Coulomb Blockade in Superconducting Quantum Point Contacts

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The amplitude of the Coulomb blockade oscillations is calculated for a single-mode Josephson junction with arbitrary electron transparency D. It is shown that a mechanism related to chiral anomaly completely suppresses the Coulomb blockade in ballistic junctions with $D = 1$. At finite reflection probability, the suppression process is described quantitatively in terms of the Landau-Zener transition. [S0031-9007(99)09032-8]

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Coulomb blockade phenomena in mesoscopic conductors have been actively studied during the past few years [1–3]. They arise from the interplay of discreteness, in units of electron charge e, of electric charge Q of a small conductor, and tunneling into the conductor. Coulomb blockade requires for its existence the "localization" of the charge Q, the condition that implies that the transparency D of the tunnel barriers isolating the conductor is small, $D \ll 1$. In ballistic junctions with $D \to 1$ the charge can move freely in and out of the conductor, and both the charge quantization and the associated Coulomb blockade are suppressed. Until now, full quantitative understanding of such a suppression has been worked out only for nearly ballistic single-mode junctions between two normal conductors [4,5]. It has been shown that for conductors with the quasicontinuous energy spectrum, the amplitude of the Coulomb blockade oscillations vanishes as the junction reflection coefficient approaches zero: $R = 1 - D \to 0$. The aim of this work was to study this problem for superconducting junctions, where the situation appears to be different. Coulomb blockade oscillations arise in this case [6] from the formation of Bloch bands in the Josephson potential $U(\varphi)$ periodic in the Josephson phase difference φ. Since the ballistic junctions also have periodic Josephson potential, one could expect that the Coulomb blockade exists even in the ballistic regime. It is shown below that this expectation is incorrect and, similarly to the normal case, the Coulomb blockade is completely suppressed when $R \to 0$.

Coulomb blockade in superconducting junctions can be conveniently discussed as the quantum dynamics of the Josephson phase difference φ. The standard Hamiltonian for quantum dynamics of φ (see, e.g., [1,7]) consists of the coupling energy $H_c(\varphi)$ of the junction electrodes, which in the case of low-transparency junctions reduces to a simple Josephson potential $U(\varphi)$, and the charging energy $(Q - q)^2/2C$, where C is the junction capacitance, q is the charge injected into the junction from the external circuit, and Q is the charge transferred through the junction. The Coulomb blockade manifests itself as periodic oscillations of the junction characteristics as a function of the charge q with the period $2e$. These oscillations can take place either in time [6,8], when the junction is biased with a dc current I and $q = It$, or as thermodynamic oscillations [9], if one of the junction electrodes is an isolated island and the charge q is induced on the junction capacitance by external gate voltage V_g coupled through a gate capacitance C_g: $q = C_g V_g$. In both situations, oscillation amplitude is the same and can be found from the junction free energy $F(q)$.

For a single-mode junction with quasicontinuous energy spectrum of the electrodes, studied in this work, the coupling energy $H_c(\varphi)$ can be represented similar to the normal case [4] as a sum of the energies $H_{L,R}$ of electrons with momenta $\pm k_F$ moving forward and backward through the junction, and a potential V responsible for scattering between these two directions of propagation. The energy of the forward-moving electrons in a superconductor can be written in the standard matrix form:

$$H_L = \int dx\, \Psi_L^\dagger(x) \begin{pmatrix} -i\hbar v_F \partial/\partial x & \Delta(x) \\ \Delta^*(x) & i x v_F \partial/\partial x \end{pmatrix} \Psi_L(x)\,, \quad (1)$$

$$\Delta(x) = \begin{cases} \Delta, & x < 0\,, \\ \Delta e^{i\varphi}, & x > 0\,, \end{cases}$$

where $\Psi_L^\dagger = (\psi_{L\uparrow}^\dagger, \psi_{L\downarrow})$ is the creation operator for quasiparticles with momentum k_F, and v_F is the Fermi velocity. H_R is given by the same expression with $v_F \to -v_F$. The pair potential $\Delta(x)$ can be written in the steplike form (1) under the assumption that the characteristic junction length d is much smaller than the superconductor coherence length $\hbar v_F/\Delta$.

