Quantum Effect of the Aharonov-Bohm Type for Particles with an Electric Dipole Moment

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In the effects of the Aharonov-Bohm type the topological properties of the phase shift are directly related to those of the linear and angular momentum of the electromagnetic fields. This interpretation leads to the formulation of a nonlocal topological effect for particles possessing an electric dipole moment. The experimental observation of this effect is within reach of atom or molecular interferometry. [S0031-9007(99)09141-3]

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The electromagnetic (em) interaction is an important feature of several quantum effects such as the Aharonov-Bohm (AB) [1] effect for charged particles and the Aharonov-Casher (AC) [2] effect for magnetic dipoles. Recently, effects for electric dipoles have been proposed by Wilkens [3], Wei *et al.* [4], and Spavieri [5]. Differently from the AB effect, in the effects for magnetic and electric dipoles the particles move in the presence of external em fields.

All these effects foresee an observable displacement of the interference pattern related to the phase shift $\Delta\phi$ of the wave function of the system. Most of them have either been already tested [6] or are within the possibility of experimental verification. Effects for quadrupoles or higher orders are unfeasible because they either require that the particle move in a medium or field strengths well beyond experimental reach [7].

One of the aims of this Letter is to provide a unitary description of the topological properties of the em interaction involved in these effects. The main purpose is to formulate, using this description, a nonlocal, topological effect for electric dipoles where, as in the AB effect, no fields act on the particle.

The interaction carries em energy, linear and angular momentum which all add to that of the matter waves and, by modifying the phase of the wave function, generate the phase shift. A general expression for the phase of these and other effects reads $\phi = \hbar^{-1}\int U\,dt$ where the term U represents the em interaction energy. This expression has the inconvenience of not evidencing the topological (or geometrical) properties of the effects for which the interaction energy has the form $U = \mathbf{Q}\cdot\mathbf{v} = -V$, where $\mathbf{Q} = \partial U/\partial\mathbf{v}$ is the canonical momentum due to interaction and V the potential energy. By means of the relation $d\mathbf{x} = \mathbf{v}dt$ valid on the path of the particle, ϕ is usually expressed as a path integral, which in the AB effect reads $c^{-1}q\int\mathbf{A}\cdot\mathbf{v}dt = c^{-1}q\int\mathbf{A}\cdot d\mathbf{x}$, where $\mathbf{A}(\mathbf{x})$ is the vector potential of a solenoid. Placing the solenoid along the z axis, $\mathbf{A} = 2\mu(-\hat{\mathbf{i}}y + \hat{\mathbf{j}}x)/r^2$ where $r^2 = x^2 + y^2$ and μ is the magnetic dipole moment linear density. The topological properties become apparent by writing $\mathbf{A} = 2\mu\nabla\theta$ for $r \neq 0$, where θ is the multivalued function $\theta(\mathbf{x}) = \tan^{-1}(x/y)$. Because of the singularity at $r \to 0$, $\nabla\times\mathbf{A} = \mathbf{B} = \hat{\mathbf{k}}2\mu\delta(r)/r$ and the AB phase shift takes on the usual form

$$\Delta\phi_{\mathrm{AB}} = \frac{1}{\hbar}\oint\mathbf{Q}\cdot d\mathbf{x} = \frac{q}{\hbar c}\oint\mathbf{A}\cdot d\mathbf{x} = \frac{q2\mu}{\hbar c}\oint d\theta = \frac{q}{\hbar c}\oint\mathbf{B}\cdot d\mathbf{S} = \frac{q\Phi}{\hbar c}\,\delta n\,, \tag{1}$$

where $\Phi = BS = 4\pi\mu$ is the magnetic flux of the solenoid and δn is the difference between the topological winding numbers n of the Feynman paths encircling the singularity.

This interpretation of the AB effect in terms of Φ, points out its topological properties but still does not recognize the physical meaning of the quantity $\mathbf{Q}$ and the constant quantity $L = q\Phi/2\pi c$ which gives the angular rate of change of the phase shift $d(\Delta\phi_{\mathrm{AB}})/d\theta = L/\hbar$ in units of $\hbar$. Furthermore, the interpretation in terms of Φ does not apply to the other effects for magnetic and electric dipoles.

