Breaking Faraday Waves: Critical Slowing of Droplet Ejection Rates

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Parametrically excited surface waves (Faraday waves) are studied near the threshold for breaking. The breaking state ejects droplets from wave peaks when the applied forcing exceeds an acceleration threshold. The rate of breaking events approaches zero gradually with decreasing acceleration. Two properties of these ejections were studied around the ejection threshold. Analysis of the ejection rate dependence on acceleration allows a determination of the ejection threshold and an inference about the wave height distribution. A Poisson distribution is found for the times between ejections. [S0031-9007(99)08902-4]

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Many fluid free surfaces, including oceans and rivers, exhibit wave breaking accompanied by droplet ejection [1]. Droplet ejection is an important mechanism for a large number of fluid processes such as mechanical and chemical transport across fluid interfaces [2]. This paper experimentally supports the hypothesis put forward by Newell and Zakharov [3] that a continuous transition exists from unbroken surfaces to surfaces with droplets and spray. This is significant to understanding a broad class of two-phase flows as well as quantifying environmentally important gas (CO_2) and heat flux in air/sea interactions. Well-controlled experiments exhibiting droplet ejection are Faraday waves forced above a threshold acceleration [4]. Faraday waves (waves in a vertically oscillated tank) have been well studied and the ejection threshold has been characterized over a wide parametric range [5,6]. Singularities at the tips of these waves cause breakup into droplets. Other systems (such as collapse of cavitation bubbles and optical burnout in nonlinear optical media) also exhibit such local self-focusing [7]. Two properties of droplet ejection in Faraday waves are discussed here: the droplet ejection rate as a function of the applied acceleration and the interval between ejection events.

The ejection threshold was previously determined to depend on both forcing and fluid parameters of the system [4,6]. Two different asymptotic behaviors were observed: for low values of a dimensionless frequency [$\omega^* = \omega\nu^3/(\sigma/\rho)^2$, where ν is the kinematic viscosity, σ is the surface tension, and ρ is the density], the threshold depends only on forcing frequency and surface tension while at high values of ω^* the threshold depends only on forcing frequency and viscosity. This acceleration threshold was difficult to measure due to infrequent droplet production near the threshold. Using laser diagnostics near the threshold has improved our understanding of the ejection process.

Droplet ejection occurs in waves restored by gravitational forces (lower frequency gravity waves) and those restored by surface tension forces (higher frequency capillary waves). These two types of waves are characterized by the dispersion relation for infinite depth, small amplitude periodic waves [8], $\omega^2 = gk + (\sigma/\rho)k^3$, with angular wave frequency ω, wave number k, surface tension σ, density ρ, and gravitational acceleration g. A crossover wavelength and a corresponding frequency, $k_0 = \sqrt{g\rho/\sigma}$ and $\omega_0 = (4g^3\rho/\sigma)^{1/4}$, can be found where gravitational and capillary effects are equal. Below the crossover frequency ($\omega < \omega_0$), gravity is the dominant restoring force, while above the crossover frequency ($\omega > \omega_0$), surface tension effects dominate. The crossover frequency for water is $\omega_0/2\pi = 13.5$ Hz [9].

Droplet ejecting Faraday waves are produced by vertically oscillating a fluid surface with sufficient acceleration. The final wave state is determined by interacting modes of the container for a given frequency. Wave states with wavelengths comparable to the container dimensions are greatly affected by the boundary conditions of the tank. These states can have simple forms consisting of superpositions of a few modes [10,11]. We have restricted our analysis here to capillary wave states. These higher frequency wave states have smaller wavelengths, are consequently less influenced by the boundary conditions of the tank, and are turbulent superpositions of many modes. Chaotic and turbulent bifurcations leading to droplet ejecting states have been observed and analyzed [12]. Droplet formation in these waves is driven by the Rayleigh instability [13], which causes the wave tips to break under the influence of surface tension forces. Similar droplet formation has been studied in other fluid systems such as liquid jets propelled from a nozzle, fluid dripping from a faucet, and in Couette systems [14].

The droplet ejecting wave states were generated using a Unholtz-Dickie TA100-20 electrodynamic shaker with a maximum force of 1100 N. The fluid was contained in a 19.5 cm diameter plexiglass tank with a depth of

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8.5 cm [15]. The applied acceleration was measured using a calibrated Omega ACC103 piezoelectric accelerometer mounted to the armature plate of the shaker.

