Formation Rate of Semilocal Strings

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We carry out three-dimensional numerical simulations to investigate the formation rate of semilocal strings. We find that the backreaction of the gauge fields on the scalar field evolution is substantial, and leads to a significant formation rate in the parameter regime where the semilocal strings are classically stable. The formation rate can be as large as one-third of that for cosmic strings, depending on the model parameters. [S0031-9007(99)09087-0]

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It is expected that the early Universe underwent a series of phase transitions as it cooled down. Typically, such transitions lead to the formation of a network of defects [1], which may be of relevance to a number of phenomena including structure formation in the Universe and baryogenesis [2]. The best known examples are topological defects such as cosmic strings, whose stability is guaranteed by the topological structure of the symmetry breaking at the phase transition, but it is also possible for nontopological defects to form. An example of the latter is semilocal strings [3,4]; the semilocal string model is simply the Weinberg-Salam model without fermions or W bosons, in the limit in which the SU(2) coupling constant is set to zero. The only parameter in the theory is $\beta \equiv m_s^2/m_v^2$, the ratio between the scalar and vector masses (squared). Its vacuum manifold is the three-sphere S^3, and has no noncontractible loops. Despite this, Nielsen-Olesen vortices [5] may form, and are classically stable if $\beta < 1$ [3,4]. Semilocal strings are closely related to electroweak strings [6], which can be formed in the electroweak phase transition, and so understanding the formation and evolution of these nontopological defects is an important task.

The semilocal model is characterized by the gauge fields having insufficient degrees of freedom to be able to completely cancel the scalar field gradients, even away from the core of any strings which form. While their stability depends on the parameter β, *stable* semilocal strings are stable not only to small perturbations but also to semiclassical tunneling [7]. Further, although they can, unlike topological strings, come to an end (in what is effectively a global monopole), they will not decay by breaking into smaller segments [8]. On the contrary, Hindmarsh [9] has conjectured that the long-range interaction between these monopoles should lead to short pieces of strings growing into longer ones.

We have recently shown [10], using a toy model with parallel strings, that semilocal strings can be identified by studying the pattern of magnetic flux in a simulation. For topological strings, an estimate of the formation rate in systems with planar symmetry is sufficient to determine the three-dimensional rate, since topology prevents the strings from having an end [11]. But nontopological strings can terminate, with the flux spreading out. The closest to a three-dimensional analytic estimate for semilocal string is Hindmarsh's calculation [9] of the average magnetic flux through a correlated area at $\beta = 0$, with the conclusion that vortices are rare. For the electroweak string, Nagasawa and Yokoyama [12] proposed a technique based on studying the scalar field alone and concluded that the initial density would be negligibly small. However, both approaches neglect the gauge fields, which we have found to play a key role [10].

In this paper, we aim to estimate the density of semilocal strings at formation. The usual argument for the formation of (topological) cosmic strings relies on the vacuum manifold having noncontractible loops which can force the existence of closed or infinite lines in which the Higgs scalar must have zero value, confining the magnetic field to these vortex structures where the symmetry has been restored. The lack of topology prevents such arguments being employed for semilocal strings, and numerical methods must be employed. This requires two parts—first, a plausible initial configuration where the field configuration captures the essence of a thermal phase transition (principally, the existence of a correlation length) and, second, dynamical evolution to allow the strings to "condense out" and be counted. The numerical simulation of a network of defects is a difficult problem because of the large range of scales involved in the dynamics, but it has come within the capability of modern supercomputers. It is often tacitly assumed that only topological defects are sufficiently robust to form in a phase transition through the Kibble mechanism [1]. Given that it is not presently known whether semilocal strings form at a comparable density to cosmic strings,

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or with a completely negligible density, our target is an order-of-magnitude estimate.

The simulations.—We work in flat space-time throughout. The Lagrangian for the simplest semilocal string model [3] is

$$\mathcal{L} = (\partial_\mu - iA_\mu)\phi_1^\dagger(\partial^\mu + iA^\mu)\phi_1 + (\partial_\mu - iA_\mu)\phi_2^\dagger(\partial^\mu + iA^\mu)\phi_2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\beta}{2}(|\phi_1|^2 + |\phi_2|^2 - 1)^2, \quad (1)$$

where ϕ_1 and ϕ_2 are two equally charged complex scalar fields, A_μ is a U(1) gauge field, and $F_{\mu\nu}$ is the associated gauge field strength. Notice that the gauge coupling and the vacuum expectation value of the Higgs have been set to one by choosing appropriate units (the inverse vector mass, for length, and the symmetry breaking scale, for energy). The only remaining parameter in the theory is $\beta = m_s^2/m_v^2$, whose value determines the stability of an infinitely long, straight, semilocal string with a Nielsen-Olesen profile: It is stable for $\beta < 1$, neutrally stable for $\beta = 1$, and unstable for $\beta > 1$ [3,4,13]. For $\beta = 1$, there is a family of solutions with the same energy and different core widths, of which only the semilocal string has complete symmetry restoration in the center [4].

