Acoustic Spectroscopy of Superfluid ^{3}He in Aerogel

A. Golov,* D. A. Geller, and J. M. Parpia
Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14853

N. Mulders
Department of Physics and Astronomy, University of Delaware, 223 Sharp Laboratory, Newark, Delaware 19716
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Liquid ^{3}He in 98%-porous aerogel simultaneously possesses properties of a disordered p-wave superfluid and an elastic solid. We have observed two sound modes of superfluid ^{3}He in aerogel. The first (fast) sound is a compression wave, and the second (slow) sound is the out-of-phase oscillation of the superfluid and the normal component that is coupled to the elastic aerogel matrix. A measurement of the slow sound velocity allows an accurate determination of the superfluid fraction. [S0031-9007(99)08991-7]

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In superfluid hydrodynamics, the presence of two interpenetrating "fluids," the normal and superfluid components, leads to a low velocity mode—second sound—in which the two components move independently [1]. Apart from being a manifestation of the special properties of superfluidity, it provides a valuable measurement tool with which the density of the superfluid component, ρ_s, can be directly determined [2].

Liquid ^{3}He in highly porous aerogel glass is the first realization of disordered superfluid ^{3}He [3,4]. In contrast to superfluid ^{4}He in aerogel [5], its zero-temperature superfluid fraction, ρ_s/ρ, is substantially suppressed from unity. Moreover, by changing the pressure [6] or the microscopic structure of the aerogel [7] one can suppress ρ_s/ρ and T_c to zero. Direct measurements of the superfluid fraction with a torsional pendulum exhibited interference with sound modes (including those reported in this Letter) of ^{3}He in aerogel [3,7,8]. We describe sound modes that we observed and the use of acoustic spectroscopy as an alternative technique for the accurate determination of ρ_s/ρ.

Our resonator consisted of a cylindrical aerogel sample (length $L = 1.59$ cm, radius $R = 0.48$ cm) grown inside a stainless steel tube. Brass diaphragms of 0.3 mm thickness (to which piezoceramic speaker and microphone transducers were attached) were pressed against the aerogel. At room temperature the gap between the aerogel sample and the tube was small, but the aerogel could freely slide within the tube. Sound spectra were recorded as the quadrature response of the microphone to the oscillations of the speaker while the drive frequency was swept continuously. The temperature, T, was determined with a melting curve thermometer.

The aerogel sample is a conglomeration of $\sim$50 Å silica particles that form a disordered network of strands with a distribution of interstrand spacing between $\sim$50 Å and $\sim$1000 Å, porosity $\phi = 0.98$, density $\rho_a = \rho_{\mathrm{SiO}_2}(1-\phi) = 0.04\ \mathrm{g/cm^3}$, and with a longitudinal sound velocity $c_a \sim 50$ m/s. In liquid ^{3}He, the viscous penetration depth, δ_v, at a frequency, f, is

$$\delta_v = (\eta/\pi f \rho_n)^{1/2}, \tag{1}$$

where η is the viscosity of ^{3}He, ρ_n is the density of the normal component, and $\rho_s + \rho_n = \rho = \phi\rho_{\mathrm{bulk}}$ is the density of ^{3}He in aerogel. At the bulk superfluid transition temperature, T_c^{bulk}, and frequencies below 10 kHz, $\delta_v \geq 0.1$ mm. Hence, in aerogel the normal component is always viscously clamped to the silica skeleton. If the latter were rigid, only the superfluid component would move and allow a pressure wave, "fourth sound," with velocity, c_4,

$$c_4^2 = \frac{\rho_s}{\rho} c_1^2 + \frac{\rho_n}{\rho} c_2^2, \tag{2}$$

to propagate. Here $c_1 = [(\partial p/\partial\rho)_T]^{1/2}$ is the velocity of the ordinary compression wave (first sound), and $c_2 = S(T\rho_s/C_p\rho_n)^{1/2}$ is the second sound velocity which, in ^{3}He-B, is less than 3 cm/s [9].