We limit ourselves to the case of adiabatic phase dynamics, when all energies, including characteristic charging energy $E_C = (2e)^2/2C$ and temperature T, are much smaller than Δ. In this case, the charge Q is carried only by Cooper pairs and can be expressed directly in terms of the Josephson phase difference φ [10]: $Q = -2ei\partial/\partial\varphi$. Even more importantly, the energy spectrum of electrons moving in the contact can be found in this regime by treating φ as stationary. The Hamiltonian $H_L + H_R$ is then reduced to a sum of the quasiparticle energies $\varepsilon_k(\varphi)$ of the occupied states, so that

$$H = \frac{1}{2C}\left(\frac{2e}{i}\frac{\partial}{\partial\varphi} - q\right)^2 + \sum \varepsilon_k(\varphi) + V\,. \quad (2)$$

The spectrum of eigenenergies $\varepsilon_k(\varphi)$ is found by solving the Bogolyubov-de Gennes (BdG) equations with the pair potential $\Delta(x)$ [Eq. (1)]. It consists of the continuum of states at energies outside the gap, $|\varepsilon| > \Delta$, and two discrete states in the gap [11–13]:

$$\varepsilon^{\pm}(\varphi) = \mp\Delta\cos\varphi/2\,, \qquad \Psi^{\pm}(x) = \sqrt{\xi/2}\begin{pmatrix} 1 \\ \mp e^{-i\varphi/2}\end{pmatrix} e^{\pm ik_F x - \xi|x|}, \tag{3}$$

where $\xi = (\Delta/\hbar v_F)\sin\varphi/2$. In all of these expressions $\varphi \in [0, 2\pi]$, and they should be continued periodically in φ beyond this interval. The subgap states merge with the continuum when $\varphi = 0 \bmod (2\pi)$. Equation (3) shows that as φ varies from 0 to 2π the state with momentum k_F moves across the energy gap from the lower half of the continuum, $\varepsilon < -\Delta$, to the upper half, $\varepsilon > \Delta$, while the $-k_F$ state moves in the opposite direction. The states in the continuum also shift up or down in a similar fashion, as can be seen from the Friedel sum rule for the density of states $\rho(\varepsilon)$ (see, e.g., [14,15]):

$$\frac{\partial\rho(\varepsilon)}{\partial\varphi} = \frac{i}{2\pi}\frac{\partial^2}{\partial\varphi\,\partial\varepsilon}\ln\det S(\varepsilon)\,, \tag{4}$$

where $S(\varepsilon)$ is the scattering matrix for scattering off the discontinuity of the pair potential $\Delta(x)$ [Eq. (1)]. The straightforward solution of the BdG equations shows that, for $+k_F$ states,

$$S(\varepsilon) = \frac{1}{e^{i\varphi} - a^2}\begin{pmatrix} |a|(1 - e^{i\varphi}) & (1 - a^2) \\ (1 - a^2)e^{i\varphi} & |a|(1 - e^{i\varphi})\end{pmatrix}, \tag{5}$$

where $a(\varepsilon) = \mathrm{sgn}(\varepsilon)[|\varepsilon| - (\varepsilon^2 - \Delta^2)^{1/2}]/\Delta$ is the amplitude of the Andreev reflection from a superconductor. From Eq. (5), we get

$$\frac{i}{2\pi}\int_0^{2\pi} d\varphi\,\frac{\partial}{\partial\varphi}\ln\det S(\varepsilon) = \begin{cases} 1, & |\varepsilon| > \Delta\,, \\ 0, & |\varepsilon| = \Delta\,. \end{cases} \tag{6}$$

Combined with Eq. (4), this equation means that as φ increases from 0 to 2π, the $+k_F$ states move up in energy, so that precisely one state is removed from the lower half of the continuum, $\varepsilon \le -\Delta$, and is added to the upper half, $\varepsilon \ge \Delta$. Together with the shift of the subgap states this means that the whole spectrum of $+k_F$ states shifts by one state up in energy. Similarly, one can show that the spectrum of $-k_F$ states shifts by one state down.

Such a motion of the energy spectrum determines the effective potential for the dynamics of φ in the Hamiltonian (2). At $\varphi = 0$, when there are no states in the gap, the equilibrium occupation of the eigenstates implies that at $T \ll \Delta$ all of the states with $\varepsilon \le -\Delta$ are filled, while those with $\varepsilon \ge \Delta$ are empty. Since the adiabatic variation of φ does not induce transitions between different quasiparticle states, the shift of the energy spectrum with these occupation probabilities gives rise to the following *aperiodic* potential for φ (Fig. 1):

$$U(\varphi) = \sum \varepsilon_k(\varphi) = \Delta[2m + (-1)^{m+1}\cos\varphi/2]\,, \qquad m \equiv \mathrm{int}(|\varphi|/2\pi)\,. \tag{7}$$

The rise of the potential (7) with φ means that the phase can increase beyond the points $\varphi = 0 \bmod (2\pi)$ only at the expense of creating quasiparticles in the junction electrodes. In the case of classical Josephson dynamics, this process generates real quasiparticles and creates a dissipative component of the Josephson current [13]. The energy relaxation then restores the 2π periodicity of all of the junction characteristics. It should be noted that the potential (7) for quantum phase dynamics cannot be obtained if one takes into account only the subgap states [16]. It is also interesting that the mechanism of the spectrum shift creating the potential (7) is very similar to the chiral anomaly in the 1D quantum electrodynamics—see, e.g., Ref. [17].

