Strong Evidence for Stochastic Growth of Langmuir-like Waves in Earth's Foreshock

Iver H. Cairns* and P. A. Robinson
School of Physics, University of Sydney, Sydney, NSW 2006, Australia
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Bursty Langmuir-like waves driven by electron beams in Earth's foreshock have properties which are inconsistent with the standard plasma physics paradigm of uniform exponential growth saturated by nonlinear processes. Here it is demonstrated for a specific period that stochastic growth theory (SGT) quantitatively describes these waves throughout a large fraction of the foreshock. The statistical wave properties are inconsistent with nonlinear processes or self-organized criticality being important. SGT's success in explaining the foreshock waves and type III solar bursts suggests that SGT is widely applicable to wave growth in space, astrophysical, and laboratory plasmas. [S0031-9007(99)08903-6]

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Earth's bow shock reflects and accelerates solar wind electrons and ions into a foreshock region whose properties are primarily determined by plasma streaming away from the shock (Fig. 1). The foreshock electrons naturally develop a bump-on-tail distribution function due to spatial gradients in the velocity required for electrons to reach particular locations in the foreshock [1–3]. As predicted by linear instability theory, these electrons drive Langmuir-like waves near the electron plasma frequency f_p [1–6]. Similar waves driven by electron beams are believed relevant in many contexts in space physics, astrophysics, and the laboratory, including type II and type III solar radio bursts, planetary foreshocks, the auroral regions and magnetospheres of Earth and Jupiter, and pulsar magnetospheres. The standard paradigm for wave growth in plasma physics involves an initially homogeneous plasma in which waves undergo exponential temporal growth that is saturated by nonlinear processes [7,8]. This paradigm cannot explain many properties of the foreshock Langmuir-like waves [5,6], including their burstiness, highly irregular and variable electric field strengths, typical weakness with respect to the threshold fields $\gtrsim 1\ \mathrm{mV\,m^{-1}}$ predicted for relevant nonlinear processes to saturate linear growth, and persistence much further from the bow shock than predicted by standard quasilinear theory [1,3–6,9]. Until recently resolved by stochastic growth theory (SGT) [5,10–12], similar problems were posed by the closely analogous Langmuir-like waves and driving electron beams associated with interplanetary type III solar radio bursts [11].

SGT treats situations in which a source of free energy interacts with driven waves and the ambient medium and evolves to a state in which (1) the particle distribution fluctuates stochastically about a state very close to marginal stability and (2) the wave gain G is a stochastic variable. Defining G, the standard energy growth rate Γ, and the wave and reference electric fields E and E_0 by $E^2(t) = E_0^2 e^{G(t)} = E_0^2 \exp[\int dt\,\Gamma]$, the theory describes the random walk in G using the standard wave equations

$$\frac{dE}{dt} = \frac{\Gamma E}{2}; \qquad \frac{dG}{dt} = \Gamma. \tag{1}$$

SGT is then a natural theory for bursty waves with widely variable fields (due to the random walk) that exist together with the driving distribution unexpectedly far from the source of unstable particles (due to the closeness to marginal stability). Via the central limit theorem, the most fundamental and testable prediction of SGT for relatively simple systems is that the probability distributions $P(G) \propto P(\log E)$ should be Gaussian in G (lognormal in E) [5,10–12]:

$$P(\log E) = (\sigma\sqrt{2\pi})^{-1} e^{-(\log E-\mu)^2/2\sigma^2}, \tag{2}$$

where μ and σ are the average and standard deviation of $\log E \equiv \log_{10} E$. This prediction is a robust and practical means to test whether SGT is relevant that requires only standard observations of wave fields [5,11]. Explanations for why the growth is stochastic are also required for a fully self-consistent theory.

