Parametric Excitation of Plasma Waves by Gravitational Radiation

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We consider the parametric excitation of a Langmuir wave and an electromagnetic wave by gravitational radiation, in a thin plasma on a Minkowski background. We calculate the coupling coefficients starting from a kinetic description. The growth rate of the instability is thus found. The Manley-Rowe relations are fulfilled only in the limit of a cold plasma. As a consequence, it is generally difficult to view the process quantum mechanically, i.e., as the decay of a graviton into a photon and a plasmon. Finally we discuss the relevance of our investigation to realistic physical situations and present an example. [S0031-9007(99)08933-4]

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The state of matter in regions where general relativistic treatments are desirable is often the plasma state. Nevertheless, plasma physics and general relativity are quite distinct areas of physics, and accordingly there are comparatively few papers using a general relativistic framework which include the electromagnetic forces in their treatments. However, there are a few exceptions to this rule (see, for example, Refs. [1–5]), where the plasma dynamics in a strong gravitational field is considered [1], relativistic transport equations for a plasma are derived [2], a general relativistic version of the Kelvin-Helmholtz theorem is derived [3], photon acceleration by gravitational radiation is considered [4], and gravitational wave effects in conducting matter are studied [5]. Omission of the electromagnetic effects for a plasma subject to gravitational forces is possible because gravity alone does not separate the charges, and in many cases the plasma can be treated as a neutral fluid, in spite of its electromagnetic properties.

In this Letter we consider a simple model problem that has two interesting properties: Firstly, the process of investigation requires a general relativistic description of the plasma dynamics to occur, and, secondly, we demonstrate the possibility of charge separation induced by the gravitational effects. We start from a monochromatic gravitational wave propagating through a thin plasma superimposed on a flat background metric, and consider the parametric excitation of a plasma wave and an electromagnetic wave. This means that we perform an expansion of the governing equations in two small parameters ϵ and η, and we only keep terms up to order $\epsilon\eta$. Here ϵ and η are proportional to the electromagnetic and gravitational wave amplitudes, respectively. The plasma wave—with an increasing amplitude due to the above mentioned mechanism—undergoes charge density oscillations. Thus the parametric excitation process—which of course has no Newtonian analog—shows that, although no *direct* charge separation can be caused by gravitational forces, *indirectly* charge density perturbations may grow due to the gravitational and electromagnetic interaction. Theoretical aspects of our problem, i.e., satisfaction of the Manley-Rowe relations and the possibility of a quantum interpretation of the interaction, and the relevance of our model problem to realistic physical situations will be discussed at the end of this Letter.

The Vlasov equation for the distribution $f = f(x^\mu, p^a)$ (where $\mu, \nu, \ldots = t, x, y, z$ and $a, b, \ldots = x, y, z$) reads [6]

$$p^t\partial_t f + p^a\partial_a f + (qg_{\mu\nu}F^{a\mu}p^\nu - p^t\mathcal{G}^a)\partial_{p^a}f = 0\,, \tag{1}$$

where $\mathcal{G}^a \equiv \Gamma^a_{\mu\nu}p^\mu p^\nu/p^t$, and this is coupled to Maxwell's equations

$$F^{\mu\nu}{}_{;\nu} = \mu_0 j^\mu \equiv \sum_{\text{p.s.}} \mu_0 q \int f(x^\nu, p^a)p^\mu |g|^{1/2}|p_t|^{-1}d^3p\,, \tag{2a}$$

$$F_{[\mu\nu,\sigma]} = 0\,, \tag{2b}$$

where p.s. stands for particle species. Here we have introduced the invariant measure $(|g|^{1/2}/|p_t|)d^3p$ on the surface in momentum space, where $g_{\mu\nu}p^\mu p^\nu = -m^2$, m being the mass of the particle in question, and we use g to denote the determinant of the metric. The gravitational field, represented by the Christoffel symbols $\Gamma^\mu_{\nu\sigma}$ in the Vlasov equation, is assumed to be generated by some outside source. The plasma itself is assumed to generate a much weaker gravitational field.

