Chaos Assisted Tunneling from Superdeformed States

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The tunneling process that governs the decay from superdeformed to normal-deformed nuclear states is shown to be enhanced by several orders of magnitude (10^4-10^6) if the normal-deformed states are chaotic at the moment of decay out. The onset of (long-range) chaos may imply that the tunneling enhancement increases with decreasing angular momentum. Experimental signatures are discussed. [S0031-9007(98)08143-5]

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Superdeformed (SD) states in nuclei may be produced at high angular momenta where they are energetically favored compared to normal-deformed (ND) states [1]. The γ decay along the SD rotational band proceeds from supercollective, stretched $E2$ transitions to considerably lower angular momenta than the point where the ND states are energetically favored. As the angular momentum decreases the SD band thus gets more excited relative to the ND states, and is embedded in a surrounding of ND states with an increasing level density. It suddenly decays to lower-lying ND states by tunneling through the barrier separating the two types of nuclear shapes. This decay occurs at a relative excitation energy of 3–5 MeV in a very large number of decay channels, making the actual decay difficult to observe, and only in very few cases have the γ rays connecting the SD and ND states indeed been measured [2–4]. States in medium-heavy nuclei around the neutron and proton resonance regions, i.e., about 6–7 MeV of excitation energy, are chaotic [5] while (rotational) states at low excitation energies are regular [6]. Consequently, as the decay along the SD band proceeds, it first passes a region of regular ND states, and then ND states with an increasing degree of chaoticity. In this Letter we shall show that the onset of chaos in the ND states implies a strong enhancement of the tunneling process from the SD states. This strong enhancement may be considered as an example of *chaos assisted tunneling* [7].

Several theoretical studies of the decay out from SD states have been performed (see, e.g., Refs. [8–11]). If changes of the barrier with angular momentum as well as changes of the number of possible decay channels are properly taken into account, the decay out occurs too smoothly compared to experimental data [8]; an additional tunneling enhancement of a factor of 2–10 is needed at angular momentum $I - 2$ compared to I (see Ref. [11], and references therein). The measured sudden decay out may be explained by a gradual increase of the tunneling probability with decreasing angular momenta that has been suggested to occur due to the onset of pairing [10]. We shall show that the onset of chaos in the ND states may also imply an enhancement of the tunnel probability, and thus provide an alternative explanation for the sudden decay out.

By considering quadrupole couplings between the different configurations along the path between the SD and ND minima, e.g., within the generator coordinate method, collective transitions between ND and SD states may be described (see, e.g., Ref. [12]). These couplings mainly take place between the SD state and the configurations along the barrier between the SD and ND minima. We shall consider the ND states connected with the SD state in this collective way as *doorway* states for the decay from the SD to the ND minimum. The energy difference between such doorway states in the $A = 150-190$ region is of the order of 1 MeV (cf. [12]) while the energy distance between neighboring ND states is 1–100 eV at the excitation energy where the decay out occurs. These states typically have mean-field configurations very different from the doorway states. Through an increasing mixing between the ND states (due to the total residual two-body interaction) with increasing excitation energy, the strength function of the doorway states is increasingly smeared out on surrounding ND states, leading to a very strong enhancement of the tunneling process.

We may estimate the chaos assisted tunneling enhancement in perturbation theory. In the limit of no mixing between the ND states, i.e., a completely regular ND system, the wave function of the SD state mixes with the doorway states only. Assuming that the tunneling coupling between the SD state and the doorway state, $|d\rangle$, is V_t and the energy distance between the doorway states is $2\Delta E$, the mixed SD state becomes $|\mathrm{SD}'\rangle \approx |\mathrm{SD}\rangle + \frac{V_t}{\Delta E}|d\rangle$, where we have assumed the SD state to be situated approximately halfway between the doorway states. The tunneling probability is then given by [13]

$$T^{\mathrm{regular}} \approx |\langle d|\mathrm{SD}'\rangle|^2 \approx \left(\frac{V_t}{\Delta E}\right)^2.$$