In order to make apparent the topological properties of the em interaction and its intrinsic physical meaning, it is convenient to introduce the classical linear momentum of the em fields $\mathbf{Q}_{\mathrm{em}}$ and the corresponding angular momentum $\mathbf{L}_{\mathrm{em}}$. Except for the sign, the momentum $\mathbf{Q}$ and its angular momentum $\mathbf{L} = \mathbf{r}\times\mathbf{Q}$, are given by

$$\begin{aligned}\mathbf{Q} &= \pm\mathbf{Q}_{\mathrm{em}} = \pm\frac{1}{4\pi c}\int(\mathbf{E}\times\mathbf{B})d^3x',\\ \mathbf{L} &= \pm\mathbf{L}_{\mathrm{em}} = \pm\frac{1}{4\pi c}\int[\mathbf{x}'\times(\mathbf{E}\times\mathbf{B})]d^3x',\end{aligned} \tag{2}$$

where the minus sign applies to the AC effect and $\mathbf{x}'$ is the polar vector with origin at the position of the charge. For all the mentioned topological effects

$$\mathbf{L} = \hat{\mathbf{k}}L = \text{const}, \quad \nabla\cdot\mathbf{Q}(\mathbf{x}) = 0\,, \quad \text{and} \quad \nabla\times\mathbf{Q} = \mathbf{L}\,\frac{\delta(r)}{r}\,. \tag{3}$$

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It follows that the vector field $\mathbf{Q}(\mathbf{x})$ is the curl of the vector potential $\mathbf{T}(\mathbf{x})$:

$$\mathbf{Q}(\mathbf{x}) = \nabla \times \mathbf{T}(\mathbf{x}),$$

with

$$\mathbf{T}(\mathbf{x}) = (1/4\pi) \int \frac{\mathbf{L}}{R} \frac{\delta(r')}{r'} d^3x' = \mathbf{L} \ln(x^2 + y^2),$$

where $R = [(x - x')^2 + (y - y')^2 + (z - z')^2]^{1/2}$ and $\nabla \times \mathbf{Q} = \nabla \times \nabla \times \mathbf{T}(\mathbf{x}) = \nabla^2 \mathbf{T}(\mathbf{x})$. Since $\mathbf{Q}(\mathbf{x}) = L(-\hat{\mathbf{i}}y + \hat{\mathbf{j}}x)/r^2$, the em momentum may be expressed also as the gradient of the multivalued function θ, i.e., $\mathbf{Q}(\mathbf{x}) = L\nabla\theta$, for $r \neq 0$.

In the terminology of fluid dynamics (or of Berry's phase [8]), we may denote the singularity of $\nabla \times \mathbf{Q} = \mathbf{L}\delta(r)/r$ as the *vorticity* (or *curvature*) of the momentum (or *connection*) $\mathbf{Q}$ and its flux through a surface S, $\oint(\nabla \times \mathbf{Q})\cdot d\mathbf{S} = 2\pi L$ as the *vortex (*or *curvature) strength*. These geometrical properties of $\mathbf{Q}(\mathbf{x})$ and $\mathbf{L}$ determine the topology of the phase shift:

$$\Delta\phi = \frac{1}{\hbar} \oint \mathbf{Q}\cdot d\mathbf{x} = \frac{|\mathbf{L}|}{\hbar} \oint \nabla\theta \cdot d\mathbf{x} = \frac{1}{\hbar} \oint (\nabla \times \mathbf{Q})\cdot d\mathbf{S} = \frac{|\mathbf{L}|}{\hbar} \oint d\theta = 2\pi n \frac{L}{\hbar}. \quad (4)$$

Equation (4) states that the angular rate of change of the phase shift is equal to the classical em angular momentum $|\mathbf{L}|$ measured in units of the quantum angular momentum $\hbar$, and the phase shift is given by the vortex strength $2\pi L$ of the singularity measured in units of $\hbar$. A quantum effect characterized by the phase shift (4) is topological in the sense that it depends only upon the topology of the path with reference to the enclosed singular em vortex.

Since for the configuration of fields of the AB effect $\mathbf{B} = \hat{\mathbf{k}} 2\mu\delta(r)/r$, using Eq. (2) to calculate $\mathbf{Q}$ and $\mathbf{L}$, one finds $\mathbf{Q}(\mathbf{x}) = L(-\hat{\mathbf{i}}y + \hat{\mathbf{j}}x)/r^2 = (q/c)\mathbf{A}(\mathbf{x}) = L\nabla\theta$ and $L = 2q\mu/c$. Obviously, Eqs. (1) coincides with Eq. (4) of which it is a special case.

