Radiatively Induced Lorentz and *CPT* Violation in Electrodynamics

R. Jackiw[1] and V. Alan Kostelecký[2]
[1]*Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139-4307*
[2]*Physics Department, Indiana University, Bloomington, Indiana 47405*
(Received 25 January 1999)

In a nonperturbative formulation, radiative corrections arising from Lorentz and *CPT* violation in the fermion sector induce a definite and nonzero Chern-Simons addition to the electromagnetic action. If instead a perturbative formulation is used, an infinite class of theories characterized by the value of the Chern-Simons coefficient emerges at the quantum level. [S0031-9007(99)09051-1]

PACS numbers: 11.30.Cp, 11.10.Gh, 11.15.Tk, 11.30.Er

Lorentz and *CPT* symmetry of conventional Maxwell electrodynamics is destroyed by adding the term $\mathcal{L}_k = \frac{1}{2} k_\mu \varepsilon^{\mu\alpha\beta\gamma} F_{\alpha\beta} A_\gamma$ to the Lagrange density [1–3]. Here, k_μ is a prescribed, constant 4-vector. The term $\mathcal{L}_k$ is of the Chern-Simons form [4]: it changes by a total derivative when the gauge potential undergoes a gauge transformation $A_\mu \to A_\mu + \partial_\mu \Lambda$. Consequently, the action and equations of motion are gauge invariant, but the Lagrange density is not. The modified theory predicts birefringence of light *in vacuo* [1,3]. Observation of distant galaxies puts a stringent bound on k_μ: it should effectively vanish [1,5].

A natural question is whether such a term would be induced through radiative corrections when Lorentz and *CPT* symmetries are violated in other sectors of a larger theory. If so, then the stringent experimental limits on $\mathcal{L}_k$ would severely restrict the viability of models with Lorentz and *CPT* breaking [6].

To study explicitly this issue, one may consider extending the quantum-electrodynamics (QED) action of a single Fermi field by including a Lorentz- and *CPT*-violating axial-vector term [2,3]:

$$I \supset \int d^4x\, \bar{\psi}(i\partial\!\!\!/ - A\!\!\!/ - m - b\!\!\!/\gamma_5)\psi\,. \tag{1}$$

Here, b_μ is a constant, prescribed 4-vector, and our γ_5 is Hermitian with $\mathrm{tr}\gamma_5\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\delta = 4i\varepsilon^{\alpha\beta\gamma\delta}$. The only other possible nonderivative *CPT*- and Lorentz-violating term in the fermion sector is uninteresting here because its properties under charge conjugation prevent it contributing to the Chern-Simons term [3].

Several calculations have been performed to determine whether radiative corrections induce the Chern-Simons term with $k_\mu \propto b_\mu$. At leading order in b_μ and the fine-structure constant, a perturbative treatment of the term in Eq. (1) has been shown to generate an ambiguous result: the coefficient k_μ of the induced Chern-Simons term is regularization dependent and can be freely selected [3]. Other claims include both a definite zero value for k_μ [7] and a definite nonzero value [8].

The purpose of this work is to clarify this situation and call attention to subtle issues, related to chiral anomalies [9], underlying the discrepancies between these various results. First, a direct calculation of k_μ is presented that is nonperturbative in b_μ. The various issues are then disentangled.

The relevant quantity for deciding whether a Chern-Simons term is induced is the vacuum persistence amplitude, or equivalently from Eq. (1) the fermion determinant $\det(i\partial\!\!\!/ - A\!\!\!/ - m - b\!\!\!/\gamma_5)$, computed to second order in the photon variables. We are thus led to examining the standard one-loop vacuum-polarization amplitude $\Pi^{\mu\nu}$, but with the usual free-fermion propagator $S(l)$ replaced by the b_μ-exact propagator from Eq. (1):

$$G(l) = \frac{i}{l\!\!\!/ - m - b\!\!\!/\gamma_5}\,. \tag{2}$$

This may also be presented as

$$G(l) = S(l) + G_b(l)\,, \tag{3a}$$

where

$$G_b(l) = \frac{1}{l\!\!\!/ - m - b\!\!\!/\gamma_5}\, b\!\!\!/\gamma_5 S(l)\,. \tag{3b}$$

With this decomposition, $\Pi^{\mu\nu}$ splits into three terms:

$$\Pi^{\mu\nu} = \Pi_0^{\mu\nu} + \Pi_b^{\mu\nu} + \Pi_{bb}^{\mu\nu}\,. \tag{4}$$