In the other extreme situation when the mixing between ND states is large, leading to quantum chaos, the tunneling strength is spread out over all ND states, $\{|\mu\rangle\}$, and is of the order $V_t' \approx \pm\frac{V_t}{\sqrt{N}}$, where N is the total number of states in the energy interval ΔE. Since the average level distance between two ND states is $\bar{d} = \rho^{-1} = \Delta E/N$, the SD state becomes, $|\mathrm{SD}'\rangle \approx |\mathrm{SD}\rangle + \frac{V_t}{\bar{d}}|\mu\rangle \approx |\mathrm{SD}\rangle + \frac{V_t\sqrt{N}}{\Delta E}|\mu\rangle$ [14], implying that the tunneling probability in the chaotic case is

$$T^{\text{chaotic}} \approx |\langle\mu|\mathrm{SD}'\rangle|^2 \approx \left(\frac{V_t}{\Delta E}\right)^2 N\,.$$

Consequently,

$$\frac{T^{\text{chaotic}}}{T^{\text{regular}}} \approx N \approx \rho_{\mathrm{ND}}(U)/\rho_{\text{doorway}}\,, \qquad (1)$$

where $\rho_{\text{doorway}} \approx 1\ \mathrm{MeV}^{-1}$ is the density of doorway states. Since the relative excitation energy between the SD and ND yrast state is $U = 3-5$ MeV when the decay out of the SD band occurs, we expect the tunneling probability to be enhanced by $\rho_{\mathrm{ND}}(U)/\rho_{\text{doorway}} \approx 10^4-10^6$ times if the ND states are chaotic.

The mechanism for the enhancement of tunneling probability in the chaotic case is similar to that causing the chaotic enhancement of parity violation around the neutron resonance region [15], where the dynamical enhancement is of the order $\rho^{1/2}$.

The gradual enhancement of the tunneling process from the SD state to the ND states may be modeled in a simple random matrix model. We write the total Hamiltonian as

$$\hat{H} = \hat{H}_{\mathrm{ND}}(\Delta) + \hat{H}_{\mathrm{SD}} + \hat{V}_{\text{coupl}}\,,$$

where the different terms describe the ND states, the SD state, and the coupling between the SD state and the ND states (tunneling), respectively. The complexity of the ND states is controlled by the parameter Δ, and the Hamiltonian may be written as [16]

$$\hat{H}_{\mathrm{ND}}(\Delta) = \sum_{n=1}^{N} \varepsilon_n \mathbf{c}_n^\dagger \mathbf{c}_n + \Delta \cdot \sum_{n>k}^{N} V_{nk}(\mathbf{c}_n^\dagger \mathbf{c}_k + \mathbf{c}_k^\dagger \mathbf{c}_n)\,,$$

where the operators, $\mathbf{c}_n$, refer to the basis states, $|n\rangle$, which, in principle, are many-particle–many-hole excitations based on the yeast normal-deformed state. In this simple model we shall, however, neglect the intrinsic structure of the states and assume a residual many-body interaction. All matrix elements are taken as Gaussian distributed random numbers with zero mean, and with standard deviations $\sqrt{2/N}$ and $\sqrt{1/N}$ for the diagonal elements, ε_n, and nondiagonal elements, V_{nk}, respectively. By the "chaoticity parameter," Δ, the structure of the ND states can be smoothly changed from regular, with $\Delta = 0$, to chaos, with $\Delta = 1$ (that is identical to the full GOE [17]), thus simulating the effect of an increasing density of ND states. The Hamiltonian describing the SD state is trivially taken as $\hat{H}_{\mathrm{SD}} = \varepsilon_{\mathrm{SD}} \mathbf{c}_{\mathrm{SD}}^\dagger \mathbf{c}_{\mathrm{SD}}$, where only one SD state (with energy $\varepsilon_{\mathrm{SD}}$) is considered. Finally, the coupling due to tunneling is described by

$$\hat{V}_{\text{coupl}} = \sum_{n=1}^{N} V_t \delta_{n,d}(\mathbf{c}_{\mathrm{SD}}^\dagger \mathbf{c}_n + \mathbf{c}_n^\dagger \mathbf{c}_{\mathrm{SD}})\,,$$

where the tunneling is assumed to occur with the strength, V_t, between the SD state and one ND doorway state, $|d\rangle$. The doorway state is thus one of the unmixed ND states described by $\hat{H}_{\mathrm{ND}}$. We generally assume V_t to be small ($\ll \bar{d}$), and put the SD state at the center of the ND states, and the doorway state as state number $N/4$.