Because the skeleton is elastic, the normal component can oscillate with the silica and will exhibit a renormalized mass and stiffness due to the aerogel's presence. This system is directly connected to the concept of a "supersolid" in which a persistent mass current (for example, the coherent flow of mobile vacancies in a quantum crystal [10]) can be established relative to the lattice. As a consequence of conservation of the momentum of the solid and the mass current, four collective modes (sounds) are predicted for this system, resembling one longitudinal and two transverse elastic waves in the solid, together with a superfluid density wave [10,11].

McKenna *et al.* [12] modified the conventional two fluid hydrodynamic equations to take into account coupling of the normal component to the aerogel mass and elasticity. Their equations contain a vector restoring force and no explicit shear stress tensor. They are therefore exact only for longitudinal waves in infinite media and do not allow transverse wave solutions. The velocities of both

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longitudinal sounds (fast, c_f, and slow, c_s), $c_x = (c_f, c_s)$, satisfy the equation [12]

$$(c_x^2 - c_1^2)(c_x^2 - c_2^2) + \frac{\rho_a}{\rho_n}(c_x^2 - c_a^2)(c_x^2 - c_4^2) = 0. \tag{3}$$

They also observed the propagation of both the fast and the slow modes in a ^{4}He filled cylinder of aerogel. Using the bulk values for $\rho_n(T)$ (which are close to those in aerogel [5]) they found good agreement of the model with the observed temperature dependence of the sound modes.

The fast mode is a compression wave in helium-filled aerogel. In contrast, the slow mode is the analog of second sound in bulk helium and fourth sound in porous media—it exists only in the superfluid state and its velocity vanishes at T_c. This mode can be visualized as a small oscillation of a deformation of the aerogel combined with a simultaneous out-of-phase motion of the superfluid component. The analysis of this slow mode in terms of the superfluid fraction provides new insights. In the limit $c_2^2 \ll c_s^2 \ll c_1^2$ Eq. (3) takes the form

$$\frac{\rho_s}{\rho} = \left\{\frac{\rho_a}{\rho}\left[\left(\frac{c_a}{c_s}\right)^2 - 1\right] + 1\right\}^{-1}, \tag{4}$$

which can be further simplified for $c_s^2 \ll c_a^2$ as

$$\frac{\rho_s}{\rho} = \frac{\rho}{\rho_a}\left(\frac{c_s}{c_a}\right)^2. \tag{5}$$

Using Eqs. (4) or (5) ρ_s/ρ can be determined from the measured ratio of sound velocities, c_s/c_a.

Unlike bulk second sound (a pure thermal wave [13]) in which the superfluid and normal components also oscillate out of phase, in the slow mode the restoring force exerted on the normal component is mainly determined by the elastic energy of the aerogel and not the thermal energy of the ^{3}He. The scale of c_s is set by the longitudinal sound velocity in aerogel, $c_a \gg c_2$ [14] [Eq. (5)]. This is also different from fourth sound in a rigid porous medium (a pure pressure wave), where the scale is set by the sound velocity in liquid ^{3}He, $c_1 \gg c_a$ [Eq. (2)]. In all previous studies of ^{3}He in aerogel, the aerogel played the role of quenched disorder that suppresses the amount of the superfluid component. In the present study, we observe that the aerogel also changes the dynamic properties of the normal component, giving rise to an unusual effect—the coexistence of two sound modes, each of which is a pressure wave.

With our apparatus we observed fundamental resonances of the fast and slow modes at frequencies, f_0, in the range 10 Hz–10 kHz. Providing they correspond to a standing wave of wavelength $\lambda = 2L$, we define the effective sound velocity, $c_x^* = 2Lf_0$. The latter is related to the sound velocity in an infinite medium, c_x, by $c_x^* = A_x c_x$, where the numerical factor, $A_x = (A_f, A_s, A_a)$, depends on the mode (fast, slow, or in empty aerogel, respectively) and boundary conditions [16]. For a pure compression wave (as would be the case of liquid in a tube), $c_x^* = c_x$, and hence $A_x = 1$.