In the AC effect a particle possessing a magnetic dipole moment $\mathbf{m}$ moves in the presence of an external electric field $\mathbf{E}$ produced by a line charge of linear density λ. In this effect, the canonical momentum coincides with the so-called *hidden momentum* of the magnetic dipole $\mathbf{Q} = \mathbf{Q}_h = \mathbf{m} \times \mathbf{E}/c = -\mathbf{Q}_{\mathrm{em}}$ [9]. Because of conservation of total momentum, $\mathbf{Q}_h + \mathbf{Q}_{\mathrm{em}} = 0$, and this explains the minus sign in Eq. (2). For the AC effect, $\mathbf{Q}$ and $\mathbf{L}$ may be calculated by writing in Eq. (2) $\mathbf{B} = -\nabla\Phi_m + 4\pi\mathbf{M}$, where Φ_m is the scalar magnetic potential, $\mathbf{M}$ the magnetization, and $\mathbf{m} = \int \mathbf{M} d^3x'$ the total magnetic moment. The result is $\mathbf{Q} = L(-\hat{\mathbf{i}}y + \hat{\mathbf{j}}x)/r^2 = \mathbf{m} \times \mathbf{E}/c = L\nabla\theta$ and $L = 2\lambda m/c$.

In these effects there is an elementary interaction involving $\mathbf{m}$ and q where the magnetic field $\mathbf{B}_q = \pm\mathbf{v} \times \mathbf{E}$, produced by q in relative motion, is experienced by $\mathbf{m}$ in its rest frame. For the discussion about locality (or nonlocality), it is useful to derive a general expression for U in terms of the magnetic flux linked by $\mathbf{m}$, namely, $U = \mathbf{Q} \cdot \mathbf{v} = \pm q\mathbf{A} \cdot \mathbf{v}/c = \pm q(\mathbf{m} \times \mathbf{x})|\mathbf{x}|^{-3}\cdot\mathbf{v}/c = \mp(\mathbf{m} \times \mathbf{E})\cdot\mathbf{v}/c = \mathbf{m} \cdot \mathbf{B}_q$. Since $m = IA/c$, where I is the dipole electric current and A its area, by summing all the contributions, $U = \sum \mathbf{m} \cdot \mathbf{B}_q = I\Phi_q/c$ where Φ_q is the total magnetic flux linked by the dipole(s). Thus, with $\mathbf{v} \cdot \nabla\theta = \dot{\theta}$, the phase reads

$$\phi = \int U\, dt = L \int \dot{\theta}\, dt = \frac{I}{c} \int \Phi_q\, dt \quad (5)$$

(Φ_q is not the flux of the solenoid of the AB effect). Thus, the phase rate of change is $d\phi/dt = \hbar^{-1}U = \hbar^{-1}L\dot{\theta} = (\hbar c)^{-1}I\Phi_q$ and $dW/dt = \dot{U} = c^{-1}I\dot{\Phi}_q$ is the work per unit of time done on the current I. Equation (5) is useful in the context of the discussion on the locality of the AC effect [10].

Before extending the above considerations to the case of an electric dipole, it is convenient to recall the general expression of its phase [5]. An electric dipole moving with a nonrelativistic velocity $\mathbf{v}$ may be thought of as being composed of two charges $\pm q$ separated by the small distance $\mathbf{r}' = \mathbf{x}_1 - \mathbf{x}_2$. Let the position of the center of mass be $\mathbf{x}$ and consider the expansion $\mathbf{A}(\mathbf{x}_i) \simeq \mathbf{A}(\mathbf{x}) + (\mathbf{x}_i - \mathbf{x})\cdot\nabla\mathbf{A}$. In the dipole approximation, $\mathbf{Q}(\mathbf{x}) = (q/c)\mathbf{A}(\mathbf{x}_1) - (q/c)\mathbf{A}(\mathbf{x}_2) \simeq (\mathbf{d} \cdot \nabla)\mathbf{A}(\mathbf{x})/c$ where $\mathbf{d} = q\mathbf{r}'$ is the electric dipole moment. By means of the principle of superposition of effects, the phase is the sum of the phases of each charge so that $\phi = \hbar^{-1} \int \mathbf{Q}\cdot d\mathbf{x} = (\hbar c)^{-1} \int (\mathbf{d} \cdot \nabla)\mathbf{A}\cdot d\mathbf{x}$.

