

## Empirical Similarity of Frequency Spectra of the Edge-Plasma Fluctuations in Toroidal Magnetic-Confinement Systems

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(Received 15 October 1998; revised manuscript received 26 January 1999)

Frequency spectra of fluctuations of the ion saturation current, floating potential, and turbulent transport measured in the plasma edge of different fusion devices (tokamaks and stellarators) have been compared. All of the spectra show the same behavior over the whole frequency range investigated, which supports universality of plasma turbulence or turbulent transport. The results obtained are an indication of edge-plasma turbulence evolving into a critical state, independent of the size and plasma characteristics of the device. [S0031-9007(99)09024-9]

PACS numbers: 52.55.-s, 52.25.Gj, 52.40.Hf

Many systems display universal characteristics whose experimental determination has led to insights furthering the understanding of their dynamics. An example of such a system is fluid turbulence [1]. In 1941, Kolmogorov showed that two-dimensional systems display features in spatial scales ( $k$ ) leading to the well known  $k^{-5/3}$  and  $k^{-3}$  regimes in 2D turbulence. Another broader group of dynamical systems are those thought to be described by the concept of self-organized criticality (SOC). These nonequilibrium systems often evolve naturally towards a state that is nearly critical [2,3]. The nature of this self-organized criticality may account for scale invariant phenomena in nature such as  $1/f$  noise [4] and fractal (self-similar) structures [5]. In SOC systems, the Fourier spectra are expected to be nearly  $1/f$  for a given range of frequencies. This behavior arises from the existence and random superposition of avalanches. It has been argued theoretically that transport processes in magnetically confined plasmas have some of the characteristics of self-organized critical systems [6–8]. Some such processes are the scaling of transport coefficients, the response to plasma perturbations [9], and the self-similar character of the electrostatic fluctuations at the plasma edge [10].

The self-similar nature of fluctuations is an indication of the existence of long-range time correlations. These are characterized by algebraic “tails” (i.e., decay at large lags) of the autocorrelation function. A comparative analysis of edge fluctuations among different magnetic confinement devices gives a Hurst exponent varying between  $H = 0.64$  and  $0.74$  [10]. The narrow range of variation of  $H$  is an indication of the similarity of the low frequency range of the spectrum among these devices. In this Letter, we take a step further in this research and explore the properties of the complete frequency spectra of edge plasma fluctuations and turbulent transport in different tokamaks and stellarators.

Even in complex systems that exhibit scale invariant fluctuations, one expects the distribution of avalanches to show finite size scaling. In a sand pile, the size of the pile,  $L$ , and the probability of dropping sand are the basic parameters of the running sand pile dynamics. Consequently, the spectral distribution,  $P$ , will depend on frequency,  $\omega$ , and the size of the pile,  $L$ . Techniques of finite-size scaling have been applied to interpret the data. The simplest possible form for finite-size scaling is the following:

$$P(\omega, L) = L^{-\beta} g(\omega/L^\nu), \quad (1)$$

where the fixed scaling function,  $g$ , and the exponents,  $\beta$  and  $\nu$  (related to the fractal dimension), are to be determined [11]. The function  $g$ , if it exists, reflects the universal character of the subjacent process [12,13].

For fusion experiments the number of parameters that determine the dynamics of the plasma is larger and it is very difficult to set up, *a priori*, a clear dependence. We do not know *a priori* which, or even how many, parameters are relevant in comparing spectra of plasma fluctuations between different machines. As a first stage for comparison of different fusion devices, an empirical approach has been taken. The fluctuations and turbulent transport spectra have been rescaled using the *ad hoc* expression

$$P(\omega) = P_0 g(\lambda\omega), \quad (2)$$

where  $\lambda$  and  $P_0$  are parameters to be determined for each machine and operational conditions. As in the case of the sand pile, if the function  $g$  exists, there is a universal class that the spectra belong to. The  $P_0$  parameter normalizes the maximum value of the fluctuation level between the different devices. The value of  $\lambda$  gives an indication of how to compare frequency ranges between different machines.