The evolution with temperature of the fundamental resonance of the fast mode is shown in Fig. 1. The sound velocity, c_f^*, is 80% of the bulk ^{3}He first sound velocity. In the superfluid phase, c_f^* changes by only $\sim$1% or less. When all ^{3}He in aerogel is normal, the solution of Eq. (3) is given by

$$c_f^2 = \left(c_1^2 + \frac{\rho_a}{\rho}c_a^2\right)\Big/\left(1 + \frac{\rho_a}{\rho}\right) \approx c_1^2\Big/\left(1 + \frac{\rho_a}{\rho}\right). \tag{6}$$

The aerogel contributes little to the restoring force, which is dominated by the compressibility of the helium, and only adds its mass, producing a decrease in the sound speed. The difference between c_f^* and c_f calculated using Eq. (6) was less than 1% at $p = 5$ bar and 7% at 29 bar, using bulk ^{3}He properties tabulated by Wheatley [17]. The agreement is improved if we account for the change in the sound path, L, caused by the deformation of the diaphragms with pressure [18]. We find $A_f = 1.01 \pm 0.01$.

Examples of the slow mode resonances at different temperatures are shown in Fig. 2. To determine the center-of-peak frequency, f_0, they were fitted to a Lorentzian. In Fig. 3 we plot the sound velocity, c_s^*, using data similar to that of Fig. 2 for pressures between 5 and 29 bar. The transition temperatures, $T_c(p)$, indicated with arrows in Fig. 3, are in good agreement with those from previous studies of ^{3}He in this type of aerogel [4,6,7].

We note that at a temperature higher than T_c (but below T_c^{bulk}) the apparent velocity of the slow mode does not vanish completely even though all the ^{3}He in aerogel is in the normal state [see Figs. 2(b) and 3]. A similar

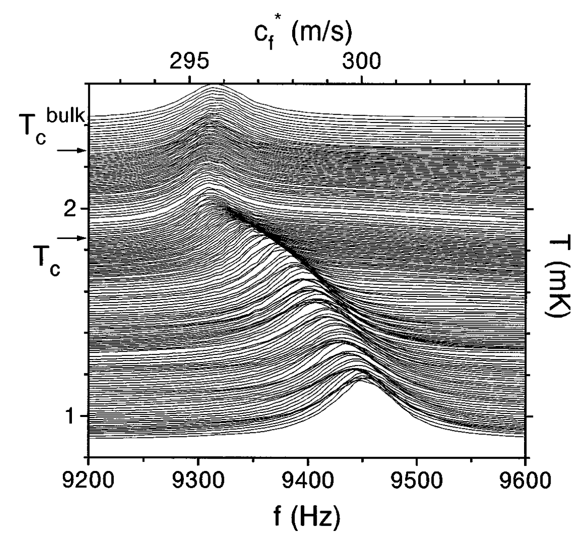

FIG. 1. Spectra of the quadrature response of the microphone (fast mode, $p = 21.7$ bar) offset vertically with temperature.