The same result has been obtained in Ref. [5] using a Lagrangian formulation applied to a nonrelativistic model of a dipole (which may be induced by a uniform field $\mathbf{E}_0$) where the two charges are held together by internal forces. If $\mathbf{d}_0$ is the dipole expectation value, the observable phase shift reads

$$\Delta\phi = \frac{1}{\hbar} \oint \mathbf{Q}\cdot d\mathbf{x} = \frac{1}{\hbar c} \oint (\mathbf{d}_0\cdot\nabla)\mathbf{A}\cdot d\mathbf{x} = \frac{1}{\hbar c} \oint [\mathbf{B} \times \mathbf{d}_0 + \nabla(\mathbf{d}_0\cdot\mathbf{A})]\cdot d\mathbf{x}, \quad (6)$$

where, by using vector identities, we have made explicit the field-dependent term $\mathbf{B} \times \mathbf{d}_0$ related to the Röntgen interaction [11]. Our result (6) for the phase shift of an electric dipole differs from that proposed by other authors [3] and [4] due to the presence of the extra term $(1/\hbar c) \oint [\nabla(\mathbf{d}_0\cdot\mathbf{A})]\cdot d\mathbf{x}$. The difference is not trivial because, if the quantity $\nabla(\mathbf{d}_0\cdot\mathbf{A})$ turns out to be proportional to the gradient of the multivalued function $\theta(\mathbf{x})$, the integral $\oint \nabla(\mathbf{d}_0\cdot\mathbf{A})\cdot d\mathbf{x} \propto \oint \nabla\theta \cdot d\mathbf{x} = \oint d\theta$ does not vanish.

Let us now consider a dipole moving in the presence of a vector potential $\mathbf{A}$ and look for a current distribution that generates a phase shift $\Delta\phi$ with a topology equivalent to that of the AB effect. For our purpose, we use here a combination of the magnetic sheet configurations used in

Refs. [3] and [5]. As shown in Fig. 1a, our sheet covers the y-z semiplane (from $y = 0$ to $y = \infty$) and is made of lines of magnetic dipoles oriented in the z direction. If the interferometric path has to encircle the z axis, a segment of the path will have to intersect the magnetic sheet and a hole in the sheet must be left through which the particles may travel undisturbed. Unless it is surrounded by a ring of permeable material as discussed below, a small hole leaves potential and field practically unchanged.

If m is the magnetic dipole per unit of volume and τ the thickness of the sheet, then the magnetic field inside the sheet ($0 \leq x \leq \tau$) is $\mathbf{B} = \hat{\mathbf{k}} 4\pi m$ and the corresponding vector potential is $\mathbf{A}_B = \hat{\mathbf{j}} 4\pi m(-\tau/2 + x)$. Outside the sheet, the magnetic field is zero. However, to obtain the total vector potential $\mathbf{A}(\mathbf{x})$ we have to add to $\mathbf{A}_B$ the contribution $\mathbf{A}_m(\mathbf{x}) = \int_V [\mathbf{m} \times (\mathbf{x} - \mathbf{x}_m)] |\mathbf{x} - \mathbf{x}_m|^{-3} \, d^3 x_m$, due to the lines of magnetic dipoles. Actually, what one needs are only the derivatives $\partial \mathbf{A}_m / \partial x$ and $\partial \mathbf{A}_m / \partial y$ which turn out to be $\partial \mathbf{A}_m / \partial x = -2m\tau(\hat{\mathbf{i}} x + \hat{\mathbf{j}} y)/r^2$, $\partial \mathbf{A}_m / \partial y = 2m\tau(-\hat{\mathbf{i}} y + \hat{\mathbf{j}} x)/r^2$, where $\nabla \times \mathbf{A}_m = 0$ and $\nabla \times \mathbf{A} = \nabla \times (\mathbf{A}_B + \mathbf{A}_m) = \mathbf{B}$.