Next we assume the presence of a small amplitude gravitational wave, and we use the transverse traceless gauge. The line element then takes the form [7]

$$ds^2 = -dt^2 + (1 + h_+)\,dx^2 + (1 - h_+)\,dy^2 + 2h_\times\,dx\,dy + dz^2, \tag{3}$$

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where $h_+ = h_+(u)$, $h_\times = h_\times(u)$, with $u \equiv z - t$, and $|h_+|, |h_\times| \ll 1$. The mass of a particle with momentum p^μ is defined as $m \equiv |g_{\mu\nu} p^\mu p^\nu|^{1/2}$. From this, p^t may be expressed as $p^t = \{m^2 + \delta_{ab} p^a p^b + h_+[(p^x)^2 - (p^y)^2] + 2h_\times p^x p^y\}^{1/2}$. Computing the Christoffel symbols of the gravitational wave (3), the Vlasov equation (1) for the unperturbed ($F^{\mu\nu} = 0$) distribution $f = f(t, x^a, p^b)$ becomes

$$\frac{\partial f}{\partial t} + \frac{p^a}{p^t} \frac{\partial f}{\partial x^a} - \mathcal{G}^a \frac{\partial f}{\partial p^a} = 0, \tag{4}$$

where $\mathcal{G}^x \equiv (-1 + p^z/p^t)(\dot{h}_+ p^x - \dot{h}_\times p^y)$, $\mathcal{G}^y \equiv (-1 + p^z/p^t)(\dot{h}_\times p^x - \dot{h}_+ p^y)$, $\mathcal{G}^z \equiv \{[(p^y)^2 - (p^x)^2]\dot{h}_+ - 2p^x p^y \dot{h}_\times\}/(2p^t)$, and $\dot{h} \equiv dh/du$. Expanding p^t according to $p^t \approx p^t_{(0)} - \mathcal{F}$, where $\mathcal{F} \equiv \{h_+[(p^y)^2 - (p^x)^2] - 2h_\times p^x p^y\}/(2p^t_{(0)})$, and $p^t_{(0)} = (m^2 + \delta_{ab} p^a p^b)^{1/2}$, the Vlasov equation becomes

$$\frac{\partial f}{\partial t} + \left(1 + \frac{\mathcal{F}}{p^t_{(0)}}\right) \frac{p^a}{p^t_{(0)}} \frac{\partial f}{\partial x^a} - \mathcal{G}^a \frac{\partial f}{\partial p^a} = 0. \tag{5}$$

For simplicity we let the gravitational wave be monochromatic, i.e., $h = \tilde{h} \exp[i(k_0 z - \omega_0 t)] + \text{c.c.}$, with the dispersion relation $k_0 = \omega_0$. We divide f according to $f = f_{\text{SJ}}(p^a) + f_{\text{g}}(t, z, p^a)$, where the equilibrium distribution f_{SJ} in the absence of a gravitational wave is taken to be the Synge-Jüttner distribution [8], and $f_{\text{g}} = \widetilde{f_{\text{g}}} \exp[i(k_0 z - \omega_0 t)]$ is the perturbation induced by the gravitational wave—which is assumed to fulfill $f_{\text{g}} \ll f_{\text{SJ}}$. From Eq. (5) we find

$$\widetilde{f_{\text{g}}} = -\frac{\mathcal{G}^a (\partial f_{\text{SJ}}/\partial p^a)}{i(\omega_0 - k_0 p^z/p^t_{(0)})} \tag{6}$$

to first order in the amplitude.

Next we assume the presence of electromagnetic perturbations with frequency and wave number $(\omega_1, \mathbf{k}_1)$ as well as electrostatic perturbations $(\omega_2, \mathbf{k}_2)$. The frequencies and wave numbers are supposed to satisfy the resonance conditions $\omega_0 = \omega_1 + \omega_2$ and $\mathbf{k}_0 = \mathbf{k}_1 + \mathbf{k}_2$. The vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ span a plane, and we choose the y axis to be perpendicular to this plane, i.e., $\mathbf{k}_1 = k_1^x \hat{\mathbf{x}} + k_1^z \hat{\mathbf{z}}$ and $\mathbf{k}_2 = k_2^x \hat{\mathbf{x}} + k_2^z \hat{\mathbf{z}}$. Including the electromagnetic field, expanding Eq. (5) to first order in h, and writing all terms proportional to h on the right-hand side, Eq. (5) becomes