It is important to note that the shape of frequency spectra is affected by plasma rotation and that it can

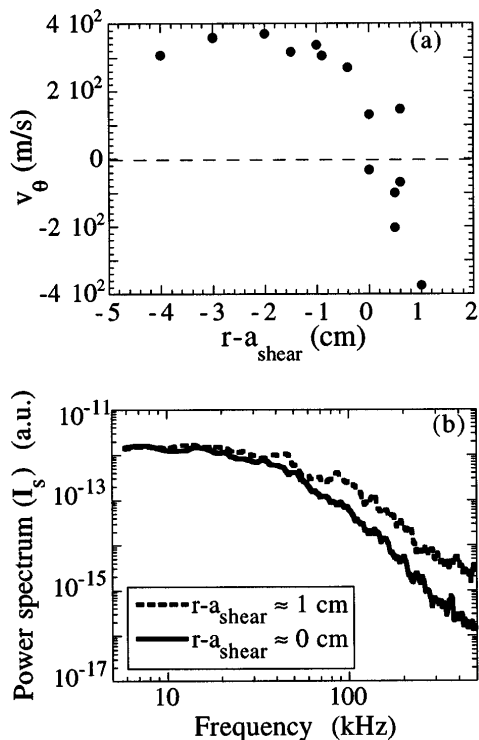


FIG. 1. (a) Radial profile of the poloidal phase velocity of fluctuations in the TJ-IU. (b) Spectrum of the TJ-IU ion saturation current fluctuations obtained in two different radial probe positions.

also be influenced by the change in the radial position. Figure 1(a) shows the radial profile of the poloidal phase velocity of the fluctuations in the stellarator TJ-IU. In Fig. 1(b), the spectra of the ion saturation current measured by Langmuir probes at two different radial positions are compared [14]. This modification of the spectrum is consistent with the expected change because of the existence of a poloidal rotation. Using data from numerical calculations, we have calculated the spectrum of the fluctuations in different reference frames,

each characterized by a different poloidal velocity. The spectrum is distorted in a similar way in the case of Fig. 1(b). A more detailed discussion of these effects will be presented elsewhere. What is relevant for the spectrum rescaling studies is that the local shift of the spectrum at a given frequency  $\omega$  is  $\langle k_\theta \rangle_\omega v_\theta$ . Here, the angular brackets refer to an average over the poloidal wave-number spectrum at constant  $\omega$  and  $v_\theta$  is the poloidal velocity of the reference frame. In rescaling among different devices, the local frequency shift will be affected by the rescaling of the wave-number spectrum. Therefore, for two devices the value of the poloidal velocity at which the spectra are similar depends on this rescaling factor. This introduces a serious problem in carrying out the rescaling of the fluctuation frequency spectrum because the poloidal wave-number spectra are not always available. In this situation, only spectra measured in the plasma rest frame can be rescaled. Therefore, the comparative studies of fluctuation spectra in different devices are done in the proximity of the velocity shear layer location where  $v_\theta \approx 0$ . Since the lowest frequencies to be considered are about 10 kHz, an uncertainty in the velocity of about 50 m/s is tolerable. This uncertainty is well within the measurement errors.

Plasma edge fluctuation measurements have been carried out by means of Langmuir probes in tokamak (TJ-I [15] and JET [16]) and stellarator (TJ-IU [14] and W7-AS [17]) fusion devices. Table I shows the main characteristics of the devices and plasmas under investigation. Edge fluctuations and turbulent transport have been characterized by means of measurement of the ion saturation current ( $I_s$ ) and floating potential ( $V_f$ ) signals using the experimental techniques described elsewhere [14]. Ion saturation current fluctuations are related with density and electron temperature fluctuations ( $I_s \propto nT_e^{1/2}$ ) whereas floating potential fluctuations depend on plasma potential and electron temperature fluctuations ( $V_f \approx V_p - aT_e$ ,  $a \approx 3$  for hydrogen plasmas). The time evolution of the

TABLE I. General characteristics of devices and plasmas under study.

| Device               | $a$<br>(m) | $R$<br>(m) | $B$<br>(T) | $\bar{n}_e$<br>( $\times 10^{19} \text{ m}^{-3}$ ) | Other plasma characteristics                              |
|----------------------|------------|------------|------------|--|---|
| TJ-I                 | 0.09       | 0.3        | 1          | 1–3  | Ohmic heating<br>$I_p \approx 35 \text{ kA}$              |
| JET                  | 1.2        | 2.9        | 2.6        | 1–2  | Ohmic heating<br>$I_p \approx 2.5 \text{ MA}$             |
| TJ-IU                | 0.10       | 0.6        | 0.67       | 0.5  | ECR heating<br>37.5 GHz, 200 kW<br>$iota(0) \approx 0.23$ |
| W7-AS <sup>(1)</sup> | 0.17       | 2.0        | 1.25       | 1.5  | ECR heating<br>70 GHz, 200 kW<br>$iota(0) \approx 0.243$  |
| W7-AS <sup>(2)</sup> | 0.18       | 2.0        | 2.5        | 3  | ECR heating<br>70 GHz, 300 kW<br>$iota(0) \approx 0.355$  |

radial turbulent particle flux has been computed as  $\Gamma(t) = \tilde{n}(t)\tilde{E}_\theta(t)/B$  where  $\tilde{n}(t)$  and  $\tilde{E}_\theta(t)$  are the fluctuating components of the density and the poloidal electric field, respectively, and  $B$  is the magnetic field. The poloidal electric field  $E_\theta$  has been deduced from floating potential signals measured by poloidally separated probes.

