Central Peak Position in Magnetization Loops of High-T_c Superconductors

D. V. Shantsev,[1,2] M. R. Koblischka,[1,*] Y. M. Galperin,[1,2] T. H. Johansen,[1] L. Půst,[3,†] and M. Jirsa[3]
[1]*Department of Physics, University of Oslo, P.O. Box 1048 Blindern, 0316 Oslo, Norway*
[2]*A. F. Ioffe Physico-Technical Institute, Polytechnicheskaya 26, St. Petersburg 194021, Russia*
[3]*ASCR, Institute of Physics, Na Slovance 2, CZ-18040 Praha, Czech Republic*
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Exact analytical results are obtained for the magnetization of a superconducting thin strip with a general behavior $J_c(B)$ of the critical current density. We show that within the critical-state model the magnetization as a function of applied field, B_a, has an extremum located exactly at $B_a = 0$. This result is in excellent agreement with presented experimental data for a $YBa_2Cu_3O_{7-\delta}$ thin film. After introducing granularity by patterning the film, the central peak becomes shifted to positive fields, $B_{cp} > 0$, on the descending field branch of the loop. Our results show that a positive B_{cp} is a definite signature of granularity in superconductors. [S0031-9007(99)08821-3]

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The investigation of magnetic hysteresis loops (MHLs) is a widely used tool to characterize superconducting samples, and, in particular, to estimate the critical current density and its dependence on magnetic field. An ever-present feature of the MHLs is a peak in the magnetization located at an applied field B_{cp} near zero. The central peak is formed due to a field dependence of the critical current density, $J_c(B)$, monotonously decreasing at small fields. When the sample is a long cylindrical body placed in a parallel applied field, the peak position can be calculated analytically within the critical-state model for several $J_c(B)$ dependences [1]. One always finds on the descending field branch that $B_{cp} < 0$, and similarly $B_{cp} > 0$ on the ascending branch of large-field loops. In simple terms, the shift is a consequence of the local flux density, B, lagging behind the applied field.

Also experimentally there are abundant observations of negative B_{cp} on the descending field branch. However, in some cases one finds B_{cp} very close to zero or even shifted to the positive side so that the peak occurs before the remanent state is reached [2–6]. The explanations for this behavior are controversial. On one hand, a decreasing sample thickness is known from experiment to shift the peak toward $B_{cp} = 0$ [7,8]. This is also in agreement with recent numerical results obtained for the critical-state model with B-dependent J_c [8,9]. However, the numerical calculations have not predicted sufficiently large shifts to bring the peak to the anomalous side of the loop.

A different explanation of the shift in B_{cp} was suggested for granular materials [10], where a demagnetization effect of the grains can be important. The magnetization of a granular superconductor is determined by both intragrain and intergrain currents. The latter represent large-size current loops, and can give a major contribution to the magnetic moment. These intergrain currents are essentially determined by the magnetic field just at the grain boundaries. Those local fields are, in turn, strongly influenced by the intragrain currents, and can vary *ahead* of the local B expected without granularity. As a result, the central peak is encountered earlier in the MHL. Models allowing quantitative MHL calculations with account of the grain-induced demagnetization effect [4,11] successfully reproduced the peak at $B_{cp} > 0$.

Although the significant influence of both sample shape and sample granularity on the MHL is generally recognized, the interplay between them has never been addressed. In particular, it is not known if a positive B_{cp} is accessible for thin enough samples, or if $B_{cp} > 0$ on the descending field branch is a definite signature of sample granularity. The present Letter aims to answer this question. We show first analytically that for *any* $J_c(B)$ the central peak is located exactly at $B_{cp} = 0$ for a thin uniform strip, a result we also confirm by experiment. The key role of granularity is then demonstrated by measurements on an artificially granular thin film, where we find $B_{cp} > 0$.

Consider a long thin superconducting strip with edges located at $x = \pm w$, the y axis pointing along the strip, and the z axis normal to the strip plane (see Fig. 1). The magnetic field, B_a, is applied along the z axis, so screening currents flow in the y direction. Throughout the paper, B is the z component of magnetic induction in the strip plane. The sheet current is defined as $J(x) = \int j(x,z)\,dz$, where $j(x,z)$ is the current density and the integration is performed over the strip thickness, $d \ll w$.