$$\begin{aligned}
&\frac{\partial f_{\text{ng}}}{\partial t} + \frac{p^a}{p^t_{(0)}} \frac{\partial f_{\text{ng}}}{\partial x^a} + q\left(-F^{at} + \frac{1}{p^t_{(0)}} \delta_{bc} F^{ac} p^b\right) \frac{\partial f_{\text{ng}}}{\partial p^a} \\
&\quad = -\frac{\mathcal{F}}{(p^t_{(0)})^2} p^a \frac{\partial f_{\text{ng}}}{\partial x^a} + \mathcal{G}^a \frac{\partial f_{\text{ng}}}{\partial p^a} - q\left(-F^{at} + \frac{1}{p^t_{(0)}} \delta_{bc} F^{ac} p^b\right) \frac{\partial f_{\text{g}}}{\partial p^a} \\
&\quad\quad - \left\{\frac{q}{p^t_{(0)}} [-h_+(F^{ax} p^x - F^{ay} p^y) + h_\times (F^{ax} p^y + F^{ay} p^x)]\right\} \frac{\partial f_{\text{SJ}}}{\partial p^a},
\end{aligned} \tag{7}$$

where we have introduced f_{ng} defined by $f_{\text{ng}} = f - f_{\text{g}}$. Furthermore, we need two components of Faraday's law. Expanding Eq. (2b) to first order in the amplitude h, we obtain

$$\begin{aligned}
\partial_t F^{xy} - \partial_y F^{tx} + \partial_x F^{ty} &= h_\times(-\partial_x F^{tx} + \partial_y F^{ty}) \\
&\quad + h_+(\partial_y F^{tx} + \partial_x F^{ty}),
\end{aligned} \tag{8a}$$

$$\begin{aligned}
&\partial_t F^{yz} - \partial_z F^{ty} + \partial_y F^{tz} \\
&\quad = h_\times(\partial_z F^{tx} - \partial_t F^{xz}) + h_+(\partial_z F^{ty} + \partial_t F^{yz}) \\
&\quad\quad + \dot{h}_\times(F^{tx} + F^{xz}) - \dot{h}_+(F^{ty} + F^{yz}).
\end{aligned} \tag{8b}$$

In what follows, we will put $h_+ = 0$; i.e., we choose a specific polarization of the gravitational wave. It turns out that such a gravitational wave does not couple to a linearly polarized electromagnetic wave with magnetic field in the y direction. Thus we assume the electromagnetic wave to have the opposite polarization, i.e., F^{yt}, F^{xz}, and F^{yz} are the only electromagnetic components to be different from zero in the linear approximation. Similarly, F^{zt} and F^{xt} are the only nonzero electrostatic components of the field tensor. We then divide f_{ng} according to $f_{\text{ng}} = f_{\text{SJ}}(p^a) + \widetilde{f_{\text{em}}}(t, p^a) \exp[i(k_1^x x + k_1^z z - \omega_1 t)] + \widetilde{f_{\text{es}}}(t, p^a) \exp[i(k_2^x x + k_2^z z - \omega_2 t)]$, where the time dependence of the amplitudes, which is due to the parametric interaction with the gravitational wave, is assumed to be slow, such that $\partial_t \ll \omega$. In the linear approximation (i.e., no gravitational coupling) the slow time dependence vanishes, and Eq. (7) gives

$$\widetilde{f^{\text{l}}_{\text{em}}} = -\frac{q\widetilde{F}^{yt}}{i\hat{\omega}_1} \frac{\partial f_{\text{SJ}}}{\partial p^y}, \tag{9a}$$

$$\widetilde{f^{\text{l}}_{\text{es}}} = -\frac{q\widetilde{F}^{at}}{i\hat{\omega}_2} \frac{\partial f_{\text{SJ}}}{\partial p^a} \tag{9b}$$

for the electromagnetic and electrostatic perturbations respectively. Here we have $\hat{\omega}_i \equiv \omega_i - \mathbf{k}_i \cdot \mathbf{p}/p^t_{(0)}$, where the scalar product is Euclidean. Considering the part of Eq. (7) varying as $\exp[i(k_2^x x + k_2^z z - \omega_2 t)]$, using linear approximations for the factors on the right-hand side, we obtain $\widetilde{f_{\text{es}}} = \widetilde{f^{\text{l}}_{\text{es}}} + \widetilde{f^{\text{nl}}_{\text{es}}}$. The linear contribution is given by (9b), by replacing ω_2 by $\omega_{2\text{op}} \equiv \omega_2 + i\partial_t$ (since $\partial_t \ll \omega_2$, division by $\omega_{2\text{op}}$ can be calculated by a first order Taylor expansion) and the nonlinear contribution is found to be $\widetilde{f^{\rm nl}_{\rm es}} = C_{\rm es}\tilde{h}_\times \tilde{F}^{yt*}$, where