The droplets were detected by monitoring the light scattered by a droplet when it enters an illuminated volume above the wave surface. The intensity of the scattered light is dependent on the size, shape, and orientation of the droplet within the illuminated volume. A Motorola MRD500 photodiode was used to detect the flash from these droplets. A SpectraPhysics Argon Ion laser with an output of 1.5 W at a wavelength of $\lambda = 488$ nm served as the light source. The laser was expanded and directed horizontally across the tank 2 cm above the surface of the unexcited liquid. The tank was surrounded by a mask with two apertures: a 2.5 cm by 3 cm window which allowed the laser light to enter the tank and a 5 cm by 8 cm window which allowed the scattered light to reach the photodetector. The illuminated volume (150 cm^3) projected onto a 50 cm^2 area of the wave surface. In order to minimize the signal from any reflected light, a dark cloth was attached to the inside surface of the far side of the tank to act as a beam dump. Figure 1 shows the tank-photodiode system. The laser was oriented at a 40° angle to the photodiode. An individual droplet could clearly be detected from the scattered light. The states we discuss typically have at most only a single droplet in the illuminated volume at any given time. We determined the droplet ejection rate for distilled water [1 centistoke (cS)], glycerin-water solutions (15 cS), and silicon oil (5 cS) in a range of frequencies (35–55 Hz) in order to characterize the rate as a function of the applied acceleration. The silicon oil measurements are shown here as they lack the noise and systematic errors induced by droplets (which scatter some laser light) clinging to the side of the container. The measurements in other fluids and frequencies all shared a continuous transition with comparable exponents.

The photodiode signal was sampled at 1 kHz by an analog-to-digital acquisition board. The acceleration was initially set at a level 10%–15% less than the previously published threshold [6] for droplet ejection and then increased in small increments. The number of droplets was determined in 300 s intervals at each acceleration. The acceleration was stepped up in 0.5%–1.2% steps. The acceleration was increased until simultaneous droplets produced indistinguishable and overlapping signals. Analysis of the probability distribution of times between ejections was performed at an acceleration 20%–40% above the ejection threshold and droplets were observed for a 1000 s interval. The applied acceleration remained effectively constant during these data runs with less than 1% drift. The signal value which corresponds to a droplet was determined for each data set and typically was set at two or three acquisition units above the top of the noise floor. We interpreted each peak above this level as corresponding to a droplet.

Wave states at a forcing frequency of 45 Hz produced in silicone oil ($\nu \sim 5$ cS and $\sigma/\rho \sim 20\ \mathrm{cm^3/s^2}$) were analyzed. These waves have sufficiently small wavelengths that any effects due to gravity, boundary conditions, or meniscus interactions are negligible and the resulting wave states are dominated by capillary forces. Data for the droplet ejection rate can be seen in Fig. 2. The measured number of droplets n is scaled with the area covered by the laser ($A = 50\ \mathrm{cm^2}$), the measurement time interval ($T = 300$ s), and the wave period $1/\omega$ to get the droplet ejection rate Φ, $\Phi = n/AT\omega$. Φ is

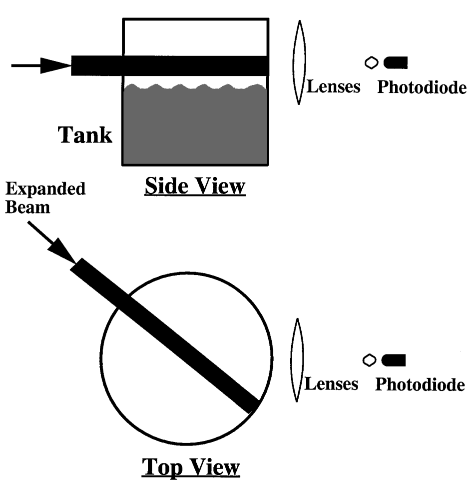

FIG. 1. A diagram of the experimental apparatus: The laser is directed at a 40° angle to the photodiode in order to maximize the signal from the ejected droplets while avoiding direct illumination and wall scatter.