The primary aim of this Letter is to establish that SGT provides a quantitative theoretical explanation for the growth and properties of the Langmuir-like waves in a large fraction of Earth's foreshock. Some initial support for SGT is provided by the qualitative properties of the waves [5,6] and a preliminary analysis of $P(\log E)$

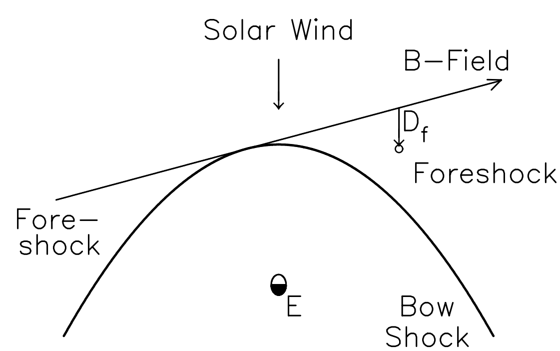

FIG. 1. Geometry of Earth's bow shock, the tangent magnetic field line, and the foreshock; D_f is the distance downstream from the tangent line along the solar wind direction.

distributions for two short time periods with limited statistics and small spatial extent in the foreshock that were selected subjectively [5]. More objective and stringent tests over a larger foreshock volume are required to establish the detailed relevance of SGT. Moreover, the foreshock environment is complicated by significant spatial variations in the waves and electrons, especially with respect to the D_f coordinate defined in Fig. 1 [1,6]: only thermal Langmuir waves are observed in the solar wind ($D_f < 0$), but there is a rapid transition to intense nonthermal waves whose electric fields E peak at relatively small positive D_f, then decrease as D_f increases. These spatial variations make the foreshock waves a challenging and stringent test of theory. One key feature of the analysis is the extraction of power-law dependencies on D_f of specific statistical wave properties (the logarithmically averaged field $E_{\mathrm{av}} = 10^{\mu}$ and σ) so as to assess SGT throughout the majority of the foreshock in one calculation. That is, the viability of a description for the microscopic behavior of the waves (SGT) is demonstrated throughout the macroscopic foreshock consistent, simultaneously, with the observed power-law trends in μ and σ.

The Letter's second aim is to demonstrate the existence of these power-law domains in D_f for E_{av} and σ and then to assess the significance of both parameters having two domains of power-law behavior with a common breakpoint in D_f. Consistent with independent theory, these domains are interpreted in terms of different processes at the shock producing the driving electrons. The Letter's third aim is to support the suggestion that SGT is potentially a new paradigm for explaining the growth of waves in space, astrophysical, and perhaps laboratory plasmas.

In addition to describing specific phenomena, SGT is important as a member of a wide class of descriptions for instabilities in inhomogeneous systems. Different wave statistics characterize these systems, presumably related to varying degrees of randomness, inhomogeneity, and self-consistency between the ambient plasma, the driving distribution, and the waves. For instance, SGT involves lognormal statistics and is associated with the unstable distribution and waves interacting self-consistently with a prescribed, independent, and inhomogeneous ambient plasma: preexisting inhomogeneities in the ambient plasma define favored sites for wave growth after which the wave-particle interactions inject fluctuations into the particle distribution which then evolve toward an SGT state [5,6,10–12]. Systems displaying self-organized criticality (SOC) [13] have power-law distributions of properties (e.g., E) and involve the medium, waves, and unstable particles all undergoing mutually self-consistent interactions. Another example is of elementary burst (EB) systems which have exponential distributions of wave properties [14]. Clarifying the regimes separating SGT, SOC, and EB systems, and explaining why systems evolve to these states is very important.

The quantitative viability of SGT for the foreshock waves is next established using the SGT prediction (2) and data from the ISEE 1 spacecraft for the period 08:25–09:55 UT on 1 December 1977. This period had unusually constant solar wind and magnetospheric conditions, allowing accurate determination of the spacecraft's position D_f (uncertainty $\pm 0.2 R_E$) and separation of temporal and spatial variations in the wave fields [5,6]. Figure 2 is a scatter plot of E versus D_f for this period, showing thermal plasma noise in the solar wind ($D_f < 0$), a region with $D_f \lesssim 0.6 R_E$ in which the fields vary between the thermal noise level and $\sim 5\ \mathrm{mV\,m^{-1}}$, and the great majority of the foreshock in which the wave fields tend to decrease slowly with increasing D_f. The significant scatter in E for a given D_f is due to intrinsic time variations, not unresolved spatial variations in D_f [6]. Only the region with $D_f \geq 0.6 R_E$ is analyzed below.

