Self-Adaptation in Vibrating Soap Films

Arezki Boudaoud, Yves Couder, and Martine Ben Amar

Laboratoire de Physique Statistique, Ecole Normale Supérieure (Laboratoire associé au CNRS et aux Universités Paris VI et Paris VII) 24, rue Lhomond, F-75231 Paris Cedex 05, France

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The vibration of a soap film set into motion by a sound wave is studied experimentally and theoretically. In contrast with the well separated resonances of a solid membrane, the modes of a liquid film exist in wide ranges of frequencies and the vibration amplitude is large for all forcing frequencies. This is due to the adaptation of the film mass distribution which concentrates at the antinodes as observed by interference fringes in monochromatic light. The theoretical model takes into account the variation of surface tension with thickness and explains the experimental results. [S0031-9007(99)09106-1]

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In early times, vibrating soap films were considered as archetypes of vibrating membranes. For instance Rayleigh [1] reports experiments of this type by Melde (1876). There are also observations by Taylor [2] and more recently by Bergman [3] on circular or square membranes. Quantitative studies are more recent and revealed that the vibration of liquid films is more complex than that of solid films. A linear theory of the propagation of waves in soap films was developed by Lucassen *et al.* [4]. Experimentally two different types of behavior are observed when soap films are force into vibration. If the films are thin, strong and rapid recirculations are generated as reported by Afenchenko *et al.* [5] and studied theoretically by Vega *et al.* [6]. In contrast, if the films are thick, the mass distribution adapts itself so that the system retains large amplitude oscillations at all forcing frequencies. This self-adaptation of soap films to the forcing was first pointed out by Airiau, Couder, and Rabaud [7]. A similar effect was found in vibrating smectic films with large forcing by Brazovskaia *et al.* [8]. However no complete model for the self-adaptation of these systems was given. A better understanding of the phenomenon was reached in systems where it results from the addition of a discrete mass. This was obtained with a small ball suspended in a vibrating smectic film [9] or with a bead sliding on a vibrating string [10]. In this latter case there is a slow dynamics by which the bead adjusts its position so that the system is resonant with the forcing. A theoretical model for this effect is given in [10].

While Refs. [9,10] considered discrete systems, in the present Letter we are concerned with self-adaptation in a continuous system. For this purpose we revisit vibrating soap films. As we will show they fundamentally differ from solid membranes because of the possibility of thickness variations. The film is formed of a 1% water solution of commercial soap ("Mir multiusages") mixed with 7% of glycerol. It has a surface tension $\sigma = 22$ mN/m and a mean thickness $e \approx 5$ μm. The frame is a rectangle of length $L = 16$ cm and width $W = 2.5$ cm. It is placed horizontally and the forcing is provided by a large loudspeaker located below the film and giving a spatially homogeneous forcing. Using a function generator and a hi-fi amplifier the excitation frequency can be tuned continuously. The whole system is placed in a perspex box to avoid drafts. The film is lit with a large monochromatic sodium lamp located behind a diffusing screen. Observed in reflection the film exhibits equal thickness interference fringes, the thickness varying by 0.22 μm between two neighboring fringes. A video camera placed just above the box is used to record the evolution of the interference patterns. We used the reflexion of a plane laser sheet of light by the film across the middle of its width. The largest deflections at the antinodes give a measure of the amplitude of the film vibration. The transverse profile of the film in the direction z perpendicular to its plane can also be observed with a stroboscopic light.

If care is taken for the initial stretching of the film, reproducible results are obtained [11]. After it has been set into vibration, an organized interference pattern appears after approximately 10 sec (Fig. 1). It is due to spatial variations of the thickness related to the vibration of the membrane. The fringes form concentric ellipses showing that the film has become thicker at the antinodes. By counting the fringes, we estimate the film thickness to vary from about 0.2 μm near the frame and at the nodes to more than 200 μm at antinodes.

