Spin-Pseudospin Coherence and CP^3 Skyrmions in Bilayer Quantum Hall Ferromagnets

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We analyze bilayer quantum Hall ferromagnets, whose underlying symmetry group is SU(4). Spin-pseudospin coherence develops spontaneously when the total electron density is low enough. The quasiparticles are CP^3 Skyrmions. One Skyrmion induces charge modulations on both layers. At the filling factor $\nu = 2/m$ one elementary excitation consists of a pair of Skyrmions and its charge is $2e/m$. Recent experimental data due to Sawada *et al.* [Phys. Rev. Lett. **80**, 4534 (1998)] support this conclusion. [S0031-9007(99)08992-9]

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The quantum Hall (QH) effect is a remarkable macroscopic quantum phenomenon in the two-dimensional electron system [1]. Attention has recently been paid to quantum coherence in QH systems. The kinetic and Coulomb Hamiltonians have the spin SU(2) symmetry. Its spontaneous breakdown leads to a spin coherence and turns the system into a QH ferromagnet. The effective Hamiltonian is the SU(2) nonlinear sigma (NLσ) model [2]. Quasiparticles are CP^1 Skyrmions [3–5].

In this paper we analyze Skyrmion excitations in bilayer QH (BLQH) ferromagnets. The lowest Landau level (LLL) contains four energy levels corresponding to the spin and layer (pseudospin) degrees of freedom. The SU(4) symmetry underlies the BLQH system. Its spontaneous breakdown leads to a spin-pseudospin coherence. The effective Hamiltonian is the SU(4) NLσ model [2]. Quasiparticles are CP^3 Skyrmions [2]. They have two characteristic features: (A) One Skyrmion induces charge modulations on both of the two layers. The main part of the activation energy is the capacitive charging energy. (B) One elementary excitation consists of a pair of Skyrmions at $\nu = 2/m$ with m odd. It carries the charge $2e/m$. Our theoretical analysis accounts for recent experimental data due to Sawada *et al.* [6,7]. Throughout the paper we use the natural units $\hbar = c = 1$.

QH ferromagnets.—We analyze Skyrmion excitations at the filling factor $\nu \equiv 2\pi\rho_0/eB_\perp = 1/m$ with m odd. We use an improved composite-boson (CB) theory [8], which is proposed based on a suggestion due to Girvin *et al.* [9]. The advantage of the scheme is a direct connection between the semiclassical property of an excitation and its microscopic wave function. We start with a review of monolayer QH ferromagnets [8]. The analysis of BLQH ferromagnets is its straightforward generalization with a replacement of SU(2) by SU(4).

The kinetic Hamiltonian for planar electrons with mass M in the perpendicular magnetic field $B_\perp$ is

$$H_K = \frac{1}{2M}\int d^2x\, \Psi^\dagger(\boldsymbol{x})(P_x - iP_y)(P_x + iP_y)\Psi(\boldsymbol{x}), \tag{1}$$

where $P_j = -i\partial_j + eA_j$ is the covariant momentum with $A_j = \frac{1}{2}\varepsilon_{jk}x_kB_\perp$; $\varepsilon_{12} = -\varepsilon_{21} = 1$ and $\varepsilon_{11} = \varepsilon_{22} = 0$. The electron field $\Psi(\boldsymbol{x})$ is a two-component spinor made of the spin-up ($\psi^\uparrow$) and spin-down ($\psi^\downarrow$) field. The symmetry group of this Hamiltonian is $U(2) = U(1) \otimes SU(2)$.

When the Zeeman energy is small, a spin coherence develops spontaneously. This is described by introducing the two-component CB field by the formula [8,9]

$$\Phi(\boldsymbol{x}) = e^{-\mathcal{A}(\boldsymbol{x})}e^{-ie\Theta(\boldsymbol{x})}\Psi(\boldsymbol{x}), \tag{2}$$

where $\mathcal{A}(\boldsymbol{x})$ is the auxiliary field obeying $\nabla^2\mathcal{A}(\boldsymbol{x}) = 2\pi m[\rho(\boldsymbol{x}) - \rho_0]$; $\rho(\boldsymbol{x}) \equiv \Psi^\dagger(\boldsymbol{x})\Psi(\boldsymbol{x})$ is the electron density. The phase field $\Theta(\boldsymbol{x})$ attaches m units of flux quantum $2\pi/e$ to each electron via the relation, $\varepsilon_{ij}\partial_i\partial_j\Theta(\boldsymbol{x}) = (2\pi/e)m\rho(\boldsymbol{x})$. The effective magnetic field is

$$\mathcal{B}_{\rm eff}(\boldsymbol{x}) = B_\perp - \varepsilon_{ij}\partial_i\partial_j\Theta(\boldsymbol{x}) = B_\perp - (2\pi/e)m\rho(\boldsymbol{x}). \tag{3}$$

