Oscillatory Neurocomputers with Dynamic Connectivity

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Our study of thalamo-cortical systems suggests a new architecture for a neurocomputer that consists of oscillators having different frequencies and that are connected weakly via a common medium forced by an external input. Even though such oscillators are all interconnected homogeneously, the external input imposes a dynamic connectivity. We use Kuramoto's model to illustrate the idea and to prove that such a neurocomputer has oscillatory associative properties. Then we discuss a general case. The advantage of such a neurocomputer is that it can be built using voltage controlled oscillators, optical oscillators, lasers, microelectromechanical systems, Josephson junctions, macromolecules, or oscillators of other kinds. (Provisional patent 60/108,353)   [S0031-9007(99)08813-4]

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It is believed that a new generation of computers will employ principles of the human brain. Such a computer, often referred to as a *neurocomputer,* consists of many interconnected units (referred to here as neurons) performing simple nonlinear transformations in parallel. Unlike a von Neumann computer, the neurocomputer does not execute a list of commands (a program). Its major aim is not a general-purpose computation, but pattern recognition via associative memory. There are many neural network models that can be used as a theoretical basis for a neurocomputer; see [1] for comprehensive review. The most promising are oscillatory neural networks because they take into account rhythmic behavior of the brain [2–7].

Whether oscillatory or not, a neurocomputer consisting of n neurons needs n^2 programmable connections (see Fig. 2), so building such a computer is a major challenge when n is large. A possible way to cope with this problem was suggested by our study of thalamo-cortical systems [8–10]. We treat the cortex as being a network of weakly connected autonomous oscillators forced by the thalamic input; see Fig. 1. We find that whether or not such oscillators communicate depends on their frequencies: If two oscillators have nearly equal frequencies, then they do communicate in the sense that the phase (timing) of one of them is sensitive to the phase of the other.

In contrast, when they have essentially different frequencies, their phases uncouple. Thus, an oscillator can interact selectively with other oscillators having appropriate frequencies. In analogy with radio, we refer to such interactions as being frequency modulated (FM).

We also find that a weak thalamic input having appropriate frequencies in its power spectrum can dynamically connect any two oscillators, even those that have different frequencies and would be unlinked otherwise.

This suggests the following design of a neurocomputer: It consists of oscillators having different frequencies and connected homogeneously and weakly to a common medium (see Fig. 2). Selective communication between such oscillators can be created by the weak forcing. We illustrate some major points using Kuramoto's model in the section below, and we discuss a general case in the discussion section and in [11].

Illustration: Kuramoto's Model.—For the sake of illustration, consider Kuramoto's phase model [12]

$$\dot{\vartheta}_i = \Omega_i + \varepsilon a(t) \sum_{j=1}^{n} \sin(\vartheta_j - \vartheta_i), \qquad (1)$$

where $\vartheta_i \in \mathbb{S}^1$ is the phase of the ith oscillator, $a(t)$ is the external input, and $\varepsilon \ll 1$ is the strength of connections. We can rewrite (1) in the form

$$\dot{\vartheta}_i = \Omega_i + \varepsilon a(t) \operatorname{Im} e^{-i\vartheta_i} M(t),$$

where

$$M(t) = \sum_{j=1}^{n} e^{i\vartheta_j} \qquad (2)$$

is a complex number denoting the "mean field activity" of the network. We see that each oscillator receives identical input; that is, the oscillators are connected homogeneously.

For the sake of simplicity we require that all differences $\Omega_i - \Omega_j$ be different when $i \neq j$. We drop this requirement in section (H).
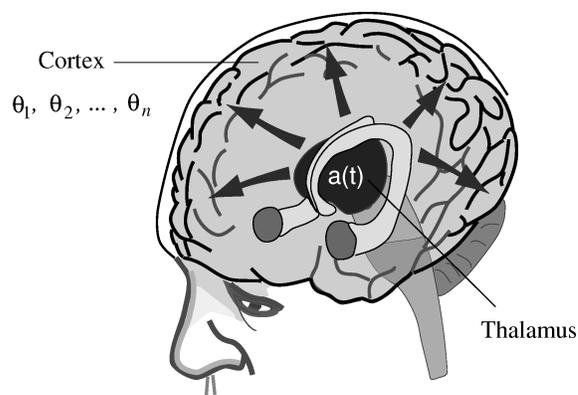
FIG. 1. We treat the cortex as being a network of weakly connected autonomous oscillators $\vartheta_1, \ldots, \vartheta_n$ forced by the thalamic input $a(t)$. (Modified from [10].)
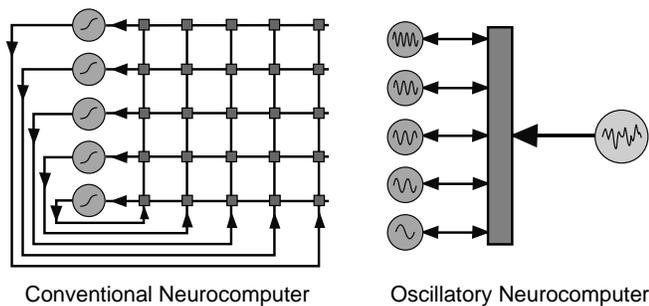
FIG. 2. A conventional neurocomputer having n neurons (circles) would have n^2 connections (squares). An oscillatory neurocomputer with dynamic connectivity imposed by the external input (large circle) needs only n connections: between each neuron (circle) and a common medium (rectangle).

