Superscaling in Inclusive Electron-Nucleus Scattering

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We investigate the degree to which the scaling functions $f(\psi')$ derived from inclusive electron-nucleus quasielastic scattering define the *same* function for *different* nuclei. In the region where the scaling variable $\psi' < 0$, we find that this superscaling is experimentally realized to a high degree. [S0031-9007(99)08962-0]

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The use of scaling and the application of dimensional analysis to inclusive scattering of a weakly interacting probe from the constituents of a composite system have been important tools in gaining new insights into physics. Examples include the scattering of keV electrons from electrons bound in atoms [1], scattering of eV neutrons from atoms in solids or liquids [2], deep inelastic scattering of GeV leptons from the quarks in the nucleon [3], and, of particular interest here, quasielastic scattering of electrons in the energy range of hundreds of MeV to several GeV from nucleons in nuclei [4]. Despite the extraordinary kinematical range for which scaling has been studied, the conceptual basis for describing this phenomenon has many common features.

Scaling allows one to represent the data in a very compact form. In many cases, the initial *experimental* observation of scaling has been the driving factor in motivating more detailed studies and has led to better understanding of the data.

The inclusive cross sections for the scattering of a weakly interacting probe in general depend explicitly on two independent variables—the energy ω and three-momentum $\vec{q}$ transferred by the probe to the constituent. *Scaling* means that, in the asymptotic regime of large $q = |\vec{q}|$ and ω, the cross sections depend on a *single* variable $z = z(q,\omega)$. This property results essentially from the kinematics of the scattering process, where a constituent is ejected nearly quasifreely from the composite system.

The interest in scaling phenomena originates from two distinct sources.

(i) The observation of the occurrence (or nonoccurrence) of scaling yields information on the domination (or not) of the quasifree scattering process or the contribution of other reaction mechanisms (which in general do not scale). These provide *experimental* reflections of the reaction mechanism which are prerequisites for a quantitative understanding of the cross section.

(ii) The function to which the data scale is closely related to the momentum distribution (or, more generally, to the spectral function) of the constituents in the composite target. This provides an interesting insight into the dynamics of the bound system.

For inclusive quasielastic electron-nucleus scattering, data are available for several nuclei and have been found to scale in a major part of the kinematical region studied. When analyzed in terms of the scaling variable y [4], which is defined as the minimal value of the momentum a nucleon can have in impulse approximation before the reaction, the data exhibit scaling for $y < 0$, that is, the region where ω is smaller than its value at the quasielastic peak. Much of the past work has concentrated on the study of the scaling properties of the response in the low-ω tail of the quasielastic peak, where y is large and negative. Detailed quantitative studies of the conditions under which scaling occurs and the impact of adverse effects such as final-state interactions (FSI) [5], the spread of the spectral function $S(k,E)$ in energy E for fixed k, and the contributions of other reaction mechanisms, have been made. For a review, see Ref. [6].

Past applications of scaling focused on individual nuclei. In this Letter, we explore a novel aspect: We compare the scaling function of *different nuclei* with mass number $A \geq 4$, and study the degree to which these scaling functions may be mapped into a *universal result* and thus to superscale.

(I) Motivation.—Discussions of scaling at intermediate energies assume that inclusive electron scattering in the quasielastic regime is dominated by the impulsive one-body knockout of nucleons. Two-body meson exchange currents (MEC), meson production, and FSI limit the range of applicability once they give sizeable contributions to the cross section.

In addition to the electron scattering angle θ_e, two variables (typically q and ω) characterize the inclusive cross section. Of course, any function of (q,ω) may be used together with q; it has been traditional to use the so-called y-scaling variable (for a review, see Ref. [6]). Upon dividing the inclusive electron scattering cross section by the single-nucleon electromagnetic cross section together with the Jacobian required in changing variables, one

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obtains a derived function $F(q, y)$. Scaling of the *first kind* means that at high enough values of q this becomes a function only of y, independent of q. Indeed, it has been found that, in the $y < 0$ region for momentum transfers of roughly 0.5 GeV/c or larger, y scaling is quite well obeyed.

In [7] (and elaborated in [8]) the idea of superscaling was introduced, motivated by the relativistic Fermi gas (RFG) model. While this model clearly does not incorporate many of the effects of initial- and final-state dynamics, it nevertheless makes an interesting prediction that warrants testing using experimental data. It suggests that, when using a dimensionless scaling variable ψ and a slightly modified version of the scaling function F, both scaling of the first kind (independence of q) and also scaling of the *second kind* (independence of the nuclear species) are expected. That is, the model predicts *superscaling* [7]. The purpose of this Letter is to see whether or not Nature obeys this extended type of scaling behavior.