It vanishes, $\langle\mathcal{B}_{\rm eff}\rangle_{\rm g} = 0$, on the ground state at $\nu = 1/m$. Substituting (2) into (1), the kinetic Hamiltonian reads

$$H_K = \frac{1}{2M}\int d^2x\, \Phi^\ddagger(\boldsymbol{x})(\mathcal{P}_x - i\mathcal{P}_y)(\mathcal{P}_x + i\mathcal{P}_y)\Phi(\boldsymbol{x}), \tag{4}$$

where $\mathcal{P}_j = -i\partial_j - (\varepsilon_{jk} + i\delta_{jk})\partial_k\mathcal{A}(\boldsymbol{x})$ is the covariant momentum. We have defined $\Phi^\ddagger(\boldsymbol{x}) \equiv \Phi^\dagger(\boldsymbol{x})e^{2\mathcal{A}(\boldsymbol{x})}$, with which $\rho(\boldsymbol{x}) = \Psi^\dagger(\boldsymbol{x})\Psi(\boldsymbol{x}) = \Phi^\ddagger(\boldsymbol{x})\Phi(\boldsymbol{x})$. An analysis of the Lagrangian shows that the canonical conjugate of $\varphi^\alpha(\boldsymbol{x})$ is not $i\varphi^{\alpha\dagger}(\boldsymbol{x})$ but $i\varphi^{\alpha\ddagger}(\boldsymbol{x})$.

We decompose the CB field as

$$\Phi(\boldsymbol{x}) = e^{-\mathcal{A}(\boldsymbol{x})}\phi(\boldsymbol{x})\boldsymbol{n}(\boldsymbol{x}), \tag{5}$$

with the U(1) component $\phi(\boldsymbol{x}) = e^{i\chi(\boldsymbol{x})}\sqrt{\rho(\boldsymbol{x})}$ and the SU(2) component $\boldsymbol{n}(\boldsymbol{x})$: It is the CP^1 field [2] subject to the constraint, $\boldsymbol{n}^\dagger(\boldsymbol{x})\boldsymbol{n}(\boldsymbol{x}) = 1$. The spin density is

$$S^a(\boldsymbol{x}) = \frac{1}{2}\rho(\boldsymbol{x})s^a(\boldsymbol{x}), \qquad s^a(\boldsymbol{x}) = \boldsymbol{n}^\dagger(\boldsymbol{x})\lambda^a\boldsymbol{n}(\boldsymbol{x}), \tag{6}$$

where λ^a are the Pauli matrices.

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At sufficiently low temperature it is reasonable to focus our attention to physics taking place within the LLL. The Hilbert space $\mathbb{H}_{\rm LLL}$ is constructed by imposing the LLL condition so that the kinetic term (4) is quenched. It has a simple expression in terms of the CB field,

$$(\mathcal{P}_x + i\mathcal{P}_y)\Phi(\boldsymbol{x})|\mathfrak{S}\rangle = -\frac{i}{\ell_B}\frac{\partial}{\partial z^*}\Phi(\boldsymbol{x})|\mathfrak{S}\rangle = 0. \quad (7)$$

The complex number is $z = (x + iy)/2\ell_B$ with ℓ_B the magnetic length. Hence, the wave function for composite bosons is analytic and symmetric in all N coordinates,

$$\Omega[z] = \langle 0|\Phi(\boldsymbol{x}_1)\cdots\Phi(\boldsymbol{x}_N)|\mathfrak{S}\rangle. \quad (8)$$

The wave function for electrons is $\mathfrak{S}[\boldsymbol{x}] = \Omega[z]\mathfrak{S}_{\rm LN}[\boldsymbol{x}]$, where $\mathfrak{S}_{\rm LN}[\boldsymbol{x}]$ is the familiar Laughlin wave function. Here, $[\boldsymbol{x}] = (\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_N)$ and $[z] = (z_1, z_2, \cdots, z_N)$. Any excitation confined to the LLL is described by a choice of the analytic spinor factor $\Omega[z]$.