Fourier spectra of the ion saturation current, floating potential fluctuations, and turbulent transport have been determined using the experimental measurements in the plasma edge region of the above mentioned devices. Figure 2 shows the frequency spectra of fluctuations for the ion saturation current measured in different devices. Clearly, the shape of the frequency spectra changes from machine to machine and with plasma conditions.

Frequency spectra of plasma fluctuations and turbulent transport measured in the TJ-I, TJ-IU, W7-AS, and JET fusion devices has been rescaled using the TJ-I tokamak as a reference (i.e.,  $\lambda_{TJ-I} \equiv 1$ ). Figure 3 shows the rescaled frequency spectra of fluctuations for the ion saturation current, floating potential, and radial turbulent transport for the different devices studied. The rescaled spectra for  $I_s$  fluctuations (i.e., signals mainly related to density fluctuations),  $V_f$  fluctuations (i.e., signals related to plasma potential and temperature fluctuations), and  $\Gamma$  show the same behavior in the entire frequency range investigated for all devices. Although there is a high degree of similarity among the spectra from different devices, the form of the scaling assumed in Eq. (2) might be an oversimplification. Further, turbulent decorrelation effects at the shear location might contribute to explain the similarity obtained in the frequency spectra in different devices partially (see [18] for plasma edge review).

The values obtained for  $\lambda$  are shown in Table II. It can be seen that the size of the machine is not directly related to the  $\lambda$  parameter, or at least it is not the only parameter that affects the  $\lambda$  value. This is not surprising because of the multiple parameters that may characterize the

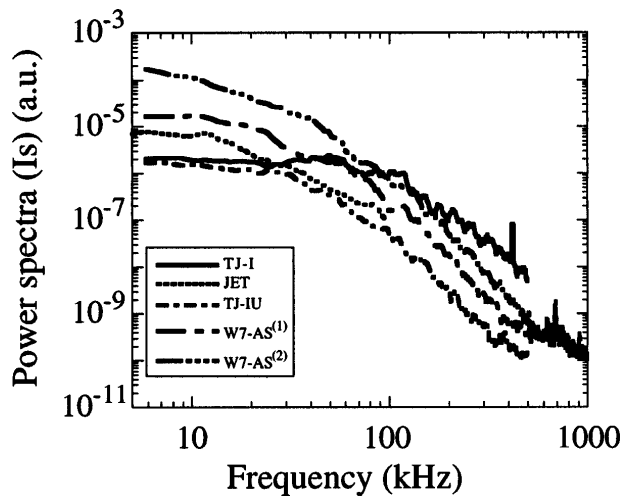


FIG. 2. Frequency spectra of fluctuations for the ion saturation current.

transport avalanches in a plasma. The different  $\lambda$  values obtained for  $I_s$  and  $V_f$  fluctuations may be due to the presence of nonlinear effects that redistribute the energy supplied, and that can substantially modify the fluctuation spectra. A theoretical determination of  $\lambda$  will require a complete understanding of the underlying dynamics of the plasma turbulence.