Before putting the world's data to the test, let us first define the scaling variables and scaling functions. The traditional approach to y scaling as summarized in [6] involves dividing the inclusive cross section by some form of off-shell single-nucleon cross section, usually the CC1 prescription of De Forest. In fact, the actual form chosen is not critical, as long as it contains the necessary relativistic content (see [8] for more discussion) and it is a very minor approximation for inclusive scattering to use the on-shell single-nucleon cross section. Accordingly, the scaling function may be written in a very simple form:

$$F(q, \omega) \cong \frac{d^2\sigma/d\Omega_e\, d\omega}{\sigma_M[v_L(m_N q/|Q^2|)\widetilde{G}_E^2 + v_T(|Q^2|/2m_N q)\widetilde{G}_M^2]}, \tag{1}$$

where $\widetilde{G}_E^2(Q^2) \equiv ZG_{Ep}^2 + NG_{En}^2$ with $\widetilde{G}_M^2(Q^2)$ defined similarly; here $Q^2 \equiv \omega^2 - q^2$, σ_M is the Mott cross section, and $v_{L,T}$ are the familiar Rosenbluth kinematical factors. Scaling of the first kind occurs for this function using experimentally determined cross sections and plotting the results versus the familiar y-scaling variable.

Additionally, the RFG model suggests making dimensionless scaling variables and scaling functions using the Fermi momentum k_F as the scale. In [7] the former was denoted ψ and given approximately by

$$\psi \cong \frac{1}{k_F}(\sqrt{\omega(2m_N + \omega)} - q), \tag{2}$$

where m_N is the nucleon mass. The result here is expanded only to leading order in k_F/m_N, which is small; the exact RFG expression may be found in [7]. Clearly this has a similar behavior to the usual y variable in that it reaches zero at the quasielastic peak. Additionally, to allow for the fact that nucleons are knocked out of all shells in the nucleus (and therefore that some aspects of the missing energy dependence in the spectral function may be incorporated) we follow the spirit of [9], shifting from ω to $\omega' \equiv \omega - E_{\mathrm{shift}}$ [see section (II) for values of the shift] and hence defining a derived variable ψ' by making this substitution in Eq. (2).

It may be shown [10] that the variable ψ' so-defined is close to y/k_F—that is, for the conditions of the present analysis it is not important which is employed, and upon plotting the function in Eq. (1) versus ψ', one continues to observe scaling of the first kind. It is not our purpose here to elaborate on the origins of the commonly used scaling variables (see, for example, Refs. [7–10]), but simply to draw from the RFG model the idea of using dimensionless quantities and a momentum scale k_F that has physical meaning. It is not our intent to justify the RFG as a model of quasielastic scattering. A decade ago [7] it provided the first motivation to investigate superscaling (and thus the motivation for the following analysis), but otherwise it is not used in this paper. Given the limitations of the RFG, one could ignore all reference to it and simply equate the variable ψ' used here with y/k_F for some characteristic momentum scale k_F.

If the function in Eq. (1) is *also* made dimensionless by multiplying by k_F to define $f(q, \psi') \equiv k_F \times F(q, \omega)$ and is plotted versus ψ', the RFG model suggests that it will also exhibit scaling of the second kind and therefore will superscale (i.e., scale in both ways). Below, we treat the world's data in this way to test whether or not Nature superscales.

(II) Results.—For these studies we concentrate on nuclei with $A \geq 4$, as the lightest nuclei are known to have spectral functions that are very far from the "universal" one which is at the basis of the superscaling idea. Data on inclusive electron-nucleus scattering for a series of nuclei $A \geq 4$ are available in the region of low momentum transfers $q \sim 0.5$ GeV/c [11–24] data extending to much higher q are available from other experiments [25–28]. Not all of these data can be used, however, as some have not been corrected for radiative and Coulomb distortion effects, are known to have problems such as "snout scattering" or the inclusion of false signals from π^-'s in the electron spectrometer, or are only available in the form of figures. Some are at very low momentum transfer and excluded as scaling is known to break down there due to large FSI and Pauli blocking effects.

In a first step, we have taken the data which meet our criteria for the nuclei $A = 12$–208 and have analyzed them in terms of scaling in the variable ψ'. For k_F we use 220, 230, 235, and 240 MeV/c for C, Al, Fe, and Au, with intermediate values for the intermediate nuclei; for E_{shift}, which has a minor effect, we use 15, 15, 20, and 25 MeV for the same nuclei.