We analyze the CB theory semiclassically, where the bosonic field operator is approximated by a c-number field. It follows from (8) that the wave function is

$$\mathfrak{S}[\boldsymbol{x}] = \prod_r \langle\Phi(\boldsymbol{x}_r)\rangle\mathfrak{S}_{\rm LN}[\boldsymbol{x}], \quad (9)$$

where $\langle\Phi(\boldsymbol{x})\rangle$ is analytic. If the Zeeman energy is neglected, the ground state is determined by minimizing the Coulomb energy. The one-point function $\langle\Phi(\boldsymbol{x})\rangle$ is a constant vector pointing to an arbitrary direction in the SU(2) space, as implies a spontaneous breakdown of the SU(2) symmetry. Actually, a small Zeeman interaction fixes this direction so that $\langle s^a(\boldsymbol{x})\rangle = \delta^{az}$, or

$$\langle\varphi^\uparrow(\boldsymbol{x})\rangle_{\rm g} = \sqrt{\rho_0}, \qquad \langle\varphi^\downarrow(\boldsymbol{x})\rangle_{\rm g} = 0. \quad (10)$$

The ground-state wave function is given by (9) with (10).

The semiclassical LLL condition follows from (5),

$$\langle\varphi^\alpha(\boldsymbol{x})\rangle = e^{-\mathcal{A}(x)}e^{i\chi(x)}\sqrt{\rho(\boldsymbol{x})}\,n^\alpha(\boldsymbol{x}) \equiv \omega^\alpha(z), \quad (11)$$

where various fields are classical ones. This is solved by $\chi(\boldsymbol{x}) = 0$ and $n^\alpha(\boldsymbol{x}) = \omega^\alpha(z)/\sqrt{\sum_\alpha|\omega^\alpha|^2}$. The soliton equation [10] follows trivially from (11),

$$\frac{\nu}{4\pi}\nabla^2\ln\rho(\boldsymbol{x}) - \rho(\boldsymbol{x}) + \rho_0 = \nu Q(\boldsymbol{x}), \quad (12)$$

where $Q(\boldsymbol{x})$ is the topological charge density,

$$Q(\boldsymbol{x}) = \frac{1}{4\pi}\nabla^2\ln\left(\sum_\alpha|\omega^\alpha(z)|^2\right). \quad (13)$$

The lightest Skyrmion on the ground state (10) is

$$\langle\varphi^\uparrow(\boldsymbol{x})\rangle_{\rm sky} = z\sqrt{\rho_0}, \qquad \langle\varphi^\downarrow(\boldsymbol{x})\rangle_{\rm sky} = \kappa\sqrt{\rho_0}, \quad (14)$$

with which the wave function is given by (9). For a large Skyrmion ($\kappa \gg 1$), the soliton equation (12) is solved iteratively and agrees with the familiar expression [3]

$$\varrho(\boldsymbol{x}) \equiv \rho(\boldsymbol{x}) - \rho_0 \simeq \nu Q(\boldsymbol{x}) = -\frac{\nu}{\pi}\frac{4(\kappa\ell_B)^2}{[r^2 + 4(\kappa\ell_B)^2]^2}. \quad (15)$$

The Skyrmion is reduced to the vortex in the limit $\kappa \to 0$.

The excitation energy of one Skyrmion consists of the exchange energy $E_{\rm ex}$, the electrostatic energy E_C, and the Zeeman energy E_Z. Minimizing their sum, we determine the Skyrmion size κ, the Skyrmion energy $E_{\rm sky}$, and the flipped-spin number $N_{\rm spin}$ as follows [8]:

$$\kappa \simeq \frac{1}{2}(\beta\nu)^{1/3}\left\{\tilde{g}\ln\left(\frac{\sqrt{2\pi}}{32\tilde{g}} + 1\right)\right\}^{-1/3}, \quad (16)$$

$$E_{\rm sky} \simeq \left(\nu\sqrt{\frac{\pi}{32}} + \frac{3\beta\nu^2}{4\kappa}\right)E_C^0, \quad (17)$$

$$N_{\rm spin} \simeq 2\nu\kappa^2\ln\left(\frac{\sqrt{2\pi}}{32\tilde{g}} + 1\right). \quad (18)$$