The most natural and powerful test of SGT over the macroscopic foreshock involves extracting Fig. 2's trends from the field samples $\log E$ using the variable

$$X = [\log E - \mu(D_f)]/\sigma(D_f)\,. \tag{3}$$

Then, inserting (3) into (2),

$$P(X) = (2\pi)^{-1/2} e^{-X^2/2}. \tag{4}$$

That is, without introducing any parameters other than the trends in μ and σ with D_f, SGT predicts that the *globally aggregated* distribution $P(X)$ should be Gaussian with unit standard deviation and zero mean. This extraction allows testing of SGT across macroscopic regions and avoids relying upon $P(\log E)$ distributions for subjective time periods with unusually small variations in D_f [5] or trying to disentangle the skews induced by gradients in D_f into the intrinsic $P(\log E, D_f)$ distributions formed by binning the fields in D_f. It is shown below that $E_{\mathrm{av}}(D_f) = 10^{\mu(D_f)}$ and $\sigma(D_f)$ are each described well as power laws in D_f with different indexes on either side of a common breakpoint at $D_f = D_0$. The formal

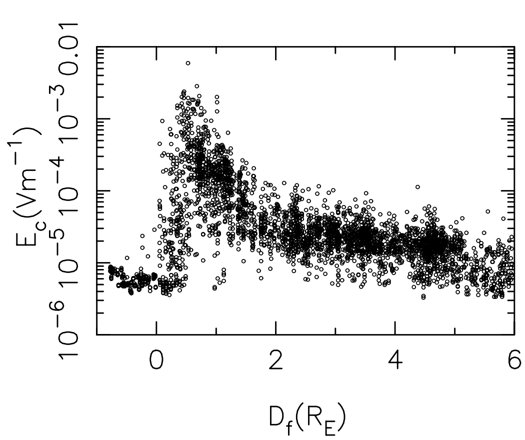

FIG. 2. Scatter plot of the wave electric fields E near f_p versus the D_f coordinates, calculated using the procedure of Ref. [6], for the period 08:25–09:55, 1 December 1977.

statistical agreement between the $P(X)$ distribution and the SGT prediction (4) depends both on the field data and the power-law trends; accordingly, it is most appropriate to simultaneously fit the trends and the $P(X)$ distribution by minimizing the deviations χ^2 between the calculated and predicted $P(X)$ distributions for varying power laws in $E_{\rm av}$ and σ. The minimizations for this seven parameter system are performed numerically using a geometric simplex method [15], yielding the degree of statistical agreement with SGT, the power-law trends (four spectral indexes and two normalizations), and the breakpoint D_0.

Figure 3 compares the $P(X)$ distribution for minimized χ^2 with the SGT prediction (solid line) for the domain $0.6 \le D_f \le 4.0R_E$. (The data have $N^{1/2}$ error bars and the bins have widths $\Delta X = 0.2$.) Excellent agreement with SGT is evident, with the observed values everywhere lying within a few error bars of the curve. Standard χ^2 and Kolmogorov-Smirnov tests [15] show that the formal statistical significance of the fit is also very high: the χ^2 significance probability (that a fit with larger χ^2 would occur by chance if the model were correct) is $P(\chi^2) = 82\%$, while the corresponding significance probability for the Kolmogorov-Smirnov test is $P(K - S) = 20\%$. It is emphasized that robust agreement with SGT is still obtained (not shown) when the D_f domain and the power-law fits for $E_{\rm av}$ and σ are varied (within the least-squares error bounds found below), although the statistical significances $P(\chi^2)$ and $P(K - S)$ can vary substantially. The excellent agreement between the SGT prediction and the $P(X)$ distributions based on data implies strongly that SGT can explain the intrinsic burstiness and highly variable amplitudes of the waves, together with their persistence far from the bow shock, in this domain of the foreshock. A plausible model for why the growth is stochastic exists already [5], based on the driving of Langmuir waves by electron beams in plasmas with preexisting density irregularities and consistent with independent data for type III bursts [11].