The term $\Pi_0^{\mu\nu}$ is the usual lowest-order vacuum-polarization tensor of QED, which we shall not discuss further. The term $\Pi_{bb}^{\mu\nu}$ is at least quadratic in b; it is at most logarithmically divergent and suffers no ambiguity in routing the internal momenta [10]. The b_μ-linear contribution to the Chern-Simons term arises from $\Pi_b^{\mu\nu}$, which is given explicitly by

$$\Pi_b^{\mu\nu}(p) = \int \frac{d^4l}{(2\pi)^4}\,\mathrm{tr}\{\gamma^\mu S(l)\gamma^\nu G_b(l+p) + \gamma^\mu G_b(l)\gamma^\nu S(l+p)\}\,. \tag{5}$$

There are several important features of this expression. Each of the two integrals is (superficially) linearly divergent. However, the divergences cancel when the two terms are taken together and the traces are evaluated. As a consequence, there is no momentum-routing ambiguity in the summed integrand: when the integration momentum

 0031-9007/99/82(18)/3572(4)$15.00

is shifted by the same amount in *both* integrands the value of the integral does not change, even though shifting separately by different amounts in each of the two integrands changes the value of the integral by a surface term. It follows that different momentum routings leave unchanged the value of $\Pi_b^{\mu\nu}$ because they produce a simultaneous shift of integration variable by the same amount in each of the integrands in Eq. (5). Therefore, a unique value can be calculated for $\Pi_b^{\mu\nu}$, which we shall show leads to a finite Chern-Simons term.

We next evaluate $\Pi_b^{\mu\nu}$ to lowest order in b, by replacing $G_b(l)$ with $-iS(l)\not{b}\gamma_5 S(l)$. This gives

$$\Pi_b^{\mu\nu} \simeq \Pi^{\mu\nu\alpha} b_\alpha \,, \tag{6}$$

where

$$\begin{aligned}\Pi^{\mu\nu\alpha}(p) &= -i \int \frac{d^4 l}{(2\pi)^4}\, \mathrm{tr}\{\gamma^\mu S(l)\gamma^\nu S(l+p)\gamma^\alpha\gamma_5 S(l+p) + \gamma^\mu S(l)\gamma^\alpha\gamma_5 S(l)\gamma^\nu S(l+p)\} \\ &\equiv I^{\mu\nu\alpha}(p) + \tilde{I}^{\mu\nu\alpha}(p)\,.\end{aligned} \tag{7}$$

A shift of integration variables in the second term reduces it to a crossed form of the first plus a contribution arising from shifting variables in a linearly divergent integral:

$$\tilde{I}^{\mu\nu\alpha}(p) = I^{\nu\mu\alpha}(-p) + \Delta^{\mu\nu\alpha}(p)\,, \tag{8}$$

where

$$\Delta^{\mu\nu\alpha}(p) = -i \int \frac{d^4 l}{(2\pi)^4}\, \mathrm{tr}\{\gamma^\mu S(l)\gamma^\alpha\gamma_5 S(l)\gamma^\nu S(l+p) - \gamma^\nu S(l)\gamma^\mu S(l-p)\gamma^\alpha\gamma_5 S(l-p)\}\,. \tag{9}$$

Since the tensorial form of $I^{\mu\nu\alpha}$ must be $\varepsilon^{\mu\nu\alpha\beta}p_\beta$, the crossed term coincides with the uncrossed term. The variable shift produces a surface term, and $\Delta^{\mu\nu\alpha}(p)$ is evaluated as

$$\Delta^{\mu\nu\alpha}(p) = -\frac{1}{8\pi^2}\,\varepsilon^{\mu\nu\alpha\beta}p_\beta\,. \tag{10}$$

Thus, we have

$$\Pi^{\mu\nu\alpha}(p) = -\frac{1}{8\pi^2}\,\varepsilon^{\mu\nu\alpha\beta}p_\beta - 2\int \frac{d^4 l}{(2\pi)^4}\, \mathrm{tr}\gamma^\mu \frac{1}{\not{l}-m}\gamma^\nu \frac{1}{\not{l}+\not{p}-m}\gamma^\alpha\gamma_5 \frac{1}{\not{l}+\not{p}-m}\,. \tag{11}$$