The numerical calculations are performed with $N = 400$ and ensemble averages are studied within 50 realizations. In the first step, $\hat{H}_{\mathrm{ND}}$ is solved for a given value of the parameter Δ, giving $\hat{H}_{\mathrm{ND}}(\Delta)\,|\mu\rangle = e_\mu|\mu\rangle$, where the eigenfunctions are expressed in unperturbed states as

$$|\mu\rangle = \sum_{n=1}^{N} a_{\mu n}|n\rangle\,. \qquad (2)$$

Through this mixing, the strength of the doorway state is spread out over several surrounding ND states, and the tunneling matrix element between the SD state and an arbitrary ND state, $|\mu\rangle$, becomes $V_{\mathrm{SD},\mu} = \langle \mathrm{SD}|\hat{V}_{\text{coupl}}|\mu\rangle = a_{\mu d}V_t$. The doorway spreading width increases with increasing Δ, and $V_{\mathrm{SD},\mu}$ finally reaches a constant value of approximately $\pm V_t \frac{1}{\sqrt{N}}$ at $\Delta = 1$, corresponding to the GOE limit.

In the next step of the calculation the energy of ND states is renormalized to have a constant average level density equal to one. Finally, the full matrix, including all of the ND states and the SD state, is diagonalized. The wave function of the SD state can then be expressed as

$$|\mathrm{SD}'\rangle = a'_{\mathrm{SD}}|\mathrm{SD}\rangle + \sum_{\mu=1}^{N} a'_\mu|\mu\rangle\,, \qquad (3)$$

where the ND eigenfunctions, $|\mu\rangle$, are given by Eq. (2). The probability of tunneling from the SD state to the ND states is calculated as $T_{\mathrm{SD}\to\mathrm{ND}} = \sum(a'_\mu)^2$, and is shown versus Δ in Fig. 1. It is seen how the tunneling probability drastically increases by about 400 times ($= N$)

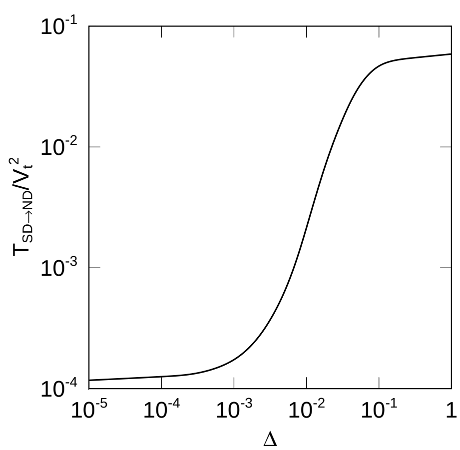

FIG. 1. Tunneling probability, $T_{\mathrm{SD}\to\mathrm{ND}}$, from SD to ND states (ensemble median value) vs the chaoticity parameter Δ, utilizing the simple model with dimension $N = 400$ (log-log scale). Notice the drastic increase in tunneling probability as the system changes from regular (small Δ values) to chaotic (large Δ values).

as Δ changes from about 0.001 to about 0.1, i.e., when the properties of the ND states change from regular to more or less chaotic. This behavior is qualitatively understood from the above discussion based on perturbation theory [see Eq. (1)].

Notice, however, that chaos, in terms of GOE behavior of the nearest-neighbor energy spacings (NND), sets in already when $\Delta \approx 0.01$ in the simple model (see Ref. [16]). Long-range energy correlations, as, e.g., measured by Δ_3 statistics [18], on the other hand, show GOE behavior only in the limited energy interval up to $L_{\max}/\rho \approx 2.5\Gamma_\mu$, where ρ is the level density and $\Gamma_\mu = 2\pi\rho\Delta^2$ is the spreading width of the wave function on unperturbed states [16]. The enhancement of the tunneling probability seen in Fig. 1 is thus connected to the onset of chaos in terms of *long-range spectrum correlations* that, in principle, could be measured by the study of Δ_3 statistics of the ND states at the decay-out energies. Practically, this is, however, almost impossible since it involves accurate knowledge of stretches of several thousands of energy states with known parity and angular momentum. A more feasible measure of the process is the fragmentation of γ decay that will be discussed below.