$$C_{\rm es} = \frac{q}{i\hat{\omega}_2}\left\{\frac{p^x p^y \mathbf{k}_1\cdot\mathbf{p}}{(p^t_{(0)})^3\hat{\omega}_1}\frac{\partial f_{\rm SJ}}{\partial p^y} + \omega_0 \mathcal{G}_{\rm op}\left(\frac{1}{\hat{\omega}_1}\frac{\partial f_{\rm SJ}}{\partial p^y}\right) + \left[\left(1-\frac{1}{p^t_{(0)}}\left(\frac{\mathbf{p}\cdot\mathbf{k}_1}{\omega_1}\right)\right)\frac{\partial}{\partial p^y} + \frac{p^y}{\omega_1 p^t_{(0)}}\left(k_1^x\frac{\partial}{\partial p^x} + k_1^z\frac{\partial}{\partial p^z}\right)\right]\left[\frac{\omega_0}{\hat{\omega}_0}(\mathcal{G}_{\rm op}f_{\rm SJ})\right] + \frac{k_1^z p^x}{p^t_{(0)}\omega_1}\frac{\partial f_{\rm SJ}}{\partial p^z}\right\}, \tag{10}$$

and $\mathcal{G}_{\rm op} \equiv (\mathcal{G}^a/i\omega_0 h_\times)\partial_{p^a} = (1 - p^z/p^t_{(0)})(p^y\partial_{p^x} + p^x\partial_{p^y}) + (p^x p^y/p^t_{(0)})\partial_{p^z}$. By combining Eqs. (10) and (2a) we then find

$$\varepsilon_L(\omega_{2\rm op}, k_2)\tilde{F}_{\rm es} = C_1\tilde{h}_\times \tilde{F}^{yt*}, \tag{11}$$

where the longitudinal dielectric permittivity ε_L is given by [9]

$$\varepsilon_L = 1 + \frac{\mu_0 q}{k_2}\sum_{\rm p.s.}\int \frac{1}{\hat{\omega}_{2\rm op}}\frac{\partial f_{\rm SJ}}{\partial p}\, d^3p\,, \tag{12}$$

the coupling coefficient C_1 is found to be

$$C_1 = \frac{\mu_0 q}{ik_2}\sum_{\rm p.s.}\int C_{\rm es}d^3p\,, \tag{13}$$

$p = [(p^x)^2 + (p^y)^2 + (p^z)^2]^{1/2}$, $k_2 = [(k_2^x)^2 + (k_2^z)^2]^{1/2}$, and $\tilde{F}_{\rm es} = [(\tilde{F}^{xt})^2 + (\tilde{F}^{zt})^2]^{1/2}$. In deriving Eqs. (11) and (12) we have used the electrostatic mode as longitudinal, i.e., $\tilde{F}^{xt}/\tilde{F}^{zt} = k_2^x/k_2^z$.

Next we turn to the part of (7) varying as $\exp[i(k_1^x x + k_1^z z - \omega_1 t)]$. Again, using linear approximations for the right-hand side, we obtain $\widetilde{f_{\rm em}} = \widetilde{f^{\rm l}_{\rm em}} + \widetilde{f^{\rm nl}_{\rm em}}$, where $\widetilde{f^{\rm l}_{\rm em}}$ is given by (9a) (naturally by replacing ω_1 with $\omega_{1\rm op}$), and $\widetilde{f^{\rm nl}_{\rm em}}$ is found to be $\widetilde{f^{\rm nl}_{\rm em}} = C_{\rm em}\tilde{h}_\times \tilde{F}^*_{\rm es}$, where