(A) *Averaging:* Let $\vartheta_i(t) = \Omega_i t + \varphi_i$, then

$$\dot{\varphi}_i = \varepsilon a(t) \sum_{j=1}^{n} \sin(\{\Omega_j - \Omega_i\}t + \varphi_j - \varphi_i). \quad (3)$$

If ε is sufficiently small [see (8) in section (H)], then one can average this system to obtain

$$\dot{\varphi}_i = \varepsilon H_i(\varphi_1, \ldots, \varphi_n) + o(\varepsilon), \quad (4)$$

where

$$H_i = \lim_{T \to \infty} \frac{1}{T} \int_0^T a(t)$$

$$\times \sum_{j=1}^{n} \sin(\{\Omega_j - \Omega_i\}t + \varphi_j - \varphi_i) \, dt$$

is the average of the right-hand side of (3).

(B) *Constant external input:* First, consider Kuramoto's model without oscillatory external input; that is, when $a(t) = a_0$ is a constant. Then each $H_i = 0$ because the oscillators have distinct frequencies Ω_i, $i = 1, \ldots, n$. This implies that the phase variables in (4) do not interact, at least on the long time scale of order $1/\varepsilon$. Therefore, neither do the Kuramoto oscillators (1).

There are many examples of biological and engineering systems that use distinct frequencies in order to avoid cross-interference between oscillators. For example, electric fish, such as *Eigenmannia* [13], have jamming avoidance response (JAR) that allows them to choose different frequencies to conavigate through water. Radio stations use different frequencies to transmit through the same airspace.

(C) *Quasiperiodic external input:* Now suppose we are given a matrix of connections $C = (c_{ij})$. Let

$$a(t) = a_0 + \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} \cos(\{\Omega_j - \Omega_i\}t) \quad (5)$$

be a time dependent external input, which is a quasiperiodic function of t. Since all differences $\Omega_j - \Omega_i$ are

different for all i and j, we find that

$$H_i = \sum_{j=1}^{n} \frac{c_{ij} + c_{ji}}{2} \sin(\varphi_j - \varphi_i).$$

If we denote $s_{ij} = (c_{ij} + c_{ji})/2$, use the slow time $\tau = \varepsilon t$, and disregard the small-order term $o(\varepsilon)$, then we can rewrite system (3) in the form

$$\varphi_i' = \sum_{j=1}^{n} s_{ij} \sin(\varphi_j - \varphi_i), \quad (6)$$

where $' = d/d\tau$. We see that the external input of the form (5) can dynamically connect any two oscillators provided that the corresponding c_{ij} is not zero.

(D) *Chaotic external input:* In general, the external input $a(t)$ can be chaotic or noisy. It can dynamically connect the ith and the jth oscillators if its Fourier decomposition has a nonzero entry corresponding to the frequency $\omega = \Omega_j - \Omega_i$, since the average, H_i, would depend on the phase difference $\varphi_j - \varphi_i$ in this case.

(E) *Oscillatory associative memory:* Since the connection matrix $S = (s_{ij})$ is symmetric, the phase model (6) is a gradient system. Indeed, it can be written in the form

$$\varphi_i' = -\frac{\partial U}{\partial \varphi_i},$$

where

$$U(\varphi_1, \ldots, \varphi_n) = -\frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} s_{ij} \cos(\varphi_j - \varphi_i)$$

is a potential function [8]. The vector of phase deviations $\varphi = (\varphi_1, \ldots, \varphi_n) \in \mathbb{T}^n$ always converges to an equilibrium on the n-torus $\mathbb{T}^n$. System (6) has multiple attractors and Hopfield-Grossberg-like associative properties [4,14]; see Fig. 3. Therefore, Kuramoto's model (1) with external forcing has oscillatory associative memory. We stress that this property is not built into the Kuramoto's network, but it is dynamically induced by the external input of the form (5). Numerical simulations show that the storage capacity of such an oscillatory network is approximately the same as that of the Hopfield model [4].