We work in temporal gauge $A_0 = 0$. Splitting the scalar fields into four real scalars via $\phi_1 = \psi_1 + i\psi_2$, $\phi_2 = \psi_3 + i\psi_4$, the equations of motion are

$$\ddot{\psi}_a - \nabla^2\psi_a + \beta(\psi_1^2 + \psi_2^2 + \psi_3^2 + \psi_4^2 - 1)\psi_a + A^2\psi_a + (-1)^b(2A\cdot\nabla + \nabla\cdot A)\psi_b = 0, \quad (2)$$

(where b is the complement of a—$1 \leftrightarrow 2$, $3 \leftrightarrow 4$, and dots are time derivatives) for the scalar fields and

$$\ddot{A}_i - \nabla^2 A_i + \partial_i\nabla\cdot A + 2(\psi_1 \overleftrightarrow{\partial_i} \psi_2 + \psi_3 \overleftrightarrow{\partial_i} \psi_4) + 2A_i(\psi_1^2 + \psi_2^2 + \psi_3^2 + \psi_4^2) = 0, \quad (3)$$

for the gauge fields ($i = 1, 2, 3$), together with Gauss' law, which here is a constraint derived from the gauge choice and used to test the stability of the code,

$$2(\psi_1 \overleftrightarrow{\partial_0} \psi_2 + \psi_3 \overleftrightarrow{\partial_0} \psi_4) + \partial_i\dot{A}_i = 0. \quad (4)$$

The arrows indicate asymmetric derivatives.

This system is discretized using a standard staggered leapfrog method; however, to reduce its relaxation time we also add an *ad hoc* dissipation term to each equation ($\eta\dot{\psi}_i$ and $\eta\dot{A}_i$, respectively). This is to allow the strings to condense out and be identified. In an expanding Universe, the expansion rate would play such a role, though η would typically not be constant. We tested a range of strengths of dissipation, and checked that it did not significantly affect the number densities obtained. Further, our results are always compared to the cosmic string case where dissipation would have the same effect. The simulations we display later used $\eta = 0.5$.

We set up initial conditions for the scalar field by placing the field in vacuum with random phases on a subgrid. We then iteratively interpolate by bisection onto the full grid; after each bisection, the field is shifted into the vacuum before the next bisection takes place.

Having fixed the initial scalar field configuration, we need to set the initial gauge field. One possibility is to choose the minimum energy configuration on the (fixed) scalar field background (this does not mean that the gauge fields will cancel all scalar gradients, as there are insufficient gauge degrees of freedom). This can only be done exactly numerically, as in Ref. [10], but in that paper it was also shown that an approximate analytic minimization ignoring the magnetic flux term performs perfectly well:

$$A_i(\mathbf{x}) = \psi_1\partial_i\psi_2 - \psi_2\partial_i\psi_1 + \psi_3\partial_i\psi_4 - \psi_4\partial_i\psi_3. \quad (5)$$

All field momenta are set to zero initially.

These initial conditions are one possible choice out of an infinity of possible ways in which one might try to represent conditions resembling a thermal phase transition. Rather than attempt a highly accurate description of the transition, which is not necessary since we are aiming at order-of-magnitude estimates of the formation rate, we need only be confident that reasonable changes to the initial conditions will not significantly alter the results. This we have already tested in two-dimensional simulations [14], where we considered a variety of initial conditions, including ones which may be closer to the sort of thermal environment considered in Ref. [15].

The formation rate.—To minimize any systematic errors in our analysis, we always compare our results to the case of cosmic strings, which is obtained by simply ignoring one of the (complex) scalar fields, setting $\psi_3 = \psi_4 = 0$. This makes the defects topological, and the flux tubes formed now map out the locations of winding in the scalar field configurations. In these field theory simulations of cosmic strings, we can follow the early stages of cosmic string network evolution by displaying the density of magnetic flux, and we do the same with the semilocal string simulations, as was done in two dimensions in Ref. [10].