$$C_{\rm em} = \frac{q}{i\hat{\omega}_1}\left\{\omega_0\mathcal{G}_{\rm op}\left(\frac{\hat{k}_2^x}{\hat{\omega}_2}\frac{\partial f_{\rm SJ}}{\partial p^x} + \frac{\hat{k}_2^z}{\hat{\omega}_2}\frac{\partial f_{\rm SJ}}{\partial p^z}\right) - \left(\hat{k}_2^x\frac{\partial}{\partial p^x} + \hat{k}_2^z\frac{\partial}{\partial p^z}\right)\left[\frac{\omega_0}{\hat{\omega}_0}\mathcal{G}_{\rm op}f_{\rm SJ}\right] - \frac{p^x p^y \mathbf{k}_2\cdot\mathbf{p}}{(p^t_{(0)})^3}\left(\frac{\hat{k}_2^x}{\hat{\omega}_2}\frac{\partial f_{\rm SJ}}{\partial p^x} + \frac{\hat{k}_2^z}{\hat{\omega}_2}\frac{\partial f_{\rm SJ}}{\partial p^z}\right)\right\}. \tag{14}$$

Eliminating linear terms proportional to F^{yz} and F^{xy} with the help of Eqs. (8b) and (8a), Eq. (2a), together with (14), gives

$$D(\omega_{1\rm op}, |\mathbf{k}_1|)\tilde{F}^{yt} = C_2\tilde{h}_\times \tilde{F}^*_{\rm es}\,, \tag{15}$$

where the dispersion function $D(\omega_{1\rm op}, |\mathbf{k}_1|)$ for the electromagnetic wave is given by [9]

$$D(\omega_1, |\mathbf{k}_1|) = 1 - \frac{\mathbf{k}_1^2}{\omega_1^2} + \frac{\mu_0 q^2}{\omega_1}\sum_{\rm p.s.}\int \frac{p^y}{\hat{\omega}_1}\frac{\partial f_{\rm SJ}}{\partial p^y}\, d^3p\,, \tag{16}$$

and the coupling coefficient C_2 is found to be

$$C_2 = \frac{i\mu_0 q}{\omega_1}\int\left[\frac{p^y}{p^t_{(0)}}\left(C_{\rm em} + \frac{qp^x p^y}{i(p^t_{(0)})^2}\left(\frac{k_2^x}{k_2\hat{\omega}_2}\frac{\partial f_{\rm SJ}}{\partial p^x} + \frac{k_2^z}{k_2\hat{\omega}_2}\frac{\partial f_{\rm SJ}}{\partial p^z}\right)\right)\right]d^3p + \frac{k_{2x}k_1^2}{k_2\omega_1^2}\,. \tag{17}$$

The operators $\varepsilon_L(\omega_{2\rm op}, k_2)$ and $D(\omega_{1\rm op}, |\mathbf{k}_1|)$ can be expanded according to $\varepsilon_L(\omega_{2\rm op}, k_2) = \varepsilon_L(\omega_2, k_2) + (\partial_{\omega_2}\varepsilon_L)i\partial_t$ and $D(\omega_{1\rm op}, |\mathbf{k}_1|) = D(\omega_1, |\mathbf{k}_1|) + (\partial_{\omega_1}D)i\partial_t$. Assuming that the dispersions relations for the Langmuir wave and the electromagnetic wave are exactly fulfilled, we let $\varepsilon_L(\omega_2, k_2) = D(\omega_1, |\mathbf{k}_1) = 0$. Equations (11) and (15) are then written as

$$\frac{\partial\tilde{F}_{\rm es}}{\partial t} = C_1\frac{\tilde{h}_\times \tilde{F}^{yt*}}{(\partial\varepsilon_L/\partial\omega_2)}\,, \tag{18a}$$

$$\frac{\partial\tilde{F}^{yt}}{\partial t} = C_2\frac{\tilde{h}_\times \tilde{F}^*_{\rm es}}{(\partial D/\partial\omega_1)}\,. \tag{18b}$$

As can be seen, generally the expressions (13) and (17)for the coupling coefficients are very complicated. In order to get transparent formulas, we present the result for a cold electron plasma with immobile ions constituting a neutralizing background. Letting $f_{\rm SJ}\to\delta^3(\mathbf{p})$, taking the cold limit of the dispersion relations (12) and (16), and using the resonance conditions, the coupling coefficients become $C_1 = C_2 \equiv C = -ik_{2x}\omega_0/k_2\omega_1$. Applying these expressions, combining (18a) and (18b), and noting that $\tilde{h}_\times$ is a constant, the growth rate γ for the Langmuir wave and the electromagnetic wave is given by

$$\gamma^2 = \left(\frac{k_{2x}\omega_0}{k_2\omega_1}\right)^2\frac{\omega_1\omega_{\rm p}}{4}|\tilde{h}_\times|^2. \tag{19}$$

where $\omega_{\rm p} = (n_0\mu_0 e^2/m)^{1/2}$ is the plasma frequency, and n_0 is the unperturbed number density of electrons. Since we have not included dissipation of the wave modes, the threshold value for the instability is zero. However, it is straightforward [10] to calculate the threshold by including appropriate damping mechanisms for the decay products.