From the Biot-Savart law for the strip geometry, the flux density is given by [12]

$$B(x) - B_a = -\frac{\mu_0}{2\pi}\int_{-w}^{w}\frac{J(u)\,du}{x-u}. \qquad (1)$$

Assuming that the strip is in a fully penetrated state, i.e., the current density is everywhere equal to the critical one, $J(x) = \mathrm{sgn}(x)J_c[B(x)]$. This state can be reached after applying a very large field, and then reducing it to some much smaller value. The field distribution then satisfies the following integral equation:

$$B(x) - B_a = -\frac{\mu_0}{\pi}\int_0^w\frac{J_c[B(u)]}{x^2-u^2}\,u\,du. \qquad (2)$$

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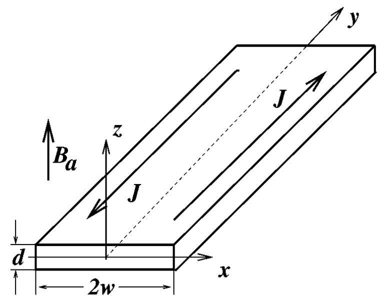

FIG. 1. Superconducting strip in an applied magnetic field.

In the remanent state, $B_a = 0$, the flux density profile $B(x)$ has an interesting symmetry. This is seen by changing the integration variable in Eq. (2) from u to $v = \sqrt{w^2 - u^2}$. We then obtain

$$B(x) - B_a = \frac{\mu_0}{\pi} \int_0^w \frac{J_c[B(\sqrt{w^2 - v^2})]}{w^2 - x^2 - v^2} v\, dv\,. \quad (3)$$

Substituting x by $\sqrt{w^2 - x^2}$ into Eq. (2), one obtains a similar equation for $B(\sqrt{w^2 - x^2})$:

$$B(\sqrt{w^2 - x^2}) - B_a = -\frac{\mu_0}{\pi} \int_0^w \frac{J_c[B(u)]}{w^2 - x^2 - u^2} u\, du\,. \quad (4)$$

By comparing Eqs. (3) and (4) at $B_a = 0$, we conclude that

$$B(x) = -B(\sqrt{w^2 - x^2}) \quad (5)$$

is generally valid if J_c depends only on the absolute value of the magnetic induction, $J_c(|B|)$. This symmetry is immediately evident for the case $J_c =$ const, i.e., the Bean model, when Eq. (2) reduces to

$$B(x) = B_a + B_c \ln \frac{\sqrt{w^2 - x^2}}{x}, \qquad B_c = \frac{\mu_0 J_c}{\pi}. \quad (6)$$

In a similar way, one can prove also another symmetry relation valid at $B_a = 0$, namely,

$$\mathcal{D}(x) = \mathcal{D}(\sqrt{w^2 - x^2}), \quad (7)$$

where $\mathcal{D}(x) = \partial B(x)/\partial B_a$.

Consider now the magnetic moment per unit length of the strip, $M = 2\int_0^w J(x)x\, dx$. Differentiating M with respect to B_a and taking into account that $B(x)$ changes sign at $x = w^* = w/\sqrt{2}$, one has after splitting the integral into two parts

$$\frac{\partial M}{\partial B_a} = 2\left(\int_0^{w^*} - \int_{w^*}^{w}\right) \frac{\partial J_c[|B(x)|]}{\partial |B|} \mathcal{D}(x) x\, dx\,. \quad (8)$$

Then, replacing in the second integral x by $\sqrt{w^2 - x^2}$ and using Eq. (5), we come to

$$\frac{\partial M}{\partial B_a} = 0 \quad \text{at } B_a = 0\,. \quad (9)$$

Consequently, on a major MHL the magnetic moment has an extremum in the remanent state for *any* $J_c(B)$ dependence. This is illustrated in Fig. 2, showing the central part of MHLs obtained by an iterative numerical solution of Eq. (2) for three $J_c(B)$ dependences. The $M(B_a)$ extremum depends on the type of $J_c(B)$ dependence. It is a maximum for a decreasing $J_c(B)$, and a minimum if $J_c(B)$ has a pronounced second peak, the so-called fishtail behavior.

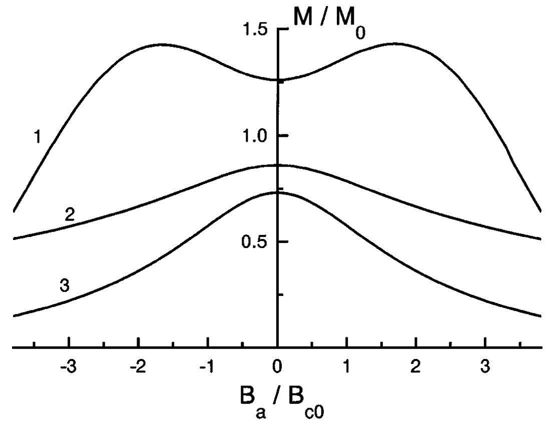

FIG. 2. Central part of descending branches of the MHL for different $J_c(B)$ dependences, $J_c/J_{c0} = \mathcal{F}(|B|/B_{c0})$. (1) $\mathcal{F}(b) = (1 + 2.5b^2)e^{-b^2/4}$; (2) $\mathcal{F}(b) = (1 + 0.25b)^{-1}$; (3) $\mathcal{F}(b) = e^{-b/2}$. Here, $B_{c0} = \mu_0 J_{c0}/\pi$, $M_0 = w^2 J_{c0}$.