The starting configurations obtained by our described procedure initially yield a complicated mess of flux. However, after a few time steps this resolves itself into loops and open segments of string. We observed a clear interaction between nearby segments which join to form longer segments. Figure 1 shows two time slices from a single large $\beta = 0.05$ simulation, in a 256^3 box, carried out on the Cray T3E at NERSC; these are close-ups showing only part of the simulation box [16]. We see a collection of short string segments and loops; visually this is very different from a cosmic string simulation where strings cannot have ends. As time progresses, the short segments either disappear or link up to form longer ones.

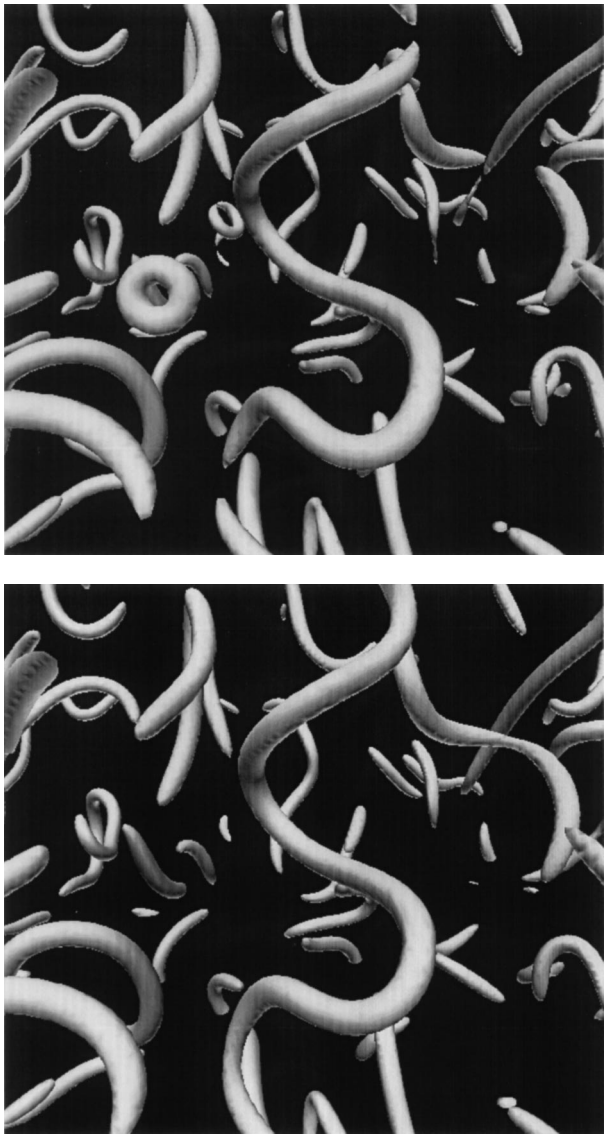

FIG. 1. Part of the large simulation, shown at time 60 and time 70. Note several joinings of string segments, e.g., two separate joinings on the long central string, and the disappearance of some loops. The different apparent thickness of strings is entirely an effect of perspective.

We can immediately conclude from these images that the formation rate of semilocal strings is not extremely close to zero; the fact that flux tubes are observed in our simulations implies that the formation rate cannot be much smaller than one per correlation volume.

Nagasawa and Yokoyama [12] studied the related case of electroweak defects and concluded that the initial density would be totally negligible. However, this is not in contradiction with our results because our semilocal strings arise during the evolution due to backreaction on the gauge fields from the scalar field gradients. This enables initially short pieces of string to join up to form semilocal strings of reasonable size, an effect not included in their analysis. Further, the electroweak string resides in a part of parameter space where the defects are dynamically unstable, and in this case we find that all the flux dissipates soon after the phase transition.

In order to quantify the formation rate, we compute the total length of string in the simulations, always comparing the semilocal string density to that of a cosmic string simulation with the same properties (including dissipation). We determine the length by setting a magnetic flux threshold and computing the fractional volume of the box which exceeds it. In Fig. 2, we plot the length of semilocal string relative to the length found in cosmic string simulations, as a function of time and with $\beta = 0.05$. We see that, after a transient during which the initial tangle of flux sorts itself out, the system settles down to a reasonable equilibrium. During that initial period, the ratio of semilocal strings to cosmic ones grows, as cosmic ones are there right from the start due to topology while the semilocal ones need time to form. Even at late times there is a modest upward trend; we identify this as being caused by the periodicity of the simulation box, which freezes-in any string crossing the box, favoring cosmic string annihilation because of their higher density. We take the relative densities of semilocal and cosmic strings to be that at time 50 in these simulations. We shall investigate this trend more thoroughly in future work using large simulations. There is a modest dependence on the choice of flux threshold, and we set it at one-half the flux density of a Nielsen-Olesen vortex.