Figure 1 shows the scaling function $f(\psi')$ for all kinematics suitable for the present study and all A available. We clearly observe a scaling behavior for values of $\psi' < 0$: While the cross sections at a given ψ'

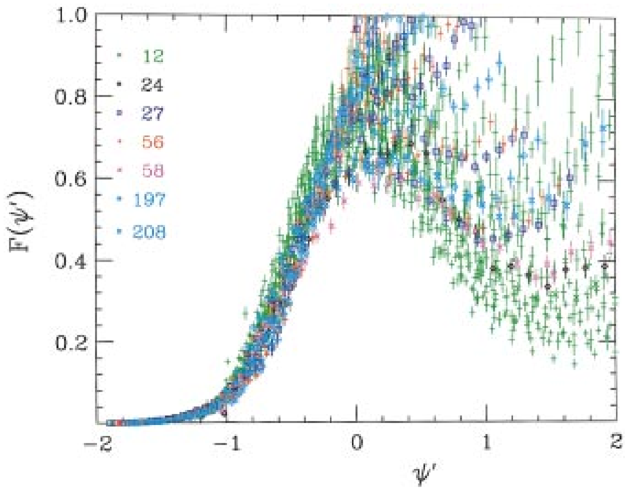

FIG. 1(color). Scaling function $f(\psi')$ as function of ψ' for all nuclei $A \geq 12$ and all kinematics. The values of A corresponding to different symbols is also shown.

vary over more than 3 orders of magnitude, the values of $f(\psi')$ are essentially universal. For $\psi' > 0$, on the other hand, the scaling property is badly violated, as expected, since here processes other than quasielastic scattering—meson exchange currents, Δ excitation, and deep inelastic scattering—contribute to the cross section. The scaling as discussed in this paper applies only to processes having the behavior of electron-nucleon quasifree scattering.

In order to separate some of the effects leading to less-than-perfect scaling at negative ψ', in Fig. 2 we show the function $f(\psi')$ for the series of nuclei $A = 12$–197, but for fixed kinematics (3.6 GeV, 16°, and hence nearly constant q). The quality of the scaling in the region $\psi' < 0$ is quite amazing. This shows that the removal of the A dependence, i.e., scaling of the second kind, actually is *better* realized in Nature than ordinary scaling. The deviations from scaling observed in Fig. 1 are *not* from an A dependence.

A part of the A-dependent increase of $f(\psi')$ at positive ψ' results from the increase of k_F with A, yielding an increase of the width of the quasielastic and Δ peaks, and a correspondingly increased overlap with non-quasi-free scattering processes (Δ excitation, π production, …). At the same time, the increasing average density of the heavier nuclei also leads to an increase in contributions of two-body MEC processes which are strongly density dependent (i.e., do not scale with k_F in the same way the one-body knockout processes do [29]).

Figure 3 shows the data for $A = 4$, 12, 27, 56, and 197 on a logarithmic scale for the kinematics of Fig. 2 and demonstrates that the scaling property extends to large negative values of ψ', corresponding to large momenta of the initial nucleon. This feature clearly cannot be predicted within the RFG model, since there the response is restricted to $|\psi'| < 1$. However, there are indications of this behavior from theoretical studies of the nuclear matter spectral function as a function of density. For different nuclear matter densities and large k, the spectral functions are similar in shape [30] and the tail of the momentum distribution $n(k)$ at $k > k_F$ (corresponding to $\psi' < -1$) is a near-universal function of k/k_F [31]. For finite nuclei and large momenta we can employ the local density approximation (LDA), as at large k we are dealing with short-range properties of the nuclear wave function [30]. Within LDA, the nuclear momentum distribution (spectral function) is then a weighted average over the corresponding nuclear matter distributions. This means that the large momentum tail of the nuclear spectral function also scales with k_F, a dependence that is removed when using ψ'. Previous work [5] has shown that in the extreme tail of the quasielastic peak, FSI play an increasingly important role, and lead to a slow convergence of $F(y, q)$ with q. Figure 3 indicates that the effects of FSI on scaling of the second kind are

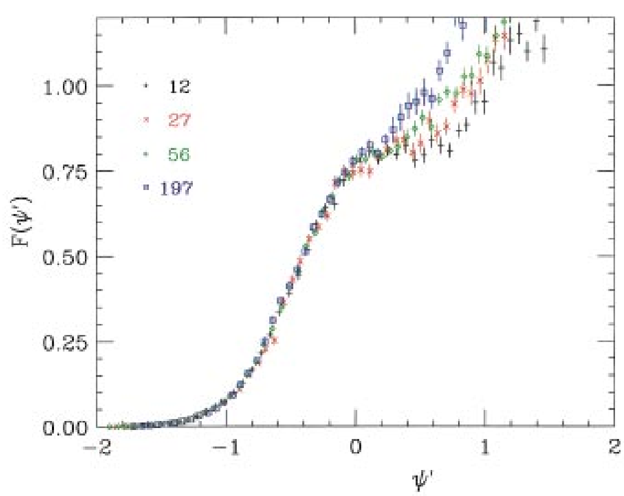

FIG. 2(color). Scaling function for C, Al, Fe, and Au and fixed kinematics [25]. The correspondence of symbol and mass number of the nucleus is also shown.