To evaluate the integral, note first that

$$\begin{aligned}\frac{1}{\not{l}+\not{p}-m}\gamma^\alpha\gamma_5\frac{1}{\not{l}+\not{p}-m} &= \frac{1}{\not{l}+\not{p}-m}\gamma^\alpha\left[\frac{-1}{\not{l}+\not{p}-m} + \frac{2m}{(l+p)^2-m^2}\right]\gamma_5 \\ &= \frac{\partial}{\partial p_\alpha}\frac{1}{\not{l}+\not{p}-m}\gamma_5 + \frac{2m}{\not{l}+\not{p}-m}\gamma^\alpha\gamma_5\frac{1}{(l+p)^2-m^2}\,.\end{aligned} \tag{12}$$

The p_α derivative contributes a term

$$-2\frac{\partial}{\partial p_\alpha}\int \frac{d^4 l}{(2\pi)^4}\, \mathrm{tr}\left(\gamma^\mu \frac{1}{\not{l}-m}\gamma^\nu\frac{1}{\not{l}+\not{p}-m}\gamma_5\right).$$

However, the above integral must vanish: no two-index pseudotensor exists involving the antisymmetric pseudotensor and depending only on a single variable p. Therefore, one is left with an entirely finite integral. We find

$$\begin{aligned}\Pi^{\mu\nu\alpha}(p) &= -\frac{1}{8\pi^2}\,\varepsilon^{\mu\nu\alpha\beta}p_\beta - 4m\int \frac{d^4 l}{(2\pi)^4}\, \mathrm{tr}\gamma^\mu\frac{1}{\not{l}-m}\gamma^\nu\frac{1}{\not{l}+\not{p}-m}\gamma^\alpha\gamma_5\frac{1}{(l+p)^2-m^2} \\ &= -\frac{1}{8\pi^2}\,\varepsilon^{\mu\nu\alpha\beta}p_\beta + \frac{im^2}{\pi^4}\,\varepsilon^{\mu\nu\alpha\beta}p_\beta\int d^4 l\,\frac{1}{l^2-m^2}\,\frac{1}{[(l+p)^2-m^2]^2} \\ &= -\varepsilon^{\mu\nu\alpha\beta}p_\beta\left(\frac{1}{8\pi^2} + \frac{2}{\pi^2}\int_{2m}^\infty da\,\frac{m^2}{\sqrt{a^2-4m^2}}\,\frac{1}{p^2-a^2+i\varepsilon}\right).\end{aligned} \tag{13}$$

The final result is

$$\Pi^{\mu\nu\alpha}(p) = \varepsilon^{\mu\nu\alpha\beta}\frac{p_\beta}{2\pi^2}\left(\frac{\theta}{\sin\theta} - \frac{1}{4}\right), \tag{14}$$

where $\theta \equiv 2\sin^{-1}(\sqrt{p^2}/2m)$ and $p^2 < 4m^2$.

The b_μ-linear contribution to the induced Chern-Simons term is determined by this expression at $p^2 = 0$. One finds a definite, nonzero, and finite result [8,11]:

$$\Pi^{\mu\nu\alpha}(p)|_{p^2=0} = \frac{3}{8\pi^2}\,\varepsilon^{\mu\nu\alpha\beta}p_\beta\,, \tag{15}$$

and

$$k^\mu = \frac{3}{16\pi^2}\,b^\mu. \tag{16}$$

This completes our calculation.

We have chosen to extract the leading-order result in b_μ. However, our calculation is in fact nonperturbative in b_μ in the sense that it has been performed keeping careful track of contributions from the b_μ-exact propagator in Eq. (3a). Thus, in this calculation, we are choosing to define the theory of Eq. (1) in a nonperturbative way.

If instead the theory in Eq. (1) is defined through its perturbation series in b_μ, the same θ dependence as in Eq. (14) emerges but the additional constant and therefore the net result for the induced Chern-Simons term is different [3]. At first order, one finds that the two integrals (7) arise from a triangle VVA graph and its crossed expression with zero axial-vector momentum. In perturbation theory, no correlation is determined *a priori* between the momentum routings in the two graphs. If the relative routings are as in (7), the resulting expression coincides with (14). Otherwise, a shift of integration variables produces the configuration (7), but generates an additional contribution. Taking the shift as proportional to the external momentum gives rise to an arbitrary multiple of $\Delta^{\mu\nu\alpha} \propto \varepsilon^{\mu\nu\alpha\beta}p_\beta$, leaving the Chern-Simons coefficient k^μ proportional to b^μ but with an *undetermined* proportionality constant.