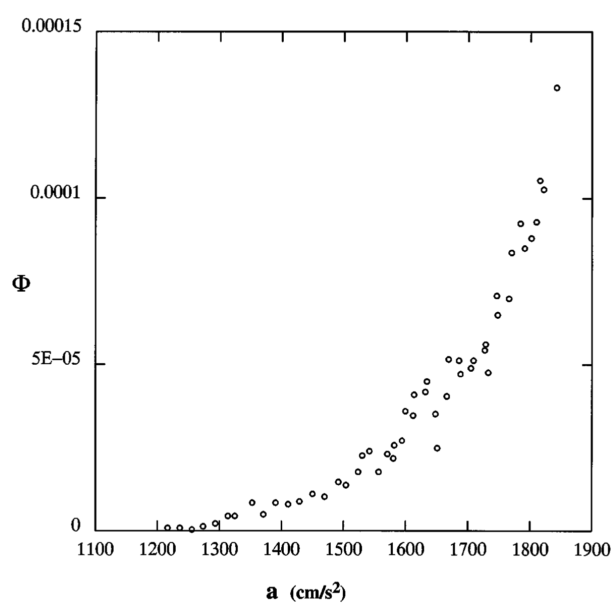

FIG. 2. Droplet rate data from $\nu = 5$ cS Silicone Oil at 45 Hz: Each point represents the number of droplets (scaled with measurement parameters) produced in a 300 s period.

independent of the particular measurement details of tank size and sampling time. There is large uncertainty in the lower acceleration data points, owing to the small number of counts at these accelerations.

The ejection rate data support several possible interpretations. One interpretation is that the rate increases as a function of acceleration scaled with the threshold value a_c, $\epsilon = (a - a_c)/a_c$. An increasing power law of ϵ works reasonably well. In order to determine the best fit power law, the characteristic acceleration a_c, exponent, and prefactor were obtained with a least square procedure. The best estimate for the threshold occurs at 1054 $\mathrm{cm/s^2}$, roughly 20% below the previously determined threshold [6]. This result is consistent with previous experimental observations of intermittent droplet ejections below the quoted threshold acceleration [6]. The experimentally derived power law for the droplet ejection rate is $\Phi = 0.039\epsilon^{2.8}$, where $\epsilon = (a - a_c)/a_c$ and $a_c = 1054\ \mathrm{cm/s^2}$. A plot of the ejection rate and scaled acceleration can be seen in Fig. 3(a). The large value of the exponent 2.8 is responsible for the extremely rare events near onset.

An alternative explanation for this dependence involves the wave height distribution. First, we hypothesize the existence of a probability distribution of the height of the free surface which scales simply with the rms waveheight:

$$\mathrm{Pr}\left(h, a, \omega, \nu, \frac{\sigma}{\rho}\right) = \mathrm{Pr}\left(\frac{h}{h_{\mathrm{rms}}}\right). \tag{1}$$

Next we assume that the rms waveheight for the capillary dominated regime depends only on the frequency and acceleration: $h_{\mathrm{rms}} = \kappa a/\omega^2$, where κ is a dimensionless prefactor. This assumption is consistent with previous models for the ejection threshold [6] and with preliminary observations. Finally the central hypothesis is that any wave whose local waveheight to wavelength ratio exceeds some fixed constant will break (i.e., waves with $h/\lambda > c$ break and eject droplets). This idea is consistent with the analytical solutions for progressive gravity and capillary waves [16,17] and past experiments on self-focusing capillary-gravity waves [4]. This formulation leads to a parameter dependence consistent with the dimensionless formulation laid out previously [6,10].

One then obtains the droplet ejection rate by integrating the tail ($h > c\lambda$) of the probability distribution of the waveheight:

$$\Phi = \frac{1}{\lambda^2} \int_{c\lambda}^{\infty} P\left(\frac{h}{h_{\mathrm{rms}}(a, \omega)}\right) dh\,. \tag{2}$$

We investigated this using a stretched exponential for the waveheight probability:

$$\mathrm{Pr}(h/h_{\mathrm{rms}}) = \frac{\gamma}{2h_{\mathrm{rms}}\Gamma(1/\gamma)}\, e^{-|h/h_{\mathrm{rms}}|^{\gamma}}. \tag{3}$$

This family of distributions has been useful for other strongly nonlinear systems [18]. This family includes both Gaussian and exponential distributions as special cases.