Here, $\rho_s = \nu e^2/(16\sqrt{2\pi}\,\varepsilon\ell_B)$, $E_C^0 = e^2/\varepsilon\ell_B$, and $\tilde{g} = g^*\mu_B B/E_C^0$. The parameter β represents the strength of the Coulomb energy; it is calculated as $\beta = 3\pi^2/64$ for a sufficiently large Skyrmion in an ideal planar system. However, an actual Skyrmion is not large, and there will also be a correction from a finiteness of the layer width. We use β as a phenomenological parameter. We fix it as $\beta = 0.24$ by requiring the Skyrmion spin $N_{\rm spin}$ to agree with the experimental data due to Barrett *et al.* [4] at $\nu = 1$: $N_{\rm spin} \simeq 3.7$ at $B = 7.05$ T, where $\kappa \simeq 1.0$ and $\tilde{g} \simeq 0.015$. Note that $N_{\rm spin} \simeq 5.3$ ($\kappa \simeq 1.10$) at $B = 3$ T and $N_{\rm spin} \simeq 2.7$ ($\kappa \simeq 0.96$) at $B = 15$ T. Our results are consistent with the previous ones [3].

The excitation energy of a Skyrmion–anti-Skyrmion pair will be given by $\Delta = 2E_{\rm sky} - \Gamma_{\rm offset}$ with a sample dependent offset $\Gamma_{\rm offset}$. In Fig. 1 we have fitted the experimental data due to Schmeller *et al.* [5] based on formula (17) with $\beta = 0.24$, where an appropriate offset $\Gamma_{\rm offset}$ is used for each curve. The theoretical curves reproduce all data remarkably well.

BLQH systems.—We proceed to analyze BLQH states at $\nu = 1/m$ and $2/m$ with m odd. There are three experimental techniques to elucidate various states. The first one is to change the total electron density ρ_0. By increasing ρ_0 the interlayer separation d is effectively increased compared with the magnetic length, $d/\ell_B \propto \sqrt{\rho_0}$. Hence, as $\rho_0 \to \infty$ the BLQH system at $\nu = 2/m$

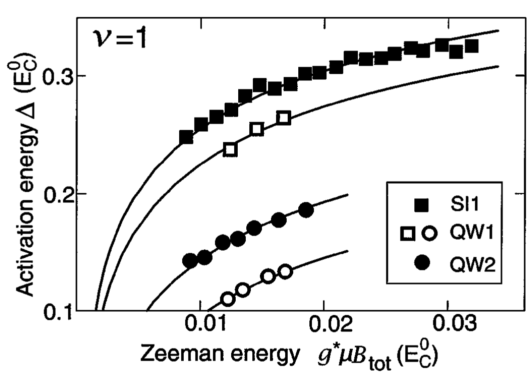

FIG. 1. Theoretical curves versus experimental data due to Schmeller *et al.* [5] for the activation energy in three samples SI1, QW1, and QW2. There are two curves for one sample (QW1) but with different mobilities. The offset $\Gamma_{\rm offset}$ increases as the mobility decreases.

will be decomposed into two independent $\nu = 1/m$ monolayer systems; as $\rho_0 \to 0$ an interlayer coherence will develop spontaneously, which has been argued [10–12] at $\nu = 1/m$ and will be argued at $\nu = 2/m$ in this paper. Consequently, we expect a phase transition at $\nu = 2/m$ between these two phases but not at $\nu = 1/m$. These two phases are clearly distinguishable by using the second technique, i.e., by applying gate bias voltage. We can control the density difference σ_0 between the two quantum wells, where $\sigma_0 = (\rho_0^1 - \rho_0^2)/\rho_0$ with ρ_0^α the density in the layer α. Only the coherent state is stable [11] against an arbitrary change of σ_0. All these features have been experimentally confirmed in recent works due to Sawada *et al.* [6] at $\nu = 1$ and 2. The coherent state at $\nu = 2/3$ has not been observed by them presumably because of a poor sample quality. The third technique is to tilt the sample with the perpendicular magnetic field $B_\perp$ fixed. In the high-density data [7] the activation energy is found to increase at $\nu = 2$ and $2/3$ as normally as in the monolayer system: Indeed, we can fit the data by the monolayer Skyrmion formula (17). We conclude that elementary excitations are monolayer Skyrmions. In the low-density data [7] it is found to decrease anomalously at $\nu = 1$ and $\nu = 2$, as is the phenomenon discovered by Murphy *et al.* [13] at $\nu = 1$: It is a behavior intrinsic to the interlayer coherent state in the BLQH system.