Coleman and Glashow have recently argued that k_μ must unambiguously vanish to first order in b_μ for any gauge-invariant *CPT*-odd interaction [7]. Their result is based on their hypothesis that one define the axial vector $j_5^\mu(x) \equiv \bar\psi(x)\gamma^\mu\gamma_5\psi(x)$ to be gauge invariant in the quantum theory at arbitrary 4-momentum, that is, at every point in x. However, a weaker condition is true: if $j_5^\mu(x)$ does not couple to any other field, then physical gauge invariance is maintained provided $j_5^\mu(x)$ is gauge invariant at *zero* 4-momentum. Equivalently, physical gauge invariance is maintained provided $\int d^4x\, j_5^\mu(x)$ and therefore the action are gauge invariant, *without* the requirement that the Lagrange density also be gauge invariant. This behavior characterizes the Chern-Simons term, so it is unsurprising that demanding gauge invariance of the Lagrange density can prevent generating the noninvariant Chern-Simons term.

The Coleman-Glashow argument is perturbative in b_μ and is taken to first order. Only in the perturbative framework does the axial vector arise as a distinct entity: it is an insertion whose gauge variance can be discussed. In contrast, with the nonperturbative definition of the theory the axial vector has no separate identity, but when the first-order contribution is extracted from our complete expression we find a nonzero result. Evidently the dynamics of the nonperturbative theory selects the weaker option: gauge invariance only for $\int d^4x\, b_\mu j_5^\mu(x)$ but not for the unintegrated quantity. Gauge invariance is preserved in the sense that $p_\mu \Pi_b^{\mu\nu} = 0$, and the induced action is gauge invariant, but the induced Chern-Simons Lagrange density is not.

The gauge anomaly vanishes for zero momentum in the axial-vector vertex, regardless of the momentum routing in the two triangle graphs. However, for *nonvanishing* momentum in the axial vertex, only a special routing of the integration momenta gives a gauge-invariant answer. This special routing is known explicitly [12], and in the limit of zero axial-vector momentum it corresponds to the result of the Coleman-Glashow assumption, giving uniquely

$$\Pi_{\rm CG}^{\mu\nu}(p) = -i\int \frac{d^4l}{(2\pi)^4}\,{\rm tr}\{\gamma^\mu S(l)\gamma^\nu S(l+p)\gamma^\alpha\gamma_5 S(l+p) + \gamma^\mu S(l+3p)\gamma^\alpha\gamma_5 S(l+3p)\gamma^\nu S(l+4p)\}$$

after an innocuous shift in both integrands. The integration momentum in the second expression must be decreased by $3p$ to bring this result into conformity with Eq. (7). Therefore, from (10) it follows that

$$\Pi_{\rm CG}^{\mu\nu\alpha}(p) = \Pi^{\mu\nu\alpha}(p) - \frac{3}{8\pi^2}\,\varepsilon^{\mu\nu\alpha\beta}p_\beta\,. \tag{17}$$

This result vanishes at $p^2 = 0$, in agreement with the Coleman-Glashow claim. However, other than the demand that gauge invariance be maintained not only for the action but also for the induced Lagrange density, there is no reason to prefer this result over any other.

A further degree of arbitrariness in the induced term appears according to the choice of regularization scheme used in the calculation. This can be true even within a nonperturbative formation.

One possible choice is Pauli-Villars regularization, which enforces gauge invariance for all axial momenta. The various values of the induced Chern-Simons coefficient are mass independent, so they are subtracted and vanish in Pauli-Villars regularization [3]. This would be true whether or not the theory is formulated perturbatively or nonperturbatively. Another possibility is dimensional regularization. This is problematic with the γ_5 matrix, and a variety of answers for k_μ can be obtained depending on how γ_5 is generalized to arbitrary dimensions. In this sense, the physics of the theory (1) depends on the choice of regularization scheme.