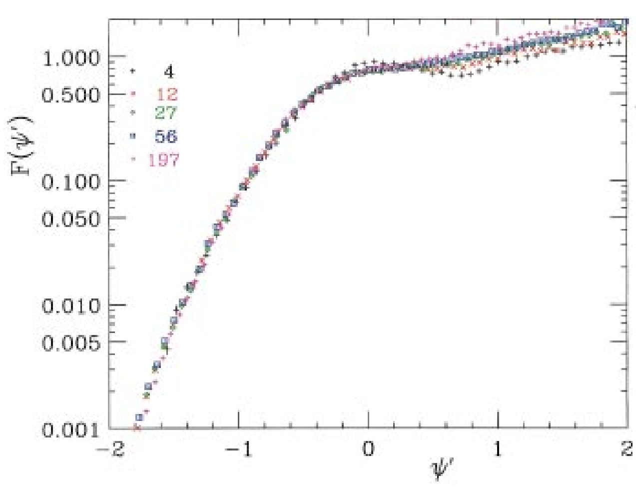

FIG. 3(color). Scaling function for nuclei $A = 4$–197 and fixed kinematics on a logarithmic scale.

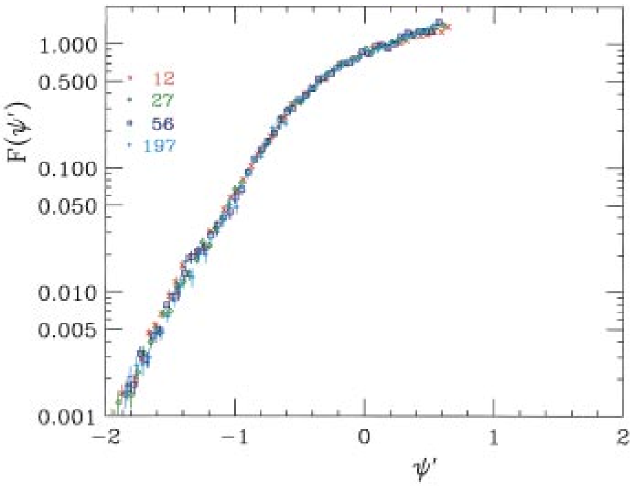

FIG. 4(color). Scaling function for nuclei $A = 4-197$ at higher momentum transfers (3.6 GeV, 25°).

less pronounced. This is presumably due to the fact that it is the FSI of the nucleon immediately after the scattering that counts, and these FSI for different A are near-universal, modulo surface effects.

In order to emphasize the quality of this superscaling in the tail, in Fig. 3 we have also included the data on ^{4}He, taken under the same kinematical conditions [25] ($k_F = 200$ MeV/c, $E_{\rm shift} = 20$ MeV). While, at $\psi' = 0$, $f(\psi')$ for ^{4}He is about 10% higher than for heavier nuclei as a consequence of the sharper peak of the spectral function at $k \sim 0$, the scaling function for ^{4}He agrees perfectly with the one for heavier nuclei for $\psi' < -0.2$.

A similar quality of superscaling is found when analyzing the data for other kinematics at both higher and lower q, where cross sections for a large range of A are available. As an example, Fig. 4 shows the scaling function for the data at 3.6 GeV and 25° [25].

(III) Conclusions.—We have analyzed data on electron-nucleus quasielastic scattering for nuclei with mass numbers $A = 4-208$ which cover a large range in (q, ω). We find that, upon use of a scaling variable which allows one to remove the "trivial" dependence on the Fermi momentum, the data on the low-ω side of the quasielastic peak show *both* the traditional scaling of the first kind—independence of q—and also scaling of the *second kind*, i.e., independence of the nuclear species where the scaling functions for different nuclear mass numbers A coincide. Indeed, this A independence of the superscaling function is much better realized than the q independence of the normal scaling function. In other words, scaling of the second kind seems to be much less sensitive to scaling violations resulting from processes such as MEC, nucleon FSI, and the spread of $S(k, E)$ in E. While this reduced sensitivity is plausible, it needs to be understood better theoretically.

In summary, what clearly has been demonstrated in detail for the first time is that scaling of the second kind and hence superscaling are well obeyed in Nature. As with studies of scaling of the first kind in previous work, the issue now is to understand which theories do or do not produce this behavior.

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