The fact that the same coupling coefficient C appears in Eqs. (18a) and (18b) for a cold plasma means that the Manley-Rowe relations [10] are fulfilled for that case. These relations usually follow from an underlying Hamiltonian structure of the governing equations, and ensures that each of the decay products takes energy from the pump wave in direct proportion to their respective frequencies. Thus fulfillment of the Manley-Rowe relation means that the parametric process can be interpreted quantum mechanically; i.e., we can think of the interaction as the decay of a graviton with energy $\hbar\omega_0$ into a photon with energy $\hbar\omega_1$ and a plasmon with energy $\hbar\omega_2$. However, we stress that this interpretation is not always possible [11]. When thermal effects are taken into account, the coefficient C_1 in (18a) is generally different from the coefficient C_2 in (18b). Thus we obtain the rather surprising result that the simple graviton interpretation (i.e., that the gravitational wave is built up of quanta with energy $\hbar\omega_0$) of the gravitational field is not always applicable.

There are two main conclusions which can be drawn from our calculations: First, wave-wave interactions can lead to energy transfer from gravitational to electromagnetic degrees of freedom, and vice versa. Second, charge separation and corresponding electrostatic fields may occur as a result of such interactions. However, as a starting point, we made quite strong idealizations, and thereby considered somewhat of a model problem. Still we think that our process can occur in a realistic physical situation, as will be demonstrated by the following example: Consider a binary system of two equal masses $m = 3M_\odot$ separated by a distance of six Schwarzschild radii $r = 12Gm/c^2$. The frequency of the emitted gravitational waves is of the order $\omega_0 \gtrsim 10^4$ rad/s. It is difficult to obtain a precise value of the amplitude $\tilde{h}_\times$, but an estimate can be obtained by considering two point masses separated by a fixed distance. Then $\tilde{h}_\times \sim Gmr^2\omega_0^2/(2c^4R)$, where R is the distance from the system. At a distance of $1/60$ a.u. (i.e., roughly 10 earth-moon distances), this implies $\tilde{h}_\times \sim 10^{-6}$. Choosing the electron number density as $n_0 = 5 \times 10^3$ m $^{-3}$ and the temperature as $T =$ 10 eV, the growth rate given by (19) is $\gamma \sim 10^{-2}$ s $^{-1}$. (The plasma parameters chosen correspond to low-density undisturbed interstellar matter (ISM). If one assumes that the black hole pair moves through the ISM with high speed (a few hundred km/s), the plasma density at the distance $1/60$ a.u. can be unaffected by a Bondi-type inflow [12].) Comparing the growth rate with the decay rate due to collisional or Landau damping [10], we note that the gravitational amplitude is orders of magnitude above threshold, in spite of the comparatively large distance from the source that was chosen.

Other examples of events that would produce considerable amounts of gravitational radiation are supernovas and black hole and neutron star formations. However, in these processes the expected gravitational radiation is very incoherent and broadband (although there are calculations predicting highly monochromatic emissions of gravitational waves for certain types of neutron star formation [13]). Some estimates of the effects due to scattering of gravitational waves in supernovas have been presented by Bingham *et al.* [14]. See the review by Thorne [15] for characteristic values of the amplitudes, frequencies, and duration times for the processes discussed above.

The parametric process considered in this Letter can be thought of as a gravitational analog of Raman scattering, where the electromagnetic pump wave has been replaced by a gravitational wave. Similarly, we can imagine that a gravitational analog of Compton scattering may occur. Such a process requires a kinetic treatment, and could be described within our formalism without very much extra difficulty. Gravitational waves may also be subject to four-wave processes such as modulational instabilities.

Finally, we note that, if we increase the plasma density, a more effective transfer of gravitational energy—as compared to our example—can be obtained. This is consistent with the resonance conditions, if we instead consider excitation of MHD waves, since the eigenfrequency of such waves decreases with density.

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