If the film behaved as a constant density solid membrane, the transverse displacement z for the eigenmodes would be

$$z = A \sin[n\pi(x/L + 1/2)] \sin[m\pi(y/W + 1/2)] \times \sin(\omega_{nm} t),$$

where $-L/2 \le x \le L/2$, $-W/2 \le y \le W/2$. The complete eigenfrequencies spectrum ω_{nm} would be given by

$$\omega_{nm} = \pi \sqrt{\frac{2\sigma}{\rho e^*}} \sqrt{\frac{n^2}{L^2} + \frac{m^2}{W^2}}. \quad (1)$$

where e^* is the effective thickness of the membrane (taking

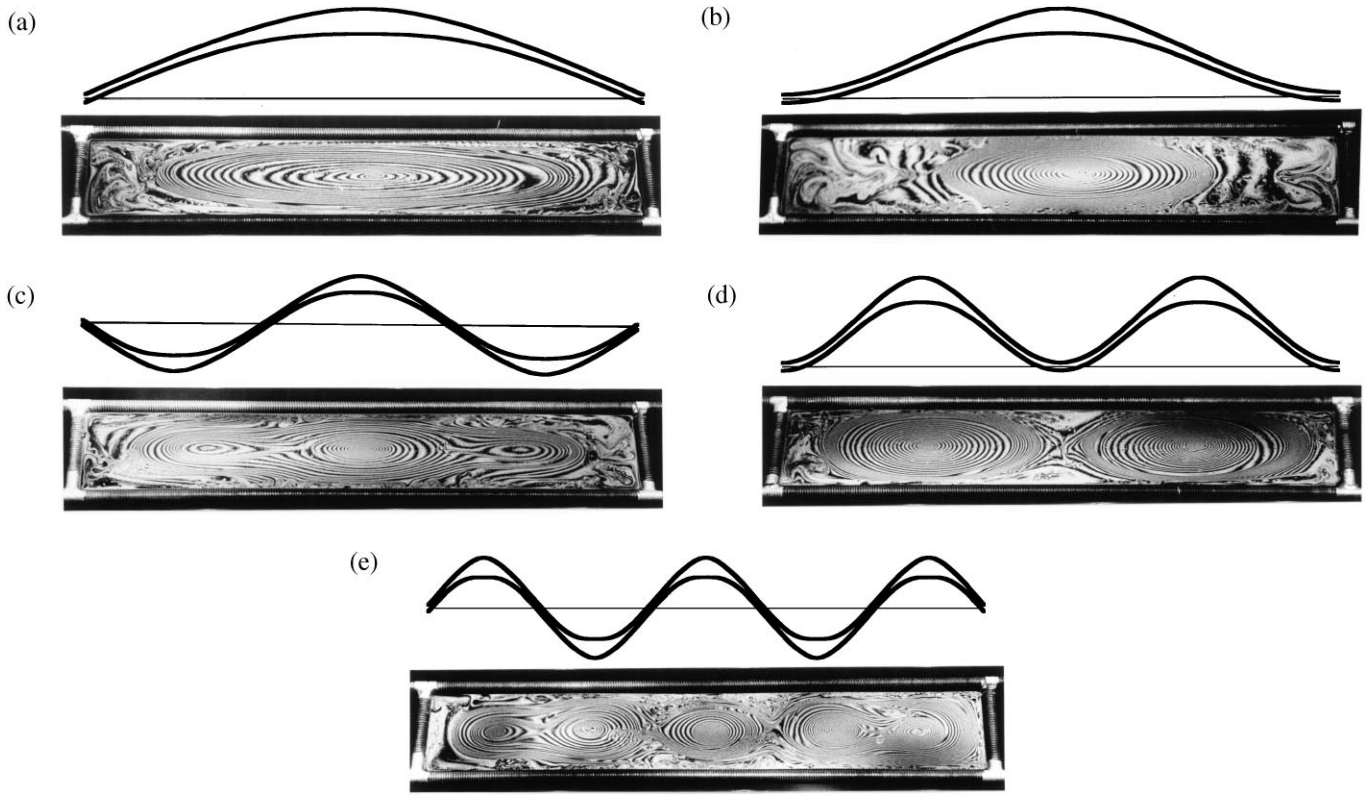

FIG. 1. The interference patterns observed on the soap films at increasing frequencies and the theoretical longitudinal cross section obtained from the model [Eq. (6)]. The interference fringes are the lines of equal thickness and mass is concentrated at the antinodes. For a better visualization of fringes, the films used for the photographs were drained to be thinner than usual. In the theoretical cross sections, the thickness is magnified. (a) Mode 1 at 21 Hz and corresponding theoretical shape at $\omega/\omega_1 = 0.99$. (b) Mode 2 at 26 Hz and the theoretical profile at $\omega/\omega_1 = 2$ (note that when the forcing is homogeneous there is only one central antinode). (c) Mode 3 at 30 Hz and the theoretical profile at $\omega/\omega_1 = 2.99$. (d) Mode 4 at 33 Hz and the predicted profile at $\omega/\omega_1 = 4$ (with a homogeneous forcing there are two antinodes). (e) Mode 5 at 38 Hz and the theoretical profile at $\omega/\omega_1 = 3.99$.

into account the inertia of the air moving with the film), ρ is the density of the intersticial fluid, and σ is the surface tension of one side of the film. When forced into vibration [1], the amplitude of response would be very small unless the forcing frequency was close to a natural frequency ω_{nm}.

Actually, the soap film has a large amplitude of oscillations for all forcing frequencies except below all natural frequencies ω_{nm} ($f < 15$ Hz) where there is hardly any vibration. The aspect of the soap film at increasing forcing frequencies is shown in Fig. 1. For $16 \leq f \leq 23$ Hz, the film organizes and concentric elliptical fringes form, surrounding