Spin-pseudospin coherence.—We analyze elementary excitations in the coherent state of the BLQH system. The electron field $\Psi(\boldsymbol{x})$ has four components $\psi^{1\uparrow}$, $\psi^{1\downarrow}$, $\psi^{2\uparrow}$, and $\psi^{2\downarrow}$, where the layer is indexed by 1 and 2. The kinetic Hamiltonian is given by (1), whose symmetry group is $U(4) = U(1) \otimes SU(4)$. When the interlayer and intralayer Coulomb energies are nearly equal and dominate the system, we expect a spin-pseudospin coherence to emerge.

Such a new phase is described in terms of the CB field defined by (2). The CB field is decomposed into the U(1) and SU(4) components by (5). Here, $\boldsymbol{n}(\boldsymbol{x})$ is the CP^3 field. The group SU(4) is generated by the Hermitian, traceless, 4×4 matrices λ^a, $a = 1, 2, \ldots, 15$, normalized as $\mathrm{Tr}\,(\lambda^a \lambda^b) = 2\delta^{ab}$. They are the generalization of the Pauli matrices in case of SU(2). The SU(4) spin density is given by (6) with such λ^a.

The Coulomb energy is decomposed into two terms,

$$E_C^\pm = \frac{1}{2} \int d^2x\, d^2y\, V^+(\boldsymbol{x} - \boldsymbol{y}) \varrho_\pm(\boldsymbol{x}) \varrho_\pm(\boldsymbol{y}) , \quad (19)$$

where $V^\pm(\boldsymbol{x}) = (e^2/2\varepsilon)[|\boldsymbol{x}|^{-1} \pm (|\boldsymbol{x}|^2 + d^2)^{-1/2}]$ with the interlayer separation d; $\varrho_+ \equiv \varrho(\boldsymbol{x}) = \rho(\boldsymbol{x}) - \rho_0$; $\varrho_-(\boldsymbol{x}) = \rho^1(\boldsymbol{x}) - \rho^2(\boldsymbol{x}) - \rho_0^1 + \rho_0^2$ with $\rho^1(\boldsymbol{x}) = \rho^{1\uparrow}(\boldsymbol{x}) + \rho^{1\downarrow}(\boldsymbol{x})$ and $\rho^2(\boldsymbol{x}) = \rho^{2\uparrow}(\boldsymbol{x}) + \rho^{2\downarrow}(\boldsymbol{x})$. The Coulomb energy E_C^+, possessing the SU(4) symmetry, is the driving force to realize the QH system. The term E_C^- describes the capacitive charging energy between the two layers. It vanishes in the limit $d \to 0$.

The Hilbert space $\mathbb{H}_{\mathrm{LLL}}$ is defined by the LLL condition (7). In the semiclassical approximation the wave function is given by (9) at $\nu \le 1$. The SU(4) spin-pseudospin coherence is shown to develop spontaneously, precisely as the SU(2) spin coherence is. The ground state at $\nu \le 1$ is given by [10]

$$\begin{aligned} \langle \varphi^{1\uparrow} \rangle_{\mathrm{g}} &= \sqrt{\rho_0/2}\sqrt{1 + \sigma_0} , \qquad \langle \varphi^{1\downarrow} \rangle_{\mathrm{g}} = 0 , \\ \langle \varphi^{2\uparrow} \rangle_{\mathrm{g}} &= \sqrt{\rho_0/2}\sqrt{1 - \sigma_0} , \qquad \langle \varphi^{2\downarrow} \rangle_{\mathrm{g}} = 0 , \end{aligned} \quad (20)$$

where $\rho_0^1 = \frac{1}{2}\rho_0(1 + \sigma_0)$ and $\rho_0^2 = \frac{1}{2}\rho_0(1 - \sigma_0)$. It is convenient to use a new set of CB fields,

$$\begin{aligned} \varphi^{S\uparrow} &= \sqrt{(1 + \sigma_0)/2}\, \varphi^{1\uparrow} + \sqrt{(1 - \sigma_0)/2}\, \varphi^{2\uparrow} , \\ \varphi^{A\uparrow} &= \sqrt{(1 - \sigma_0)/2}\, \varphi^{1\uparrow} - \sqrt{(1 + \sigma_0)/2}\, \varphi^{2\uparrow} , \end{aligned} \quad (21)$$