The fragmentation of γ decay from the SD state to the ND states can also be described within the simple model. Assume that each unmixed ND state, $|n\rangle$, γ decays to *one* specific lower-lying daughter state, $|\bar{n}\rangle$, with a matrix element, M_o, independent of n, i.e., $\langle\bar{n}|\mathcal{M}|m\rangle = M_o\delta_{n,m}$. Mixing of ND states thus creates fragmentation of the γ decay. The matrix element between the SD state and the ND daughter state $|\bar{n}\rangle$ becomes

$$M_n = \langle\bar{n}|\mathcal{M}|\mathrm{SD}'\rangle = M_o \sum_\mu a_{\mu n} a'_\mu\,,$$

where Eqs. (2) and (3) have been used. In Fig. 2 the distribution of matrix elements, $|M_n/M_o|^2$, is shown for different Δ values. In the GOE limit ($\Delta = 1$), the Porter-Thomas (PT) distribution [19] is obtained, but large deviations from this distribution are seen for smaller Δ values. For example, at $\Delta = 0.01$ that corresponds to the Δ value where $T_{\mathrm{SD}\to\mathrm{ND}}$ changes most drastically (Fig. 1), the probability for very large and very small matrix elements is noticeably larger than predicted by the PT distribution.

We now estimate the discussed effects in some realistic nuclear cases. The tunneling matrix element, V_t, connecting the SD state with the doorway state, spreads out over surrounding ND states by the residual two-body interaction, $V_{2p\text{-}2h}$, in an approximately Gaussian way [20]. The width of this Gaussian can be estimated from Fermi's golden rule, $\Gamma_\mu(U) = 2\pi\rho_{2p\text{-}2h}(U)\langle V^2_{2p\text{-}2h}\rangle$, where $\rho_{2p\text{-}2h}(U)$ is the level density of $2p$-$2h$ neighboring states to the doorway state at the excitation energy U above yrast. The tunneling matrix element at an energy distance ΔE from the doorway state can thus be estimated as

$$V^2(\Delta E, U) \approx \frac{V_t^2}{\sqrt{2\pi}\,\sigma}\exp\left[-\frac{1}{2}\left(\frac{\Delta E}{\sigma}\right)^2\right]\bar{d}(U)\,,$$

where $\sigma = \Gamma_\mu(U)/2.4$, and $\bar{d}^{-1} = \rho(U)$ is the total level density at U. Assuming V_t to be small implies that the decay proceeds through the mixing with one ND state only [9]. The tunneling probability from the SD state to the ND state is then calculated from perturbation theory as $T_{\mathrm{SD}\to\mathrm{ND}} \approx V^2(\Delta E, U)/\bar{d}^2 = V^2\rho^2$. From these relations, $T_{\mathrm{SD}\to\mathrm{ND}}$ is easily calculated vs U or, by specifying the relative energy of the ND and SD rotational bands, vs the angular momentum I.

In ^{152}Dy we calculate $U(I) = E_{\mathrm{SD}}(I) - E_{\mathrm{ND}}(I) \approx 0.003(50^2 - I^2)$ MeV, $\rho \approx 0.07\exp(5.9\sqrt{U})$ MeV^{-1}, and $\rho_{2p\text{-}2h} \approx 15U^3$ MeV^{-1} [21]. As discussed above, $\Delta E \approx 0.5$ MeV. With these assumptions we calculate $T_{\mathrm{SD}\to\mathrm{ND}}$ vs I for different values of $V^{\mathrm{rms}}_{2p\text{-}2h}$ (see Fig. 3).

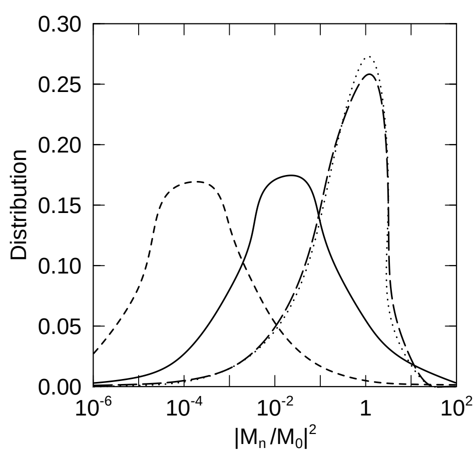

FIG. 2. Distribution of matrix elements sizes, $|M_n/M_o|^2$, calculated for $\Delta = 0.001$ (short-dashed line), 0.01 (solid line), and 0.1 (long-dashed line) (log scale). The distribution for $\Delta = 0.1$ is almost identical to that of $\Delta = 1$ (not shown), as well as the Porter-Thomas distribution (dotted line), while large deviations are seen for smaller Δ values.