(F) *Hebbian learning rule:* Suppose we are given a set of m key vectors to be memorized

$$\xi^k = (\xi_1^k, \xi_2^k, \ldots, \xi_n^k), \qquad \xi_i^k = \pm 1, \qquad k = 1, \ldots, m,$$

where $\xi_i^k = \xi_j^k$ means that the ith and jth oscillators are in-phase ($\varphi_i = \varphi_j$), and $\xi_i^k = -\xi_j^k$ means they are antiphase ($\varphi_i = \varphi_j + \pi$). First, notice that the problem of mirror images does not exist in oscillatory neural networks, since both ξ^k and $-\xi^k$ result in the same phase relations. A Hebbian learning rule of the form

$$s_{ij} = \frac{1}{n} \sum_{k=1}^{m} \xi_i^k \xi_j^k \quad (7)$$

is the simplest one among many possible learning algorithms. To get (6) it suffices to apply the external input of the form (5) with $c_{ij} = s_{ij}$ for all i and j.

Patterns to be Memorized   Pattern to be Recognized

Initialization of the Network

t=0   t=2.5   t=5   t=7.5   t=10

Pattern Recognition

t=0   t=2.5   t=5   t=7.5   t=10
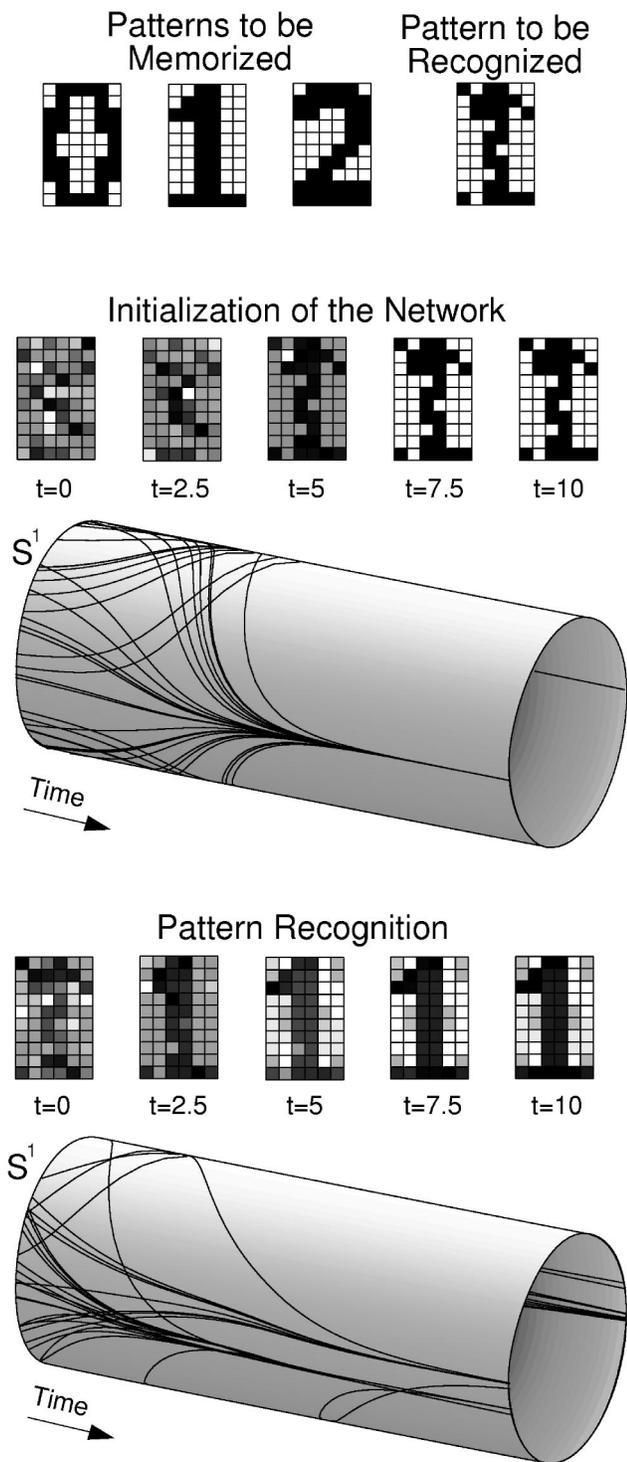
FIG. 3. Simulation of the phase deviation model (6) with Hebbian learning rule (7). Parameters: $n = 60$, $t \in [0, 10]$. The network is initialized according to the algorithm described in section G.