a single central antinode [Fig. 1(a)]. The system is in its fundamental mode. For larger frequencies $24 \leq f \leq 28$ Hz, though there is still one antinode, the fringes tend to become more circular in the central part of the film [Fig. 1(b)], and the extremities do not vibrate. Starting at about 28 Hz there is a flow by which some of the mass is transferred from the central region towards the extremities so that two new sets of elliptical fringes appear symmetrically with respect to the central one. This state with three antinodes [Fig. 1(c)] is observed in the range 28–31 Hz, with more and more mass being transferred from the central antinode to the lateral ones. Ultimately, only the two lateral antinodes remain [Fig. 1(d)] in the range 32–36 Hz. The process repeats itself when new antinodes appear in-between the existing antinodes and between them and the frame, leading to a state with five antinodes [Fig. 1(e)] in the range 37–39 Hz. This evolution continues for frequencies up to about 55 Hz where the pattern starts evolving in the transverse direction, leading at about 90 Hz to the formation of a two dimensional pattern with staggered antinodes. We will concentrate on the frequency range $f < 55$ Hz where there is exactly half a wavelength in the frame's width. So, we will consider from now on that the median region of the film width has a 1D dynamics. This is a bold hypothesis, but it permits a complete treatment of the problem.

The sequence 1, 1, 3, 2, 5, 3, 7, ..., $2p - 1$, $2p/2$, $2p + 1$, ..., for the numbers of antinodes appears in contradiction with the usual situation in 1D vibrating systems where the number of antinodes is 1, 2, ..., p, $p + 1$, ... This is due to the specificity of the homogeneous forcing discussed for strings in Morse [12]. The usual even eigenmodes have a shape which is antisymmetrical with respect to the middle of the system

while the forcing is symmetrical. For this reason they are not excited. When forced at $\omega = 2\pi f$, the shape of the film is

$$z(x,t) = A\left(\frac{\cos[(\pi\omega/\omega_1)(x/L)]}{\cos(\pi\omega/2\omega_1)} - 1\right)\sin\omega t\,. \quad (2)$$