Two $YBa_2Cu_3O_{7-\delta}$ (YBCO) epitaxial thin films were chosen for a comparative study: (A) a uniform film of regular shape and (B) a film with artificial granularity. Sample (A) was prepared by laser ablation on a MgO substrate [13]. The thickness was 200 nm, and the sample was patterned by chemical etching into a rectangular shape of dimensions 0.66×1.4 mm^2. The homogeneity of the sample was tested by magneto-optical imaging, where it showed complete absence of any visible defects [14].

Sample (B) was laser ablated to a 150 nm thickness on a $LaAlO_3$ substrate. The film was patterned by means of electron beam lithography into a hexagonal close-packed lattice of disks with $2r = 50$ μm diameter. The disks are touching each other at the circumferences in order to enable the flow of interdisk currents. The width of the contact region is $\epsilon = 3.5$ μm (see Fig. 3). The superconducting disks can be considered as grains and the connections between them as intergrain junctions. The overall size of the sample was 4×4 mm^2, comprising $\approx$8000 disks. The transition temperature after the patterning process is 83 K.

The magnetization measurements were performed using a vibrating sample magnetometer (VSM) PAR Model 155 with computerized control and data processing. The magnetic field is generated by a conventional electromagnet with $B_{a,\max}$ $\pm$ 2 T. The field was applied parallel to the c axis, which is perpendicular to the film plane. The magnetization loops were always measured after zero-field cooling of the sample. The maximum applied field for all measurements was significantly higher than the field giving a fully penetrated state, as verified by the

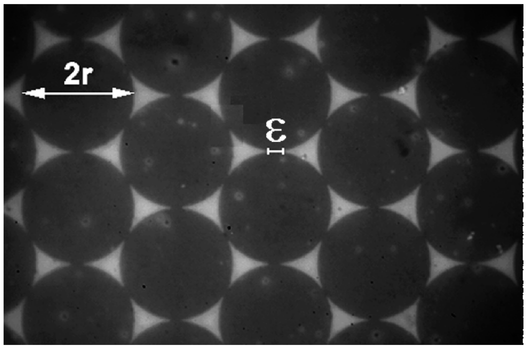

FIG. 3. Polarization image of the sample (B) with artificial granularity. The black disks are superconducting and have a diameter of $2r = 50\ \mu$m. The width of the contact region between the disks is $\epsilon = 3.5\ \mu$m.

magneto-optical method. To focus on our main issue, only the data measured on the descending field branch of the MHLs near $B_a = 0$ are presented.

Figure 4 shows the central part of the MHLs for the uniform film (A). The measurements were carried out over a wide range of temperatures in order to probe different $J_c(B)$ dependences. There is evidently here a pronounced central peak at all temperatures. Within the experimental resolution, the position of the peak is located at zero applied field. The observation that the peak remains at $B_a = 0$ over the entire temperature range is in full agreement with our general analytical result.

Note that the result was derived for an infinite strip fully penetrated by the magnetic field. In a thin sample, in contrast to long cylinders, the flux fronts move in response to a changing applied field at an exponential rate [12]. Therefore, a fully remagnetized state is reached quickly after reversing the direction of a field sweep, and our experimental conditions are consistent with the assumptions made in the theory. The only exception is the finite length of the strip. However, our results show that it is not a crucial factor for the peak position.

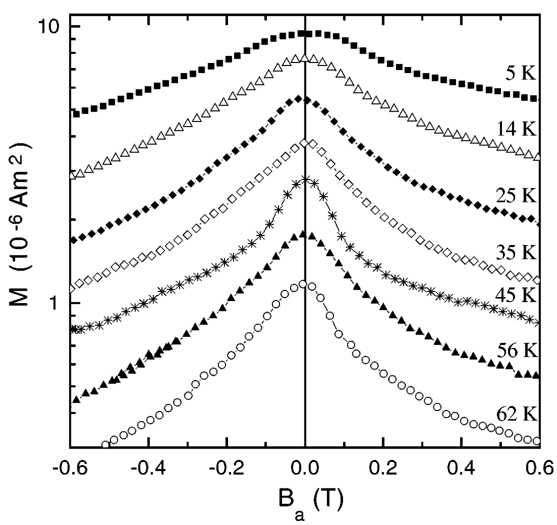

FIG. 4. Central part of descending branches of the MHL for a thin YBCO strip for different temperatures. The peak in magnetization is located always at zero applied field.