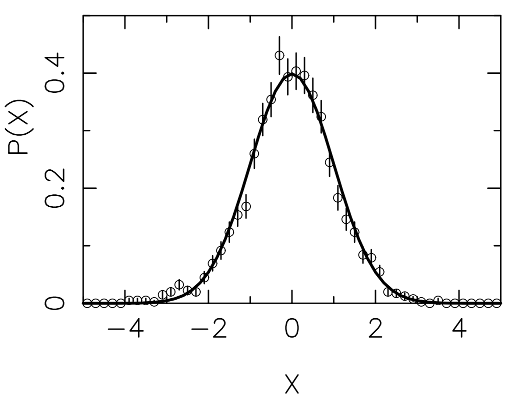

FIG. 3. Probability distributions $P(X)$ with minimized χ^2, calculated from the wave data using the procedure in the text, are plotted for the domain $0.6R_E \le D_f \le 4.0R_E$. The solid line shows the SGT prediction (4).

Importantly, the measured $P(X)$ distributions are seen to be inconsistent with constant and homogeneous linear growth/damping, SOC, or an EB system: in these cases the $P(X)$ distributions should be uniform (flat) [5,11], power-law [13], or exponential [14], respectively. Furthermore, the relevance of thermal noise effects and nonlinear processes can be directly determined since these effects modify the $P(\log E)$ and $P(X)$ distribution in known ways [12]: for instance, typically SGT effects coexist with a nonlinear process, so the pure SGT prediction (2) is relevant at fields below the nonlinear threshold E_c but nonlinear effects modify the $P(\log E)$ distribution at $E \gtrsim E_c$. Very importantly, the measured $P(X)$ distributions show no evidence for nonlinear or thermal noise effects being important in this region of the foreshock even when plotted on a log-log scale to emphasize their tails. This provides another strong argument against the standard paradigm of homogeneous linear growth followed by nonlinear saturation, as well as implying that any nonlinear Langmuir processes proceeding in this region (e.g., in connection with f_p and $2f_p$ radiation from the foreshock [1,2]) are dynamically unimportant for the statistical wave properties.

Figure 4 demonstrates that $E_{\rm av} = 10^{\mu}$ and σ are both dual power-law functions of D_f with different indexes in two domains of D_f and a common breakpoint at $D_0 = (2.1 \pm 0.2)R_E$. Shown are the power laws derived by minimizing χ^2 for the $P(X)$ distribution in Fig. 3 (solid lines), as well as separate least-squares (LSQR) fits to power laws for the domains $0.6R_E \le D_f \le 2.0R_E$ and $2.0R_E \le D_f \le 4.0R_E$ (dashed lines) that are not required to meet continuously at a common breakpoint. The indexes, constants, and statistical significances of these fits are given in Table I. The very close overlaying and the excellent statistical significance of these power laws calculated by two separate analyses with and without

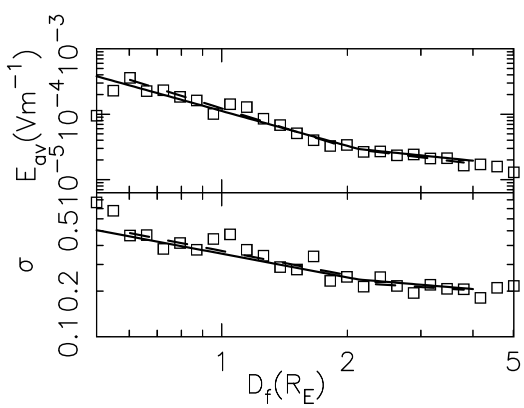

FIG. 4. Binned in $\log D_f$, the quantities $E_{\rm av}$ (upper panel) and σ (lower panel) are power-law functions of D_f, with distinct indexes and a common breakpoint in two ranges of $D_f \ge 0.6R_E$. Power-law fits obtained by minimizing χ^2 for the $P(X)$ distributions (solid lines) and by least-squares analysis (dashed) are shown.