An important consequence of aperiodicity of the potential (7) is the complete suppression of the Coulomb blockade oscillations in ballistic junctions. Since the Coulomb blockade in superconducting junctions results from the formation of Bloch bands in a periodic Josephson potential, the aperiodicity of the potential obviously suppresses the Coulomb blockade. However, the periodic nature of the potential and the Coulomb blockade are restored by finite reflection in the junction. Indeed, the aperiodicity of the potential (7) is the result of the transfer of one occupied $+k_F$ states from the energy range $\varepsilon \le -\Delta$ to $\varepsilon \ge \Delta$ and one empty $-k_F$ state in the opposite direction as phase evolves from 0 to 2π. The backscattering term V in the Hamiltonian (2) couples these states at $\varphi = \pi$ and prevents such a transfer. If the coupling is sufficiently strong, the occupied $+k_F$ state, which starts at $\varphi = 0$ from the energy $\varepsilon = -\Delta$, turns into the $-k_F$ state at $\varphi \simeq \pi$, and moves back into the energy range $\varepsilon \le -\Delta$. Similarly, the empty state starting from $\varepsilon = \Delta$ at $\varphi = 0$ returns to this energy at $\varphi = 2\pi$. In this way the backscattering couples the branch (7) of the Josephson potential with no quasiparticles at $\varphi = 0$ to the one with

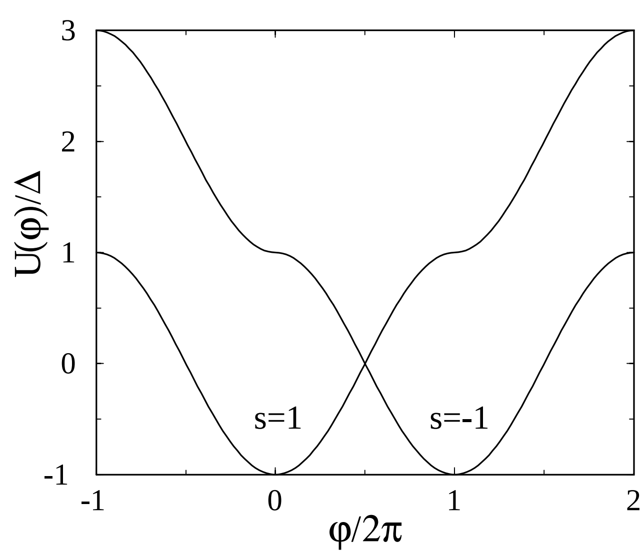

FIG. 1. Two branches of the Josephson potential in a ballistic junction with transparency $D = 1$: one that corresponds to the equilibrium occupation of Andreev states at $\varphi = 0$ ($s = 1$) and another with equilibrium at $\varphi = 2\pi$ ($s = -1$).

no quasiparticles at $\varphi = 2\pi$ [the same as (7) but shifted along the φ axis by 2π—see Fig. 1], thus creating the periodic low-energy branch of the potential.

Quantitatively, the backscattering term in the Hamiltonian (2) is $V = \int dx\, U(x)\rho(x)$, where $U(x)$ is the potential profile along the junction and

$$\rho(x) = \sum_{L,R} \Psi^\dagger_{L,R}\sigma_3\Psi_{L,R} + (\Psi^\dagger_R\sigma_3\Psi_L e^{2ik_F x} + \text{H.c.})$$

is the operator of electron density. Here and below, σ's denote the Pauli matrices. Using the fact that the characteristic range of the potential $U(x)$ is on the order of junction length d and is much smaller than the coherence length $\hbar v_F/\Delta$, we find that the only nonvanishing matrix elements of V in the basis of the subgap states (3) are those that couple the two branches of the potential:

$$\langle \Psi^-|V|\Psi^+\rangle = ir\Delta \sin\varphi/2 \,. \tag{8}$$

Here $r = -iU(2k_F)/\hbar v_F$ is the reflection amplitude of the junction [4], and $U(2k_F)$ is the Fourier component of the potential $U(x)$. At small r, the backscattering term (8) is relevant only in the vicinity of $\varphi = \pi$, where it reduces to $ir\Delta$. Then, the junction Hamiltonian (2) for $\varphi \in [0, 2\pi]$ takes the following form in the basis of two branches of the potential:

$$H = \frac{1}{2C}\left(\frac{2e}{i}\frac{\partial}{\partial\varphi} - q\right)^2 + \Delta(ir\sigma_- - ir^*\sigma_+ - \sigma_3\cos\varphi/2)\,. \tag{9}$$