Here z is the vertical displacement of the film at a position x (with $-L/2 < x < L/2$). A is a typical displacement proportional to the forcing and ω_1 is the fundamental frequency. The profiles given by Eq. (2) when ω/ω_1 is in the vicinity of an integer n are of the type displayed in Fig. 1 (for $\omega/\omega_1 = 0.99$, 2, 2.99, 4, and 4.99). When ω/ω_1 is close to an odd integer n there are n antinodes. When it is close to an even integer there are only $n/2$ antinodes. This explains the observed sequence of numbers of antinodes for our system.

In the case discussed by Morse the amplitude of vibration given by (2) diverges in the vicinity of the odd modes which are the only resonances. For the other frequencies, including when ω/ω_1 is an even integer, the amplitude is very small. What is observed here with fluid films is different: The amplitude is large in all cases (Fig. 2). The vibration amplitude varies by only about a factor 4 (from 0.5 to 2 mm) in a wide frequency range (20–55 Hz). A resonancelike behavior remains, however, for the odd modes. The amplitude versus frequency curve (Fig. 2) exhibits weak peaks at frequencies located within the range of observation of each odd mode. A jump of π of the phase of the motion relative to the phase of the forcing is observed at these peaks. The width of these phase jumps shows the quality factor to be $Q \sim 20$.

We can now measure the wavelength as a function of frequency. We define it as the mean distance between two neighboring antinodes for the even modes and as twice this distance in the odd modes. The dispersion curve (Fig. 3) shows small steps at frequencies $f = 27.5$, 31.5, 36.5, 39.5, 44.5, and 49.5 Hz accounting for a local fast variation of the wavelength. These frequencies are the extremities of the intervals where a given mode is observed. The vibrating film adapts to the forcing frequency: As long as the mass distribution can adjust to the forcing, the wavelength changes smoothly. When the mass distribution is too constrained to follow the forcing, there is a jump in the wavelength and a change of mode. This effect is not included in the model presented below as we implicitly assume that the mass concentration at an antinode is not too large. In the experiment, a drop may form at each antinode. We will model this elsewhere [13].

We now write the equations for the deflection $z(x)$ from equilibrium and for the film thickness $e(x)$. For the sake of simplicity, our model is only 1D. The wave equation reads

$$\rho e\,\partial_{tt} z = 2\partial_x(\sigma\,\partial_x z) + F\sin(\omega t)\,. \quad (3)$$

F is the forcing. The experiment shows that damping is small [14]. We will look for periodic solutions $z = Z\sin(\omega t)$. We can estimate using Eq. (5) $(\partial_x\sigma\,\partial_x z)/(\sigma\,\partial_{xx} z) \sim (\omega Z/\omega_0 L)^2 \ll 1$, so that $\partial_x(\sigma\,\partial_x z) \sim \sigma\,\partial_{xx} z$. The wave equation becomes

$$-\rho e\omega^2 Z = 2\sigma\,\partial_{xx} Z + F\,. \quad (4)$$

In the tangent plane, two forces are acting on the film. The projection of the local acceleration on the tangent $(\partial_{tt} z\,\partial_x z)$ gives a centrifugal force of temporal mean $\rho\omega^2 e Z\,\partial_x Z$. It accounts for the mass concentration at antinodes. The restoring force comes from the variations of surface tension with thickness. This effect is known as Marangoni or Gibbs elasticity of soap films [15]. It is due to the thermodynamical equilibrium between soap molecules in the bulk and at the interface. From Ref. [15], we take the force acting against thickness