We find this form to reasonably describe our measurements for $\gamma = 1.1$ and $\kappa/c = 2.5$ [19]. This implies that within the framework of these assumptions that the tail of the distribution of waveheight for strongly nonlinear waves near breaking is approximately exponential. This is similar to the behavior of fluctuations in other strongly nonlinear systems such as temperature in turbulent convection or velocity derivatives in three dimensional fluid turbulence. This is contrary to what we would expect for weakly nonlinear turbulence, i.e., Gaussian statistics [20]. A comparison of the two alternative explanations (probability based and power law) can be seen in Fig. 3(b).

The form based on the probability distribution should work well above transition. At low accelerations this model is plagued by the lack of any reference to transition. Obviously at low enough accelerations the low amplitude waves become regular in their dynamics and finally the surface becomes flat. An exact determination of the form near transition is hampered by the infrequency of these very rare events and by the likelihood of finite size effects.

Dynamical information for droplet ejection was analyzed by determining the time interval between ejections. Return maps and histograms were studied to determine the correlation between individual ejecting waves. Data taken in a state excited at 45 Hz and 1836 $\mathrm{cm/s^2}$ revealed no readily apparent patterns at any of several time scales. The data analyzed here are from two 1000 s data sets. A total of 1670 droplets were detected in this sampling period, with separation intervals ranging from 21 ms to 9.9 s. All intervals less than 20 ms were ignored to avoid any spurious signals from droplet oscillation or droplets passing through the laser twice during a ballistic trajectory. Return maps of the time intervals showed no structure.

A histogram of the probability $\mathrm{Pr}(t_0)$ constructed with 32 bins is plotted in Fig. 4. This probability distribution appears to be a Poisson distribution. Only the first data

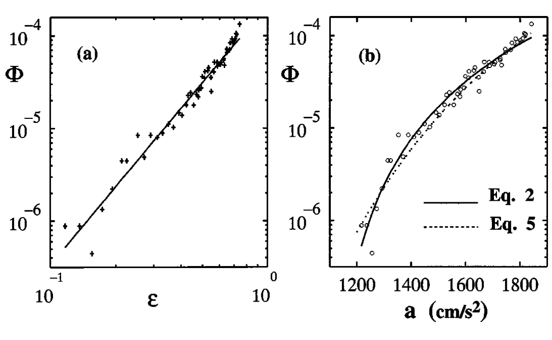

FIG. 3. (a) The nondimensional rate vs scaled acceleration illustrate a power law dependence with a best fit exponent of 2.8. This model indicates a well-defined transition. (b) A comparison of the two functional forms for the rate measurements, one probability based and one power law. Both forms collapse well to the data at high accelerations.

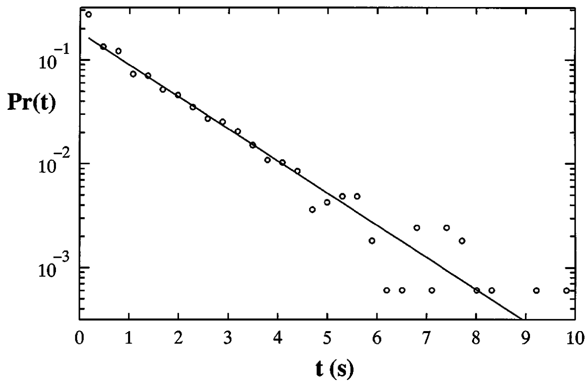

FIG. 4. A histogram of the times between ejections indicates that the distribution is a Poisson distribution and supports a hypothesis that the ejections are independent events.

point is not consistent with a Poisson distribution. We hypothesize that this is due to the unavoidable double counting of some ejections. Double counting may be caused by droplets which scatter light twice during their trajectory, a wave which ejects multiple droplets, or multiple waves ejecting simultaneously. The functional form of the interval distribution is $\mathrm{Pr}(t) = 0.18e^{-0.22t}$. This is a Poisson distribution with constants determined using linear regression. These results indicate that droplet ejection in Faraday waves is a random and uncorrelated phenomenon. Individual waves eject independently of other waves in a fashion similar to a radioactive decay. The decay time τ is related to the rate $\Phi = \langle 1/t \rangle = 1/\tau$.

Possible future experiments could explore properties of the droplets such as size and motion that may be determined from further analysis of the time series of the photodiode voltage. Another aspect that could be investigated is the energy distribution of the ejected droplets. The inferred waveheight distribution is also possibly measurable using techniques such as in the work of Wright *et al.* [21].

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