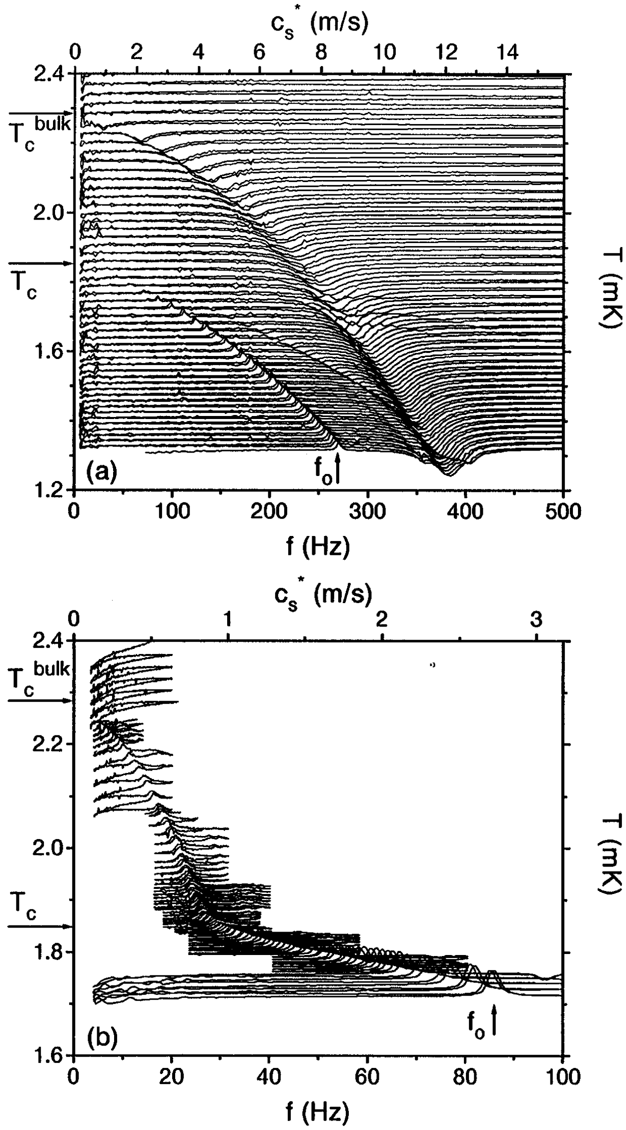

FIG. 2. (a) Spectra of the quadrature response of the slow mode ($p = 21.7$ bar) offset vertically with temperature. The sharp positive peak (f_0) disappearing at $T_c = 1.85$ mK is the fundamental slow mode resonance. The stronger and broader negative peak vanishing at $T_c^{\mathrm{bulk}} = 2.28$ mK is the Helmholtz resonance. Five higher modes with the same T_c are seen in the figure (two cross the Helmholtz mode at 1.2 and 1.4 mK and three rapidly changing modes cross just below T_c). (b) An enlarged plot of the low frequency region [at 10 times the drive used in (a)] shows the conversion of the slow mode into the slowly varying edge mode that disappears at T_c^{bulk}.

effect was reported by Mulders *et al.* [19] for superfluid ^{4}He in aerogel, and was attributed to ^{4}He in connected macroscopic voids in the aerogel. As our sample is believed to be free of voids, it is most likely that we excite an oscillatory "edge mode." In such a mode, compression and lateral deformation of the aerogel stimulate longitudinal oscillations of the bulk superfluid ^{3}He in the narrow gap between the aerogel and the walls of the surrounding tube. In this oscillatory motion, the inertial term is dominated by the superfluid density in the gap, and the

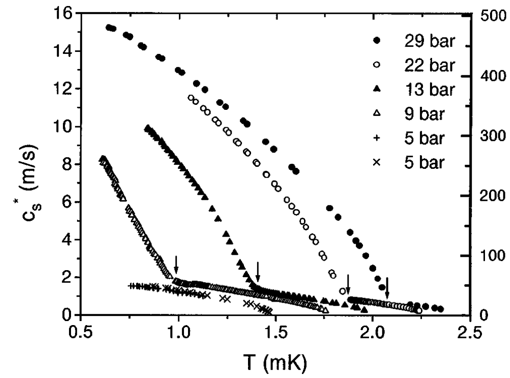

FIG. 3. The sound velocity, c_s^*, vs temperature. $p = 29.0$, 21.7, 13.3, and 9.1 bar. For $p = 5.0$ bar, (+) denotes data from the fundamental, f_0, and (×) from the first harmonic, $f_1 = 2f_0$, of the edge mode. All but the 5 bar data display the slow mode branch arising from superfluidity of ^{3}He in aerogel. The onset of superfluidity of ^{3}He in aerogel is shown by arrows.

resonant frequency should vary as the square root of bulk ρ_s/ρ. The expected temperature dependence agrees well with the observed frequency dependence and is shown as the solid line in Fig. 4.