Figure 3 shows the ratio of semilocal and cosmic string lengths, as a function of the stability parameter β. These results are derived from 700 simulations (50 semilocal and 50 cosmic at each of seven β values) carried out in boxes of dimension 64^3 using a Sun Ultra II workstation.

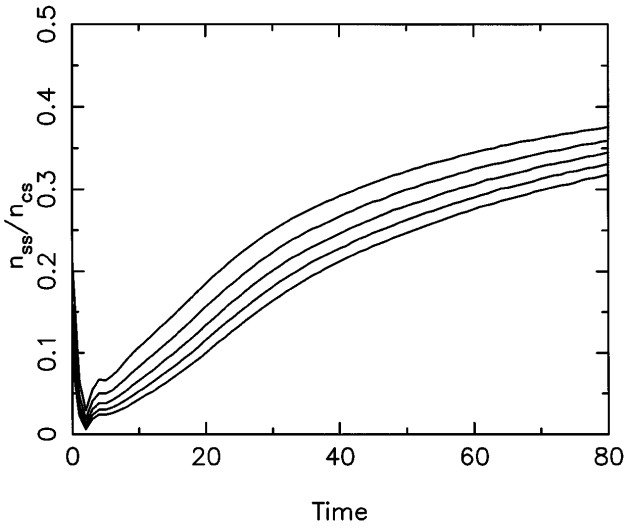

FIG. 2. This shows the ratio of total string lengths in a semilocal and cosmic string simulation, with $\beta = 0.05$. The different lines show different magnetic flux thresholds, from bottom to top they are 0.6, 0.55, 0.5, 0.45, and 0.4 times the peak flux of a Nielsen-Olesen vortex.

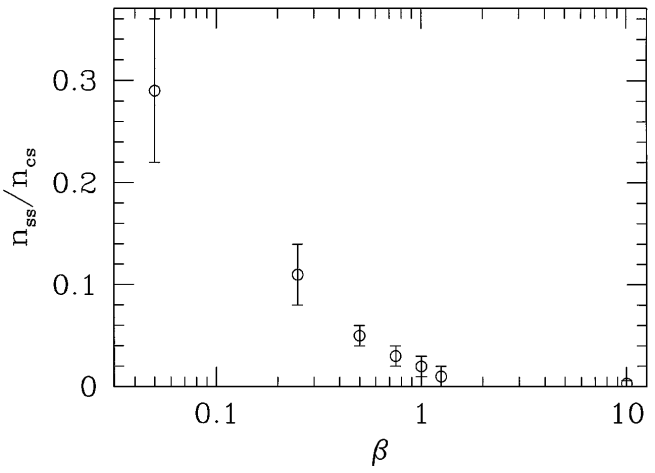

FIG. 3. The ratio of lengths of semilocal and cosmic strings.

Although the initial correlation length of 16 units is a sizable fraction of the box size, we are interested only in brief evolution to allow the strings to become identifiable and so boundary effects are not important. The error bars include the statistical spread between simulations, and an estimated 25% systematic error from the length counting algorithm (see the spread in Fig. 2) and the viscosity. Those latter uncertainties are the dominant ones. Recalling that the formation rate of cosmic strings is estimated to be of order one per correlation volume (0.88 in Ref. [11]), these results are in excellent agreement with the two-dimensional results we found in Ref. [10]. They show a significant formation rate for low β, decreasing dramatically as $\beta \to 1$, beyond which there is no evidence of semilocal string formation. The small amount of string seen in some large-β simulations is an artifact of the viscosity; the flux all dissipates if the viscosity is turned off, while it persists if $\beta < 1$.

In conclusion, our simulations have shown that dynamically stable nontopological defects, such as semilocal strings, can form with substantial density; this happens even if the configuration immediately after the phase transition has no symmetry restoration (recall our initial conditions place the scalar field in the vacuum everywhere), and is due to the backreaction of the gauge fields on the scalars. For $\beta = 0.05$, we found that the formation rate was around 0.3 times that of cosmic strings, while, as β is increased towards the stability/instability transition at $\beta = 1$, the density drops to zero.

Moreover, we have observed short segments of string joining to form long strings and loops, in agreement with Refs. [8,9], and collapsing longitudinally, as invoked in some baryogenesis scenarios [17]. It is important to study such phenomena in the related case of electroweak strings, where although the monopoles are of finite size their cores can become comparable to the separation as $\sin^2\theta_w \to 1$. The details of these dynamics and their implications deserve further study.

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