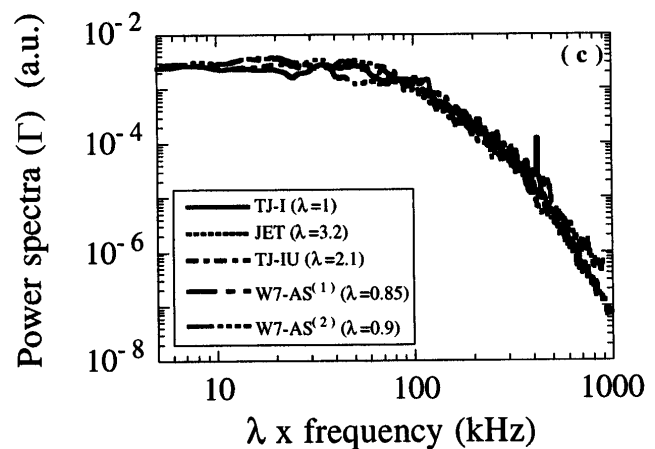
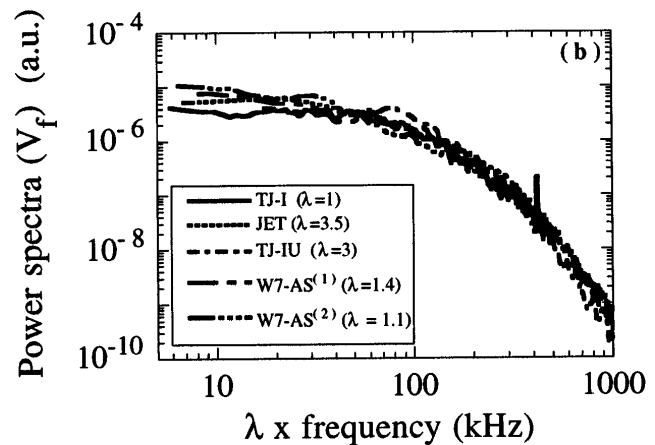
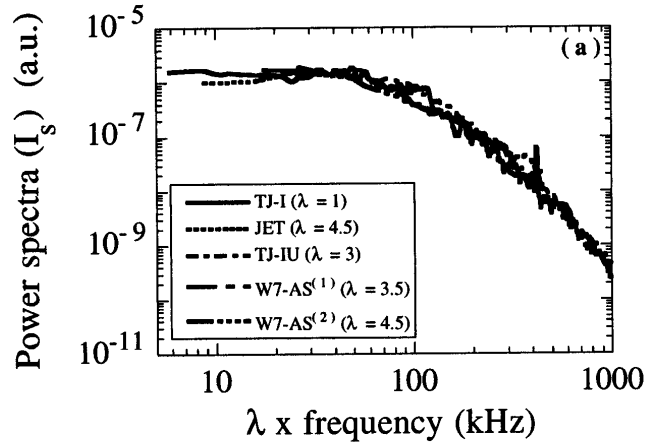


FIG. 3. Rescaled frequency spectra of fluctuations for the ion saturation current (a), floating potential (b), and turbulent transport (c).

TABLE II. Values of  $\lambda$  for different experimental results obtained using Eq. (2).

| Device               | $\lambda$<br>( $I_s$ ) | $\lambda$<br>( $V_f$ ) | $\lambda$<br>( $\Gamma$ ) |
|----------------------|------------------------|------------------------|---------------------------|
| TJ-I                 | 1                      | 1                      | 1                         |
| JET                  | 4.5                    | 3.5                    | 3.2                       |
| TJ-IU                | 3.0                    | 3.0                    | 2.1                       |
| W7-AS <sup>(1)</sup> | 3.5                    | 1.4                    | 0.85                      |
| W7-AS <sup>(2)</sup> | 4.5                    | 1.1                    | 0.9                       |

Individual fluctuation spectra are consistent with the existence of three distinct frequency ranges. For the TJ-IU stellarator, these frequency ranges are the following: a low frequency range ( $f < 10$  kHz) where the spectra are rather flat; a high frequency range ( $f > 100$  kHz) where the spectra fall approximately as  $1/f^{2-3}$ , and an intermediate frequency range where the spectrum decay index is close to  $-1$  [19]. The separation between ranges is not sharp, at least for the present available statistics. After rescaling, the separation becomes even more diffused. Better statistics are needed for a more detailed identification of these breaking points. If they exist, a rescaling of the spectra that preserves those break points may require the use of multifractal rescaling [20].

In conclusion, comparative studies of plasma-edge fluctuations carried out in different magnetic confinement devices (tokamaks and stellarators) support the view that plasma turbulence or turbulence transport displays universality. The results obtained are an indication of edge-plasma turbulence evolving into a critical state independent of the size and plasma characteristics of the device. However, it is not clear from these results what relation the scaling parameter  $\lambda$  bears to global plasma parameters (if any). In order to verify the existence of finite-size scaling (unique to critical systems) in the statistical properties of plasma fluctuations, it is important to investigate fusion plasmas with similar plasma properties (magnetic topology, collisionality, etc.) but with different plasma sizes.

This research was sponsored in part by DGICYT (Dirección General de Investigaciones Científicas y Técnicas) of Spain under Project No. PB96-0112-C02-02 and by Oak Ridge National Laboratory, managed by Lockheed Martin Energy Research Corp., for the U.S. Department of Energy under Contract No. DE-AC05-96OR22464.

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