The width of the Bloch bands and the associated amplitude of the Coulomb blockade oscillations depend on the probability amplitude w of staying on the low-energy periodic branch of the potential in the Hamiltonian (9). This amplitude is controlled by the usual Landau-Zener transition, the same as in the case of classical phase dynamics [18]. The only difference with the classical case is that now the transition should take place in the course of φ motion under the potential barrier, i.e., in "imaginary time." Indeed, in the quasiclassical approximation, the stationary Schrödinger equation with the Hamiltonian (9) and energy $\varepsilon \simeq -\Delta$ describing the evolution of φ near the level-crossing point $\varphi \simeq \pi$ is

$$2(E_C/\Delta)^{1/2}\partial\psi_s/\partial x = -sx\psi_s/2 + \sqrt{R}\,\psi_{-s}\,, \tag{10}$$

where $x \equiv \varphi - \pi$, and $s = \pm 1$ is the potential branch index (Fig. 1). In Eq. (10) we removed the phase Θ of the coupling terms in the Hamiltonian (9) by the simple unitary transformation $\psi_s \to e^{is\Theta/2}\psi_s$. Equations (10) are the imaginary-time versions of the equations describing the regular Landau-Zener transitions, and their solution is provided by the parabolic cylinder functions. From the asymptotes of these functions [19] we find that the probability amplitude w for the state $s = 1$ starting at $x \to -\infty$ to reach the state $s = -1$ at $x \to \infty$ is

$$w = \frac{1}{\Gamma(\lambda)}\left(\frac{2\pi}{\lambda}\right)^{1/2}\left(\frac{\lambda}{e}\right)^{\lambda}, \qquad \lambda \equiv (R/2)\,(\Delta/E_C)^{1/2}\,. \tag{11}$$

The amplitude w is plotted in Fig. 2. It tends to 1 at $R \gg (E_C/\Delta)^{1/2}$, while $w \simeq (2\pi\lambda)^{1/2}$ at $R \ll (E_C/\Delta)^{1/2}$. Since the amplitude of the Coulomb blockade oscillations is proportional to w, Eq. (11) shows that, similar to the normal junctions, in superconducting junctions, these oscillation vanish as $R^{1/2}$ at $R \to 0$.

The low-energy periodic branch of the potential in the Hamiltonian (9) coincides with the classical stationary Josephson potential which for arbitrary junction transparency D is [11,12] $U(\varphi) = -\Delta[1 - D\sin^2(\varphi/2)]^{1/2}$. For D larger than the small ratio E_C/Δ, the characteristic magnitude of the potential $U(\varphi)$ is larger than E_C, and one can find the first few eigenenergies ε_n for φ motion in this potential using the quasiclassical wave functions away from the potential minima at $\varphi = 0, 2\pi$ and matching them to the oscillator wave functions in the vicinity of these points. Taking into account that the wave functions should be periodic, $\psi(\varphi + 2\pi) = \psi(\varphi)$, we find that each oscillator eigenenergy acquires a small correction $-\delta_n$: $\varepsilon_n = \hbar\omega_p(n + 1/2) - \delta_n$, where

$$\delta_0 = \Delta bDw\left(\frac{E_C}{2\pi^2\Delta D}\right)^{1/4} e^{-a\sqrt{\Delta D/E_C}}\cos\frac{\pi q'}{e}\,, \qquad \delta_n = (-1)^n\delta_0\frac{b^{2n}}{n!}\left(\frac{\Delta D}{2E_C}\right)^{n/2}. \tag{12}$$

Here $\omega_p = (E_C\Delta D/2\hbar)^{1/2}$ is the frequency of small oscillations around the potential minima, and $q' = q - e\Theta/\pi$ is the induced charge shifted by the phase of the backscattering coupling. The numerical factors a and b in Eq. (12) can be expressed in terms of elliptic integrals, and are plotted as functions of the transparency D in Fig. 3. At $D \ll 1$, $a = 2\sqrt{2}$ and $b = 4$, while, at $D \to 1$, $a = 8(\sqrt{2} - 1) + R\ln\sqrt{R}$, $b = 8(\sqrt{2} - 1)$. Summing the corrections δ_n [Eq. (12)] over n, we can find the q-dependent part of the junction free energy at