(G) *Initializing the network:* To use Kuramoto's model to implement the standard Hopfield-Grossberg paradigm, as we do in Fig. 3, we need a way to present an input image as an initial condition $\vartheta(0)$, and to read the output

from the network. While the latter task poses no difficulty and can be accomplished using Fourier analysis of the "mean field" $M(t)$ given by (2), the former task requires some ingenuity since we do not have a direct access to the oscillators.

Suppose we are given a vector $\xi^0 \in \mathbb{R}^n$ to be recognized. Let us apply the external input $a(t)$ with $c_{ij} = \xi_i^0 \xi_j^0$ for a certain period of time. This results in the phase deviation system of the form

$$\varphi_i' = \sum_{j=1}^{n} \xi_i^0 \xi_j^0 \sin(\varphi_j - \varphi_i).$$

It is easy to check that if $\xi_i^0 \xi_j^0 = 1$, then $\varphi_i(t) - \varphi_j(t) \to 0$, and if $\xi_i^0 \xi_j^0 = -1$, then $\varphi_i(t) - \varphi_j(t) \to \pi$ for all i and j. Thus, the network activity converges to the equilibrium having phase relations defined by the vector ξ^0; see the middle part of Fig. 3. When we restore the original external input $a(t)$, which induces the desired dynamic connectivity, the recognition starts from the input image ξ^0. (We added noise to the image ξ^0 at the bottom of Fig. 3 to enhance the effect of convergence to an attractor during recognition.)

(H) *Network size:* One of the major disadvantages of (1) with the forcing of the form (5) is the requirement that all $\Omega_j - \Omega_i$ be distinct when $i \neq j$. Since we use averaging, the parameter ε must be much smaller than the difference between any pair $|\Omega_j - \Omega_i|$ and $|\Omega_{j'} - \Omega_{i'}|$ for $j \neq j'$ or $i \neq i'$. This imposes a severe restriction on the size of the network

$$\varepsilon n^2 \ll \Omega_{\max} - \Omega_{\min}, \tag{8}$$

where $\Omega_{\max}$ ($\Omega_{\min}$) is the maximal (minimal) frequency in the network.

To avoid restriction (8) we may use separate external inputs for each oscillator; that is, we consider the canonical model of the form

$$\dot{\vartheta}_i = \Omega_i + \varepsilon a_i(t) \sum_{j=1}^{n} \sin(\vartheta_j - \vartheta_i),$$

where Ω_i are some distinct frequencies, for example, $\Omega_i = \Omega_0 + i$. Condition (8) has a simple form, $\varepsilon \ll 1$, in this case. If in addition

$$a_i(t) = a_0 + \sum_{j=1}^{n} c_{ij} \cos(\{\Omega_j - \Omega_i\}t),$$

then the oscillator phases are governed by (6).

Discussion.— The major goal of this paper is to present a theoretical framework for a new architecture for oscillatory neurocomputers. We do not intend to devise new neurocomputational paradigms, but to devise hardware that can implement existing paradigms. Since we used the canonical model approach [8], our analysis is applicable to a broad family of oscillatory networks regardless of the nature of each oscillator. For example, dynamic

connectivity and oscillatory associative memory have been proven [8,10] to exist in a general dynamical system

$$\dot{x}_i = f_i(x_i) + \varepsilon g_i[x_1, \ldots, x_n, a(t), \varepsilon],$$

but the form of the external input $a(t)$ that imposes desired dynamic connectivity is more sophisticated than (5). Here each vector $x_i \in \mathbb{R}^m$ describes activity of the ith oscillator, and f_i and g_i are unknown functions that describe dynamics of the ith oscillator and interconnections in the network. Thus, the oscillatory neurocomputer can be built using such diverse mechanisms as VCOs (voltage controlled oscillators [15]), Josephson junctions, optical oscillators, lasers, or MEMS (microelectromechanical systems [16]).

The proposed design overcomes two challenging problems:

(i) An oscillatory neurocomputer consisting of n neurons does not need n^2 programmable connections to emulate an associative neural network having a fully connected synaptic matrix (such as required by Hopfield's network).

(ii) The oscillators can be put together in an arbitrary, random fashion, yet the network can establish a desired configuration via dynamic connectivity.

We stress that our design does not eliminate all problems, but it replaces them by another, hopefully simpler, problem of generating an appropriate external input $a(t)$. We also omitted such important issues as how dynamic connectivity degrades through unwanted interactions [term $o(\varepsilon)$ in (4)], delays, and noise. In [11] we present more detailed analysis and discuss further this approach to oscillatory neurocomputers.

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