and a similar set for the spin-down fields. They are reduced to the symmetric and antisymmetric fields at the balanced point ($\sigma_0 = 0$). We call them the "bond" and "antibond" fields. The ground-state value (20) is transformed into $\langle \widetilde{\Phi} \rangle_{\mathrm{g}} = \sqrt{\rho_0}\,(1, 0, 0, 0)$ in terms of the new fields, $\widetilde{\Phi} \equiv (\varphi^{S\uparrow}, \varphi^{S\downarrow}, \varphi^{A\uparrow}, \varphi^{A\downarrow})$.

The tunneling energy is

$$E_T = -\frac{1}{2} \Delta_{\mathrm{SAS}} \sqrt{1 - \sigma_0^2} \int d^2x\, [\varrho_-^S(\boldsymbol{x}) - \varrho_-^A(\boldsymbol{x})] , \quad (22)$$

where $\varrho_-^S(\boldsymbol{x}) = \rho^S(\boldsymbol{x}) - \langle \rho^S(\boldsymbol{x}) \rangle_{\mathrm{g}}$ with $\rho^S = \varphi^{S\uparrow\ddagger}\varphi^{S\uparrow} + \varphi^{S\downarrow\ddagger}\varphi^{S\downarrow}$ and similar equations for $\varrho_-^A(\boldsymbol{x})$. The tunneling gap is $(1 - \sigma_0^2)^{1/2} \Delta_{\mathrm{SAS}}$ on the state (20).

A Skyrmion excitation flips in general spins and pseudospins. It is a CP^3 Skyrmion [2] described by

$$\langle \widetilde{\Phi} \rangle_{\mathrm{sky}} = \sqrt{\rho_0}\,(z, \kappa_1, \kappa_2, \kappa_3) , \quad (23)$$

with constant parameters κ_i. Its classical configuration is determined by (11)–(13), and (15) with $\kappa^2 = \sum_\alpha \kappa_\alpha^2$. For definiteness we assume hereafter that the tunneling energy is much larger than the Zeeman energy. (For instance, $\Delta_{\mathrm{SAS}}/g^* \mu_B B \simeq 4$ at $B = 5$ T in the sample of Ref. [6].) Then, we have $\kappa = \kappa_1 \neq 0$, $\kappa_2 = \kappa_3 = 0$. It is identical to the CP^1 Skyrmion (14) in the spin space. At the balanced point the Skyrmion size, the Skyrmion energy, and the flipped-spin number are given by the same formulas as (16)–(18), where β now depends on the layer separation d. Here, we concentrate our attention to its dependence on the imbalance parameter σ_0. The term involving σ_0 is only the charging term (19) for $\nu \le 1$. We give a numerical estimation of the activation energy at $\nu = 1$ in Fig. 2 by using the sample parameters ($d = 231$ Å, $\ell_B = 120$ Å) of Ref. [6]. The theoretical curve explains the experimental data quite well with a reasonable Skyrmion size $\kappa \simeq 1$.

BLQH ferromagnet at $\nu = 2$.—A caution is needed to analyze the BLQH system at $\nu = 2$ since one Landau state contains two electrons. We attach one unit of flux to each electron and transform it into the CB field by formula (2) with $m = 1$. The effective magnetic field is not given by (3) but by

$$2\mathcal{B}_{\mathrm{eff}} = 2B_\perp - \varepsilon_{ij}\partial_i\partial_j\Theta(\boldsymbol{x}) = 2B_\perp - (2\pi/e)m\rho(\boldsymbol{x}) . \quad (24)$$

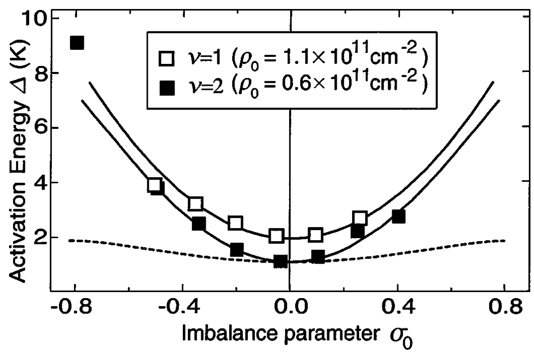

FIG. 2. Theoretical curves versus experimental data due to Sawada *et al.* [6]. The Skyrmion charge is e at $\nu = 1$ and $2e$ at $\nu = 2$. The discrepancy of the data for large $|\sigma_0|$ may indicate that genuine CP^3 Skyrmions are excited there since the tunneling gap $(1 - \sigma_0^2)^{1/2}\Delta_{SAS}$ becomes smaller. The dotted curve is for a would-be Skyrmion carrying e at $\nu = 2$.