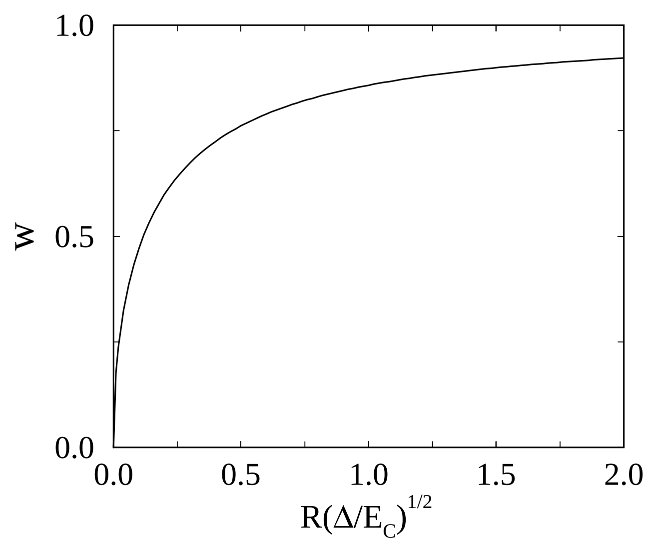

FIG. 2. The probability amplitude w [Eq. (11)] for the Josephson phase difference φ to stay in the low-energy branch of the Josephson potential in junctions with the small reflection coefficient $R \ll 1$.

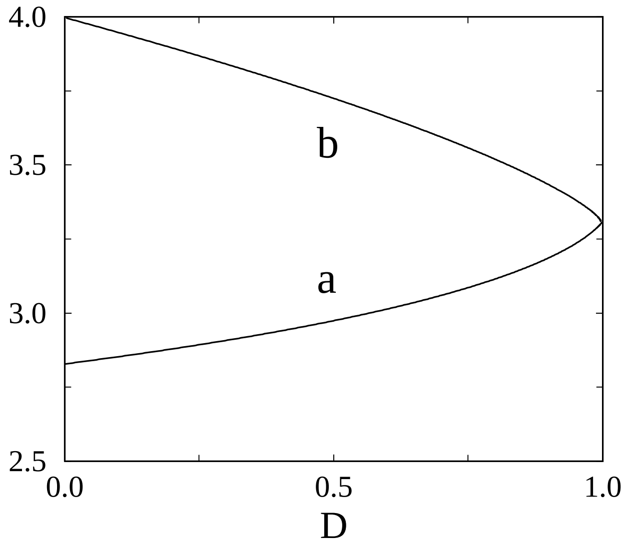

FIG. 3. Exponent a and the preexponential factor b in the amplitude of the Coulomb blockade oscillations [Eq. (12)] in a single-mode Josephson junction as functions of the junction transparency D.

finite temperatures ($T \ll \Delta$) on the order of $\hbar\omega_p$:

$$F(q) = -\delta_0(q)\,(1 - e^{-\hbar\omega_p/T}) \times \exp\left\{-b^2\left(\frac{\Delta D}{2E_C}\right)^{1/2} e^{-\hbar\omega_p/T}\right\}. \qquad (13)$$

The free energy (13) determines the amplitude of the Coulomb blockade oscillations, for instance, oscillations of the voltage across the junction: $V(q) = dF(q)/dq$. It should be possible to experimentally observe the D dependence of the Coulomb blockade oscillations either in the semiconductor/superconductor heterostructures [20] or in the controllable atomic point contacts [21,22]. Both techniques allow fabrication of the Cooper-pair box-type [9] of structures, and in both cases the junction transparency D can be varied in a controlled way. Observation of the decrease of the oscillation amplitude with D in accordance with Eqs. (12) and (13) would demonstrate the suppression of the Coulomb blockade of Cooper-pair tunneling by quantum charge fluctuations.

In summary, we have studied the Coulomb blockade oscillations in single-mode Josephson junctions with arbitrary electron transparency D in the adiabatic limit $E_C \ll \Delta$. It was shown that the amplitude of these oscillations decreases steadily with increasing D at intermediate D's and then is rapidly suppressed [on the scale $(E_C/\Delta)^{1/2}$] at $D \simeq 1$. The rapid suppression is described quantitatively by the amplitude (11) of the Landau-Zener transition between two branches of the Josephson potential.

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