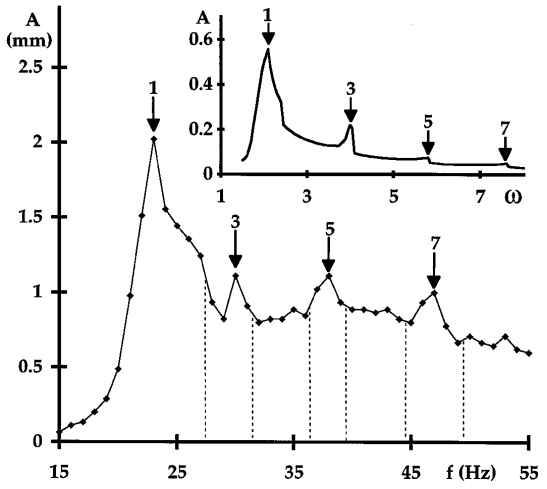

FIG. 2. Experimental amplitude of vibration versus the forcing frequency. The dotted lines mark the frequency of transition between two modes. Inset: Theoretical adimensional amplitude versus adimensional frequency. In the two cases the amplitude remains large at most frequencies.

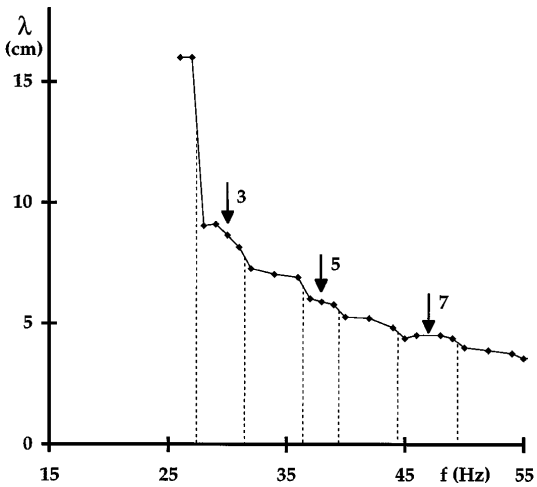

FIG. 3. The experimental wavelength versus the forcing frequency. At the transition frequencies shown by dotted lines, the curve has small steps showing a local faster evolution of λ.

variations to be $\partial_x \sigma = -E \partial_x e/e$, where E is the elasticity coefficient of the film ($E \approx 0.15$ N/m). We obtain the equation for the equilibrium in the film plane,

$$-\frac{E}{e}\,\partial_x e + \rho\omega^2 e Z \partial_x Z = 0\,. \quad (5)$$

The set (4)-(5) is integrable analytically. Equation (5) gives the thickness $e = E/(B - \frac{1}{2}\rho\omega^2 Z^2)$, where B is a constant determined through mass conservation. The general solution reads

$$x = \varepsilon \int \sqrt{\frac{\sigma}{E \ln(1 - \rho\omega^2 Z^2/2B) - FZ + A}}\, dZ\,. \quad (6)$$

The constants $\varepsilon = \mathrm{sgn}(\partial_x Z)$ and A result from the boundary conditions $Z(-L/2) = Z(l/2) = 0$. Using the experimental value $F \approx 0.15$ N/m^2, we recover both a thickness and an amplitude frequency dependence similar to the experimental ones. The mass concentrates at the antinodes (Fig. 1) and adjusts with the forcing frequency. This adaptation allows the amplitude to remain large (Fig. 2, inset) in a wide range of frequencies. The theoretical curve of the Fig. 2 inset also exhibits weak peaks with phase jumps corresponding to resonances. The qualitative agreement shows that the self-adaptive behavior can be explained only by the spatial variations of thickness. In the experiment, the first and second peaks are weaker than expected, probably because of damping by air friction.

As a conclusion, we have shown that the fluidity of soap films gives them a behavior very different from solid membranes. The liquid film can adapt its mass distribution to the forcing frequency. This adaptation is smooth within the interval of a mode existence and fast at the interval extremities. These transitions, as well as the phase jumps at the resonance, occur through the coupling between the vibration and Marangoni waves. These waves were already shown to be important in the vorticity generation [5,6]. The study of this coupling is beyond the scope of the present Letter and will be developed elsewhere [13].

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