The MHLs for the sample (B) with artificial granularity are shown in Fig. 5. The central peak displays here different behavior as compared to the uniform film (A). For all temperatures $B_{\mathrm{cp}} > 0$ on the descending field branch. The shift of the central peak becomes more and more pronounced with decreasing temperature reaching $B_{\mathrm{cp}} = +40$ mT at $T = 5$ K.

In this sample, the "grains" and their interconnections are of the same material, and, hence, have equal critical current density. Because of the patterning, the spatially averaged intergrain current density J_c^{tr} is less than the current density, J_c^{gr}, of the superconducting material itself. For the hexagonal pattern (see Fig. 3) one has $J_c^{tr} = \eta J_c^{gr}$, where η depends on the direction of J_c^{tr}, within the limits $\epsilon/2r < \eta < \epsilon\sqrt{3}r$. Furthermore, the magnetic moment M^{gr} due to intragrain currents and moment M^{tr} due to intergrain currents are related as

$$M^{gr}/M^{tr} = (J_c^{gr}/J_c^{tr})(r/R) \approx 2r^2/\epsilon R\,,$$

where R is the overall size of the sample. In the present case, this ratio is $1/6$, showing that the intergrain contribution to the total magnetic moment is dominant. A considerable grain-induced demagnetization effect is therefore expected for this sample. Its granularity, being the only essential difference from the uniform film, is the only possible origin of the observed shift to $B_{\mathrm{cp}} > 0$.

The superconducting Bi-2223 tapes are known to have a granular microstructure and the two contributions to the magnetic moment are of comparable magnitude [4]. The grain demagnetization effect has been suggested as an explanation for the observed shift of the peak to $B_{\mathrm{cp}} > 0$

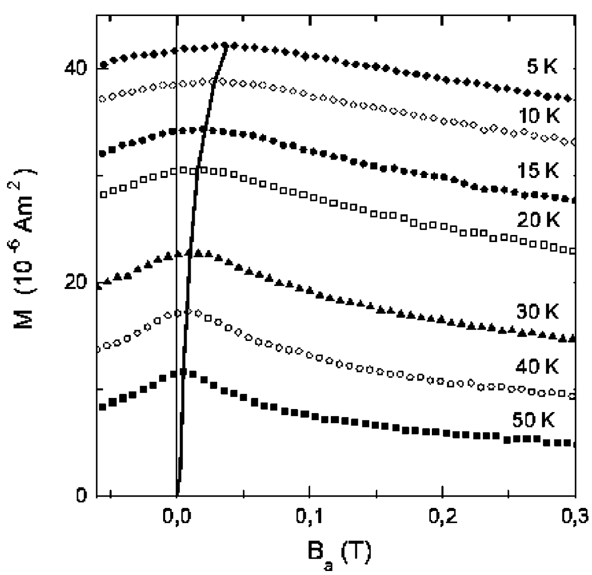

FIG. 5. Central part of the descending branch of MHLs for the sample (B) with artificial granularity. The solid line indicates position of the magnetization peak which is located at positive fields at all of the temperatures.

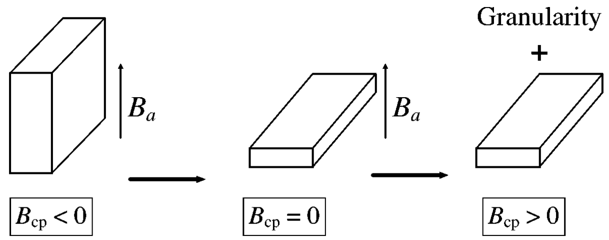

FIG. 6. A schematic drawing illustrating the main conclusion of the paper. Long samples in parallel magnetic field show the peak on the descending field branch of MHL at a negative field. As the sample height decreases the peak is shifted towards zero. It is exact zero in the limiting case of a uniform thin strip. A granular thin strip has the peak at positive fields.

on the descending MHL branch in these materials [3–6]. Our present results give strong support to this physical picture.

In conclusion, we arrive at the following general scenario concerning the central peak position (see Fig. 6). Long samples in parallel magnetic field show in their MHLs a central peak at a negative applied field, i.e., after passing through the remanent state on the descending field branch. As the sample thickness decreases the peak position is shifted towards zero. It is located exactly at $B_{\text{cp}} = 0$, in the limiting case of a uniform strip of infinitesimal thickness. Granularity always leads to a shift of B_{cp} in the positive direction on the descending field branch. Thus, for a granular thin strip the peak is located at a positive field, i.e., before the remanent state is reached. The origin of this effect is that granularity induces demagnetization fields which strongly modify the intergranular currents via their B dependence.

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*Present address: Superconducting Research Laboratory, ISTEC, Division 3, 1-16-25, Shibaura, Minato-ku, Tokyo 105, Japan.
†Present address: Seagate Technologies Inc., Bloomington, MN 55435.

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