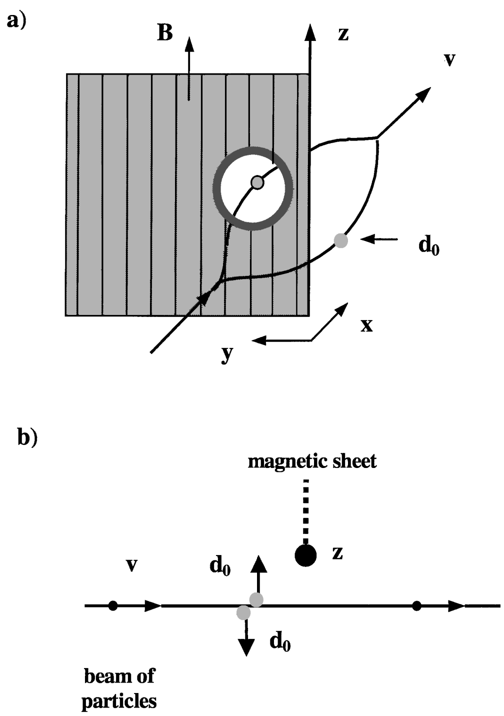

FIG. 1. (a) The beam of particles is split on the plane of motion before reaching the magnetic sheet where the field $\mathbf{B}$ is confined within the sheet. The path encircles the singularity z and one segment of the interferometric path goes through a hole where $\mathbf{B} = 0$ because of the shielding of a ring made of permeable material. Particles on opposite sides of the singularity acquire opposite phases and the phase of the outcoming beam is shifted by the observable amount $\Delta\phi$. (b) Interferometric path of particles possessing opposite electric dipole moment $\pm \mathbf{d}_0$. In this case there is no need to split the beam. Particles with opposite dipole moment acquire opposite phases and the phase of the beam is shifted by the observable amount $\Delta\phi$.

The topology of our effect depends on the geometrical properties of $\mathbf{Q} = c^{-1}(\mathbf{d}_0 \cdot \nabla)\mathbf{A}$ and its vorticity $\nabla \times \mathbf{Q}$, characterized by Eqs. (2) and (3) regardless of the distribution of sources, fields, and potentials. Taking $\mathbf{d}_0$ parallel to the $\mathbf{m} \times \mathbf{v}$ direction (here the y direction), one finds $(\mathbf{d}_0 \cdot \nabla)\mathbf{A}_B = d_0 \partial_y \mathbf{A}_B = 0$ and, for $r \neq 0$,

$$\begin{aligned} \mathbf{Q} &= c^{-1}(\mathbf{d}_0 \cdot \nabla)\mathbf{A}_m = c^{-1}\nabla(\mathbf{d}_0 \cdot \mathbf{A}_m) \\ &= c^{-1} 2 d_0 m\tau(-\hat{\mathbf{i}} y + \hat{\mathbf{j}} x)/r^2 = L \nabla\theta . \end{aligned} \tag{7}$$

The same result is obtained by calculating $\mathbf{Q}$ and $\mathbf{L}$ using expression (2) yielding, in agreement with Eq. (3), $\mathbf{Q} = \nabla \times \mathbf{T}$ and $\nabla \times \mathbf{Q} = \mathbf{L}\delta(r)/r = \hat{\mathbf{k}} c^{-1} 2 d_0 m\tau \delta(r)/r$.

Inspection of Eq. (7) shows that $\mathbf{Q}$ is not altered if at the intersection of the path with the magnetic sheet the field $\mathbf{B}$ vanishes. Thus, we may place a ring made of permeable material around the hole in the sheet so that the lines of the field are guided through the ring and $\mathbf{B} = \mathbf{A}_B = 0$ in correspondence to the hole [12]. Another way to see that $\mathbf{Q}$ is not affected by the permeable material, is to write, as in the right-hand side of Eq. (6), $(\mathbf{d}_0 \cdot \nabla)\mathbf{A} = \mathbf{B} \times \mathbf{d}_0 + \nabla[\mathbf{d}_0 \cdot (\mathbf{A}_m + \mathbf{A}_B)]$ where $\nabla(\mathbf{d}_0 \cdot \mathbf{A}_B) = \hat{\mathbf{i}} d_{0y} \partial_x A_{By} = -\mathbf{B} \times \mathbf{d}_0$. Consequently, $(\mathbf{d}_0 \cdot \nabla)\mathbf{A} = \mathbf{B} \times \mathbf{d}_0 - \mathbf{B} \times \mathbf{d}_0 + \nabla(\mathbf{d}_0 \cdot \mathbf{A}_m) = \nabla(\mathbf{d}_0 \cdot \mathbf{A}_m)$ as in (7). In conclusion, by means of the permeable material, the field $\mathbf{B}$ and the vector potential $\mathbf{A}_B$ may be eliminated at the intersection without altering the topological properties of the phase and of the em momentum $\mathbf{Q}$. In this case there are no fields on the path of the particle and $(\mathbf{d}_0 \cdot \nabla)\mathbf{A} = \nabla(\mathbf{d}_0 \cdot \mathbf{A}_m) = \nabla(\mathbf{d}_0 \cdot \mathbf{A})$ so that, from Eqs. (4), (6), and (7),

