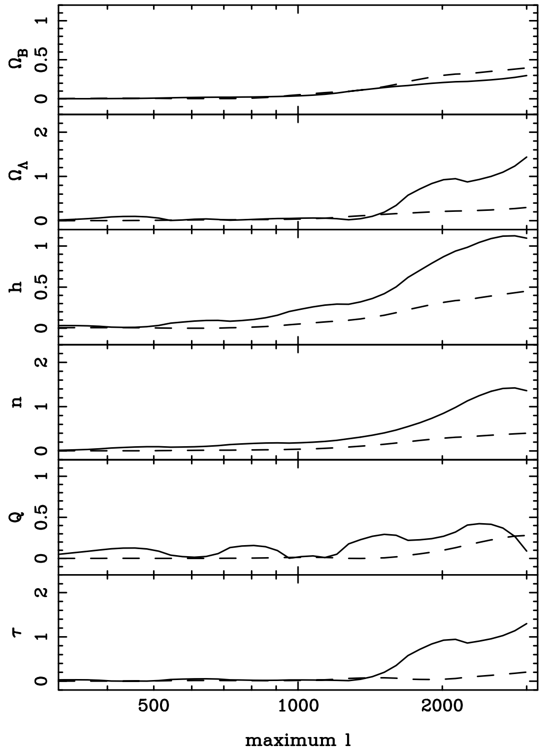

FIG. 3. The bias in the determination of the cosmological parameters due to neglecting the effect of neutrino heating. Plotted is the ratio of the bias to the statistical errors for each parameter, again as a function of the maximum l out to which we have information. In each case the solid curve includes information from polarization; the dashed curve does not.

to surprise theorists by achieving more than was once thought reasonable.

Finally, we should acknowledge that there is room to improve upon our analysis. For example, we have assumed only six cosmological parameters. Among the other parameters that might be considered are running of the spectral index, primordial magnetic field, and gravitational waves.

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