In Fig. 5 we plot the superfluid fraction of ^{3}He in aerogel vs reduced temperature T/T_c. Because of its quadratic dependence on c_s [Eq. (5)], the edge mode contribution to ρ_s/ρ is less than 0.003 at all pressures and the uncertainty of ρ_s/ρ introduced by its presence is small. The values of ρ_s/ρ are comparable to those obtained with a torsional oscillator [3,6–8]. The superfluid fraction extrapolates to $\rho_s/\rho(T = 0) \approx 0.3$ at $p = 29$ bar and to lower values at reduced pressures.

Figure 2(a) shows a Helmholtz resonance at frequency f_H associated with the volume of the cell and the filling tube of length $l \approx 4$ cm and diameter $d \approx 0.14$ cm. The ratios f_H/c_1 vs T/T_c^{bulk} fall on a pressure independent curve. Below 100 Hz, $f_H(T)$ closely follows the behavior expected for fourth sound, Eq. (2). At low frequencies, δ_v [Eq. (1)] is comparable to the diameter of the fill tube,

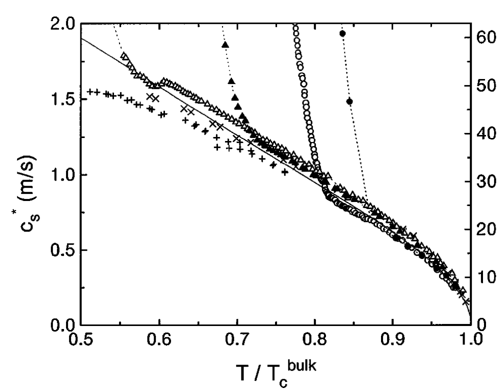

FIG. 4. The sound velocity, c_s^*, near T_c vs T/T_c^{bulk} (same notations as in Fig. 3; dotted lines guide the eye). The edge mode data collapse onto a single curve—the solid line, $c = (2.4\ \mathrm{m/s})(\rho_s^{\mathrm{bulk}}/\rho_{\mathrm{bulk}})^{1/2}$ for ^{3}He at $p = 9$ bar [20].

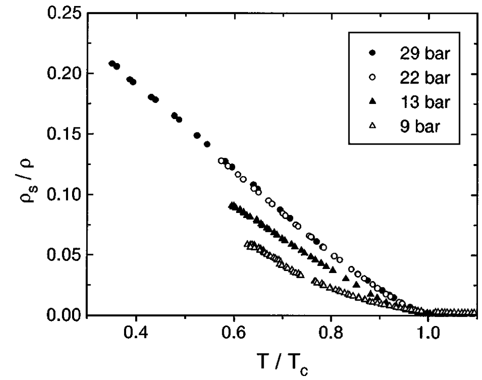

FIG. 5. The complete data set of ρ_s/ρ plotted vs the reduced temperature T/T_c.

$\delta_v \sim d$, so that only the superfluid component can oscillate. Hence the frequency, f_H, is given by [21]

$$2\pi f_H = c_4(d/D)(lL)^{-1/2} \propto c_1(\rho_s^{\text{bulk}}/\rho_{\text{bulk}})^{1/2}. \quad (7)$$

In summary, we have observed two distinct acoustic modes of superfluid ^{3}He in aerogel, one which exists only below the suppressed transition temperature of ^{3}He, and the other which shows a frequency shift below this temperature. The two modes establish the unique character of this system that simultaneously displays behavior of a disordered superfluid and an elastic solid. Tracking the slow mode frequency provides a cleaner technique for the mapping of ρ_s/ρ than the torsional oscillator because interfering resonances are excluded. We have also observed a new edge mode which occurs at the interface between a normal fluid and a superfluid. Absolute measurements of ρ_s/ρ with this technique will require computation of the exact vibrational modes of a resonator together with predetermined boundary conditions.

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*Permanent address: Department of Physics and Astronomy, Manchester University, Manchester M13 9PL, United Kingdom.

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