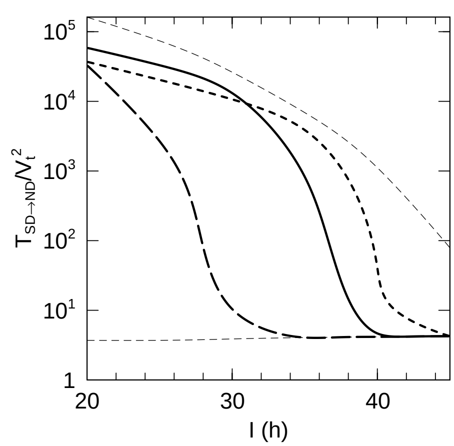

FIG. 3. Estimated tunneling probability, $T_{\mathrm{SD}\to\mathrm{ND}}$, from SD to ND states in ^{152}Dy vs angular momentum, calculated for three different assumptions on the size of the two-body interaction, $V^{\mathrm{rms}}_{2p\text{-}2h} = 5$ (long-dashed line), 10 (solid line), and 15 keV (short-dashed line). The thin dashed lines correspond to estimates of $T^{\mathrm{regular}}_{\mathrm{SD}\to\mathrm{ND}}$ (lower curve) and $T^{\mathrm{chaotic}}_{\mathrm{SD}\to\mathrm{ND}}$ [$= \rho(U(I))$; upper curve], respectively.

For $V_{2p\text{-}2h}^{\mathrm{rms}} = 10$ or 15 keV, $T_{\mathrm{SD}\rightarrow\mathrm{ND}}$ has saturated to the fully chaotic situation at the angular momenta where the decay out occurs, $I = 24$ and 26 [1]. This would imply that the decay out from the SD band is strongly enhanced by the chaotic nature of the ND states, but that the enhancement increases slowly with decreasing angular momentum, as given by $T_I/T_{I+2} \approx \rho(U(I))/\rho(U(I + 2)) \approx 1.5$ for $I = 28$. If, on the other hand, $V_{2p\text{-}2h}^{\mathrm{rms}}$ is smaller (e.g., 5 keV), the tunneling enhancement is about 10 times larger at $I = 26$ than at $I = 28$ due to the dynamical process of the onset of (long-range) chaos. The actual size of $V_{2p\text{-}2h}$ is not known, although 10–15 keV seems to be more reasonable than 5 keV. However, one should remember that what is here discussed is ensemble averages, and large fluctuations are expected to occur.

In the nucleus ^{194}Pb the excitation energy is much smaller at the decay out from the SD band. This is found to occur at $I = 6$ and 8 at $U \approx 2.7$ MeV [3]. One thus expects that the chaotic properties of the ND states at the decay out are less pronounced than, e.g., in ^{152}Dy or in ^{194}Hg, where the excitation energy may be estimated to 5.5 MeV and is measured [2] to 4.2 MeV, respectively. With similar estimates as for ^{152}Dy performed above, we find that drastic changes of the tunneling probability in ^{194}Pb indeed occur at $I = 6$ and 8, for the quite reasonable value of $V_{2p\text{-}2h}^{\mathrm{rms}} = 10$ keV.

Experimental signs of chaos assisted tunneling may be obtained by a careful statistical analysis of the strengths of the γ-ray matrix elements. A first effort to undertake such a statistical analysis has already been performed [4]. This was done with 43 observed discrete transitions from the $I = 10$ and $I = 12$ SD states in ^{194}Hg, and gave some indications for deviations from a PT distribution. With the new generation of γ-ray detectors, such as Euroball, Gamma-sphere, and, in the future, Greta it will be possible to perform much more detailed analyses. In particular, one might analyze the decay out from two consecutive spin values separately, and test if the distributions are different (and deviate from the PT distribution), as expected if the tunneling is dynamically enhanced due to the *onset* of chaos [22] (see Fig. 2).

In summary, we have shown that the tunneling process connected to the decay out of superdeformed states is strongly enhanced by chaotic properties of the normal-deformed states at the decay-out energies. The onset of chaos, as measured in terms of long-range energy correlations (e.g., by Δ_3 statistics), may give rise to an increasing tunneling enhancement as the angular momentum decreases. Careful experimental studies of γ-ray matrix elements for the first step transitions out of the SD states may reveal the chaotic nature of nuclear states at excitation energies of 3–5 MeV.

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