$$\begin{aligned} \Delta\phi &= \frac{1}{\hbar} \oint \mathbf{Q} \cdot d\mathbf{x} = \frac{1}{\hbar c} \oint \nabla(\mathbf{d}_0 \cdot \mathbf{A}) \cdot d\mathbf{x} \\ &= \frac{1}{\hbar} \oint \frac{\delta(r)}{r} \mathbf{L} \cdot d\mathbf{S} = \frac{L}{\hbar} \oint d\theta \\ &= \frac{(\delta n)}{\hbar c} 4\pi d_0 m\tau , \end{aligned} \tag{8}$$

and the phase shift possesses a nonlocal nature and a topology equivalent to that of the AB effect.

The difference between the other effects for electric dipoles [3–5] and the present, which is a generalization of the AB effect, is apparent from the inspection of Eq. (6). The interpretation given in Refs. [3–5] is that the physical origin of these effects is due to the Röntgen interaction term $\mathbf{B} \times \mathbf{d}_0$ while $\oint \nabla(\mathbf{d}_0 \cdot \mathbf{A}) \cdot d\mathbf{x} = 0$. In the present effect, instead, $\mathbf{B} = \mathbf{B} \times \mathbf{d}_0 = 0$ while $\oint \nabla(\mathbf{d}_0 \cdot \mathbf{A}) \cdot d\mathbf{x} \neq 0$. Thus, the experimental verification of result (8) is also a test of these two contrasting interpretations.

To measure the phase shift, one needs to prepare a beam of dipoles in the state $\langle \mathbf{d} \rangle = \mathbf{d}_0 \neq 0$, moving with uniform velocity and with $\mathbf{d}_0$ in the direction of the vector $\mathbf{v} \times \hat{\mathbf{k}} m$, and employ interferometers in which the incoming beam of particles (see Fig. 1a) is split into two coherent beams that pass on opposite sides of the singularity and then recombine. Because of the em interaction, particles on opposite sides of the interferometric path acquire opposite phases and the beam coming out is phase shifted by the amount $\Delta\phi$ to be measured by the interferometer.

An even simpler configuration consists of a beam of particles formed by two coherent beams, not spatially separated, possessing *opposite* electric moments $\pm\mathbf{d}_0$ and made to pass through the *same* vector potential. We do not know if this arrangement is realizable with present technology but it is worth mentioning that, in order to experimentally detect the AC phase, Sangster *et al.* [13] have developed such an arrangement for *magnetic* dipoles. In our case, particles with opposite dipole moments can be made to pass on one side of the singularity as shown in Fig. 1b. From Eq. (6), particles with $\pm\mathbf{d}_0$ acquire opposite phase leading once more to the observable phase shift (8).

Concerning the analysis of possible experimental verification of the phase shift, we mention here the related relevant aspects considered in Refs. [3–5] The phase shift (8) is conveniently expressed as

$$\Delta\phi \sim 4.0\,\frac{d_0}{(ea_o)}\,\frac{\tau}{(\mathrm{mm})}\,\frac{B}{(\mathrm{kG})} \sim \frac{2\alpha E_0 \tau B}{\hbar}\,.$$

These two equivalent expressions for $\Delta\phi$ are given in physical and mks units, respectively, and the last term has a form suitable for induced dipoles. Some atom interferometers can detect phases of 0.1 rad [14], and atomic beam splitters may reach the supermillimeter range [15]. Thus, the thickness τ may be of the order of 1 mm and both molecular ($d_0 \sim 4ea_o$) or atomic ($d_0 \leq ea_o$) interferometers may be used. For our configuration, B may reach relatively high values because the solenoids extend from $z = -\infty$ to $z = \infty$ and, as in the AB effect with a toroid, need not be open. By using material with high permeability or superconducting magnetic sheets, the field strength may be well above the kG range. For alkali atoms $\alpha \sim 10 \times 10^{40}$ Fm^2, and with $B \sim 1$ T and $E_0 \sim 10^6$ V/cm, it is possible to achieve a phase shift greater than $\pi/2$. In conclusion, this quantum effect for electric dipoles may be observed in atom or molecular interferometry and its verification is within reach of present experimental technique.

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