It vanishes on the homogeneous ground state at $\nu = 2/m$. Because of the Fermi statistics the wave function is the antisymmetric product of two wave functions (9),

$$\mathfrak{S}[\boldsymbol{x}] = \prod_r [\langle \Phi_1(\boldsymbol{x}_r)\rangle \otimes \langle \Phi_2(\boldsymbol{x}_r)\rangle - \langle \Phi_2(\boldsymbol{x}_r)\rangle \otimes \langle \Phi_1(\boldsymbol{x}_r)\rangle] \mathfrak{S}_{\text{LN}}[\boldsymbol{x}]^2, \tag{25}$$

where $\langle \Phi_1(\boldsymbol{x})\rangle$ and $\langle \Phi_2(\boldsymbol{x})\rangle$ are analytic and satisfy

$$\sum_\alpha |\langle \varphi_1^\alpha(\boldsymbol{x})\rangle|^2 = \sum_\alpha |\langle \varphi_2^\alpha(\boldsymbol{x})\rangle|^2, \tag{26}$$

as follows from the semiclassical LLL condition (11).

When $\Delta_{SAS} \gg g^*\mu_B B$, the spin-up and spin-down bond states are filled. Hence, the ground state is given by (25) with a set of two constant CB fields,

$$\langle \tilde{\Phi}_1\rangle_{\text{g}} = \sqrt{\frac{\rho_0}{2}}\,(1,0,0,0), \qquad \langle \tilde{\Phi}_2\rangle_{\text{g}} = \sqrt{\frac{\rho_0}{2}}\,(0,1,0,0)\,, \tag{27}$$

in terms of the bond and antibond fields. This might be identified with the canted state [14] for $\Delta_{SAS} \gg g^*\mu_B B$. A Skyrmion excitation flips pseudospins, or induces a coherent tunneling excitation. It is described by (25) with a set of two CB fields,

$$\begin{aligned} \langle \tilde{\Phi}_1\rangle_{\text{sky}} &= \sqrt{\rho_0/2}\,(z, \kappa_1, \kappa_2, \kappa_3)\,, \\ \langle \tilde{\Phi}_2\rangle_{\text{sky}} &= \sqrt{\rho_0/2}\,(\kappa_1', z, \kappa_2', \kappa_3')\,, \end{aligned} \tag{28}$$

with $\kappa^2 \equiv \sum_\alpha \kappa_\alpha^2 = \sum_\alpha \kappa_\alpha'^2$. It consists of two CP^3 Skyrmions (23), and the Skyrmion charge is $2e$. We emphasize that there exists no Skyrmion composed of one CP^3 Skyrmion at $\nu = 2$ because of the constraint (26).

An estimation of the excitation energy of the Skyrmion (28) is straightforward. We concentrate our attention to its dependence on the imbalance parameter σ_0. The terms involving σ_0 are the charging energy (19) and the tunneling energy (22). The charging energy increases while the tunneling energy decreases as σ_0 increases. We give a numerical estimation in Fig. 2 by using the sample parameters ($d = 231$ Å, $\ell_B = 228$ Å, $\Delta_{SAS} = 6.8$ K) of Ref. [6]. The vortex limit ($\kappa \simeq 0$) gives a best fitting of the experimental data because of a large tunneling gap ($\Delta_{SAS} = 6.8$ K). We have also given a theoretical curve for a would-be Skyrmion carrying charge e at $\nu = 2$ by using the same parameters, where the charging energy and the tunneling energy are found to cancel each other almost completely (Fig. 2).

The driving force of the SU(4) spin-pseudospin coherence is the Coulomb exchange energy arising from the SU(4)-invariant Coulomb term E_C^+ in (19). Provided the exchange energy is dominant, it is obvious that the SU(4) coherence develops also at $\nu = 2/m$ with the Skyrmion charge $2e/m$, and at $\nu = 6, 10, 14, \ldots$ with charge $2e$.

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