Referring to the dispersive representation, presented in Eq. (14), we see that the theory predicts a definite absorptive part, which is sufficiently well behaved to enter into an unsubtracted dispersion relation. Nevertheless, there remains the possibility of a real subtraction, which is perturbatively undetermined, even though we presented a nonperturbative argument for the value $1/8\pi^2$. This situation is familiar in quantum field theory. Parameters in a Lagrangian typically acquire infinite radiative corrections that must be renormalized. They flow with the renormalization scale and are determined only by comparison with experiment. Here, the radiative corrections are finite, so infinite renormalization is unnecessary, but nevertheless no definite value is determined in perturbation theory. Another instance of this phenomenon occurs in the chiral Schwinger model, which generates an undetermined mass for its vector meson [13]. The θ angle of QCD provides a further example [14]. For all these, a finite parameter must be fixed by reference to nature. For the Chern-Simons case this has already been done [1,5].

In this work, we have found that the apparently reasonable physical question "Is a Chern-Simons term induced in the theory (1)?" has no unique answer. Equivalently, there is no unique evaluation of the effective action $-i \ln \det(i\not\partial - \not A - m - \not b\gamma_5)$. We have given a nonperturbative definition of the theory (1) that induces a definite and nonzero value of the Chern-Simons coefficient. If instead a perturbative definition is used, an infinite class of theories characterized by the value of the Chern-Simons coefficient emerges at the quantum level. The choice of regularization procedure can induce further ambiguity in both nonperturbative and perturbative schemes. Although one could perhaps argue that our nonperturbative formulation is the most aesthetically satisfying, there seems no compelling reason to prefer any one definition over another.

We thank Don Colladay, Sidney Coleman, J.-M. Chung, and P. Oh for useful discussions. This work is supported in part by the United States Department of Energy under Grants No. DE-FC02-94ER40818 and No. DE-FG02-91ER40661.

[1] S. Carroll, G. Field, and R. Jackiw, Phys. Rev. D **41**, 1231 (1990).
[2] D. Colladay and V. A. Kostelecký, Phys. Rev. D **55**, 6760 (1997).
[3] D. Colladay and V. A. Kostelecký, Phys. Rev. D **58**, 116002 (1998).
[4] R. Jackiw and S. Templeton, Phys. Rev. D **23**, 2291 (1981); J. Schonfeld, Nucl. Phys. **B185**, 157 (1981); S. Deser, R. Jackiw, and S. Templeton, Ann. Phys. (N.Y.) **140**, 372 (1982); **185**, 406E (1988).
[5] M. Goldhaber and V. Trimble, J. Astrophys. Astron. **17**, 17 (1996); S. Carroll and G. Field, Phys. Rev. Lett. **79**, 2394 (1997).
[6] Even if a nonzero term is induced by a particular sector of a larger theory, the net effect can still vanish. This would occur in the standard-model extension of Ref. [3] through the anomaly-cancellation mechanism, for example.
[7] S. Coleman and S. Glashow, hep-ph/9812418.
[8] J.-M. Chung and P. Oh, hep-th/9812132. See also A. A. Andrianov, R. Soldati, and L. Sorbo, Phys. Rev. D **59**, 025002 (1999).
[9] R. Jackiw, in *Current Algebra and Anomalies*, edited by S. Treiman, R. Jackiw, B. Zumino, and E. Witten (Princeton University Press, Princeton, NJ, 1985); S. Adler, in *Lectures on Elementary Particles and Field Theory*, edited by S. Deser, M. Grisaru, and H. Pendleton (MIT Press, Cambridge, MA, 1970).
[10] In fact, the contribution from $\Pi_{bb}^{\mu\nu}$ to the induced Chern-Simons term is at least cubic in b and hence finite because all terms quadratic in b are necessarily CPT even.
[11] In the limit of zero m, the CPT-violating term in Eq. (1) can be removed altogether by a chiral field redefinition [2]. The result here remains nonzero in this limit because the chiral redefinition does not commute with quantization [D. Colladay (private communication)].
[12] R. Jackiw, in *Current Algebra and Anomalies* (Ref. [9]), pp. 118–122. Equations (4.26) show that the routing ambiguity parameter must be $a = -2$, and in (4.20) q is set equal to $-p$ when the axial-vector vertex carries zero momentum.
[13] R. Jackiw and R. Rajaraman, Phys. Rev. Lett. **54**, 1219 (1985); **54**, 2060(E) (1985); **55**, 2224 (1985).
[14] R. Jackiw and C. Rebbi, Phys. Rev. Lett. **37**, 172 (1976); C. Callan, R. Dashen, and D. Gross, Phys. Lett. **63B**, 334 (1976).