TABLE I. Power-law trends $Y = A_1 D_f^{-\alpha_1}$ for $0.6R_E \leq D_f \leq D_0$ and $Y = A_2 D_f^{-\alpha_2}$ for $D_0 \leq D_f \leq 4.0R_E$.

Y: Means:	E_{av} LSQR	E_{av} $P(X)$	σ $P(X)$	σ LSQR
α_1	-2.0 ± 0.6	-1.8 ± 0.3	-0.52 ± 0.16	-0.53 ± 0.08
$\log A_1$	-3.9 ± 0.1	-3.9 ± 0.1	-0.45 ± 0.05	-0.43 ± 0.01
α_2	-0.79 ± 1.0	-0.65 ± 0.22	-0.23 ± 0.18	-0.17 ± 0.15
$\log A_2$	-4.3 ± 0.5	-4.3 ± 0.6 [a]	-0.54 ± 0.19 [a]	-0.59 ± 0.07
D_0	$\cdots$	2.1 ± 0.2	2.1 ± 0.2	$\cdots$
χ_1^2	0.45	19	19	12
$P(\chi_1^2)$	1.0	0.82	0.82	0.44
χ_2^2	0.12	19	19	4.8
$P(\chi_2^2)$	1.0	0.82	0.82	0.44

[a]Specified by A_1, α_1, α_2, and D_0 for the $P(\chi)$ calculation.

a common breakpoint shows that the χ^2 minimization process does not bias the $P(X)$ distribution and that the common breakpoint D_0 is real.

The existence of distinct power laws either side of a common breakpoint D_0 for both E_{av} and σ suggests that the characteristics of the electron distributions driving and damping the waves differ in these two domains. It is plausible that these domains correspond to production of the driving electrons by different processes at two regions of the shock. Importantly, the standard time-of-flight model for the foreshock electron beam [1–3] predicts that the transition near $D_f \sim 2.1R_E$ for this event corresponds to cold, weak beams with speeds of $\sim$1–2.5 times the thermal speed of solar wind electrons. These beams are unlikely to satisfy the stringent conditions required by linear theory [3,16] for effective wave growth near f_p. Thus, existing independent theory also implies that a second source of driving electrons is required for wave growth to continue deeper in the foreshock. If mirror reflection and shock-drift acceleration near the shock's magnetic tangent point produce the unstable electrons at small D_f, as is widely believed [1–3], then the driving electrons at $D_f \gtrsim D_0$ are most likely produced by acceleration or magnetosheath leakage [3] from quasiparallel regions of the shock.

In conclusion, this Letter presents very strong evidence that SGT is a viable, quantitative, and robust theory for the Langmuir-like waves throughout a large fraction of the foreshock during at least the period studied. In particular, SGT is presently unique in being able to account for the burstiness, irregular levels, typical weakness, and persistence far from the shock of the waves. The $P(X)$ distributions are inconsistent with the standard wave-growth paradigm, SOC, an EB system, or nonlinear processes being important. The different power-law trends and common breakpoint for $E_{av}(D_f)$ and $\sigma(D_f)$ in neighboring foreshock regions suggest, consistent with independent theory, that the driving electrons are produced by different processes at distinct regions of the bow shock. Finally, the demonstration that SGT is a viable theory for two separate classes of wave phenomena in space (Langmuir waves/electron beams in Earth's foreshock and type III solar bursts) suggests that SGT may well be widely applicable to wave growth in space plasmas and, by analogy, in some astrophysical and laboratory plasmas.

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*Formerly at the Department of Physics and Astronomy, University of Iowa, Iowa City, IA 52242.

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